Example 5.1 Rigidity of a simple square raft

1 Description of problem

For comparison with complex foundation rigidity problems, no solution is yet available. Therefore, for judgment on the analysis of *El Gendy* (1998) to find the system rigidity of foundation, consider the simple example of raft foundation shown in Figure 5.2. The raft has dimensions of 12 [m] × 12 [m] and carries four symmetrical and equal loads, each of P = 9000 [kN]. The raft rests on a homogenous soil layer of thickness 20 [m]. *Young*'s modulus of the raft and soil materials is $E_b = 2 \times 10^7$ [kN/m^2] and E_s = 10000 [kN/m^2], respectively. *Poisson*'s ratio of the raft material is ν_b = 0.15.

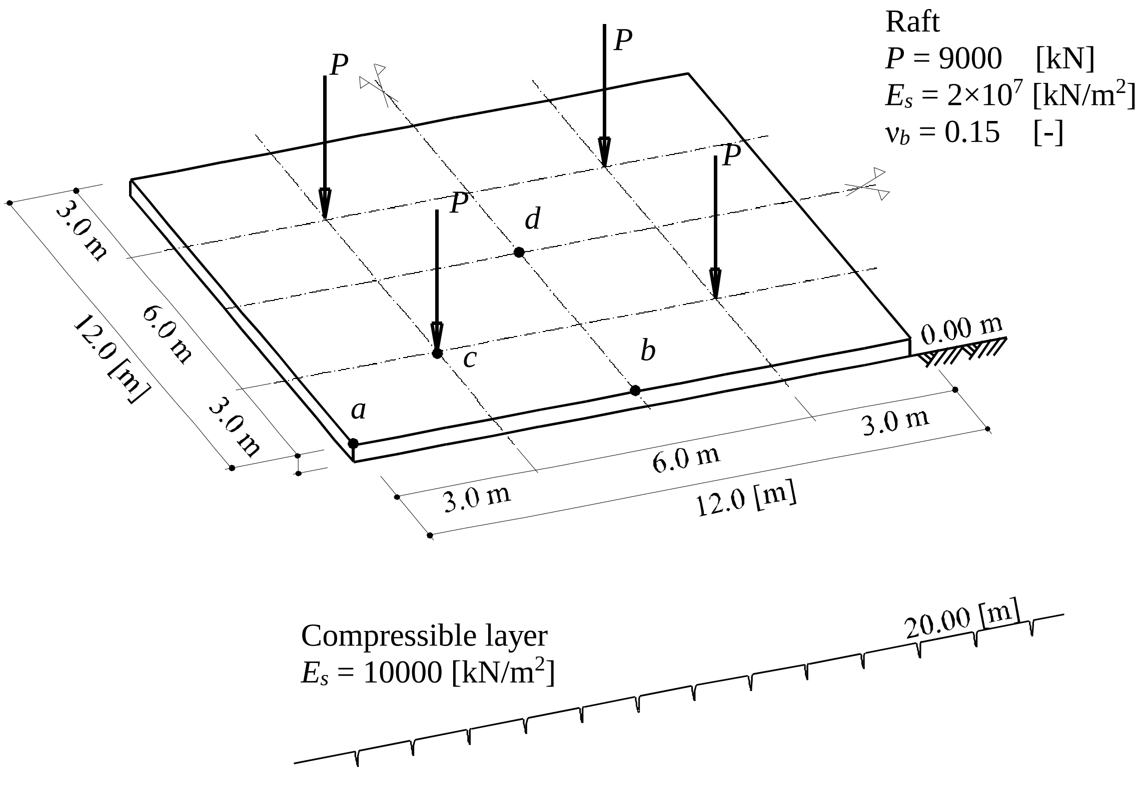

Figure 5.2 Raft dimensions, loads and subsoil

*Deninge*r (1964) studied the same example using the finite difference method by dividing the raft into 6 × 6 elements. Each element has dimensions of 2 [m] × 2 [m]. He examined the raft thickness for several values of 0.4, 0.5, 0.6, 0.8 and 2 [m].

The moment at any point on the raft foundation depends on the system rigidity of the foundation, external load values and load distributions. So, the moment m_x at the position of the concentrated load, independent of rigidity formulae, can be used to find the rigid thickness of the raft in this example. Here, the raft is considered rigid at a thickness gives moment m_x more than 90 [%] of the maximum moment that can occur at that point.

The raft in this example is considered rigid for thickness more than 0.85 [m] according to *Deninger*'s analysis. An application for Equation 5.2 to this example gives a system rigidity k_{st} = 0.71. So, the raft is considered very stiff according to system rigidity of *Graßhoff* (1987).

2 Analysis and discussion

Series of computations using the finite element method for several values of raft thickness are carried out. The moments and the settlements at some selected points are plotted against the raft thickness to describe the foundation rigidity.

First, the raft is subdivided first into 24 × 24 square elements. Each element has dimensions of 0.5 [m] × 0.5 [m]. Then, it is subdivided into 12 × 12 square elements. Each element has dimensions of 1 [m] × 1 [m] as shown in Figure 5.3. Taking advantage of the symmetry in shape, soil and load geometry about *x*- and *y*-axes, the analysis is carried out only for a quarter of the raft.

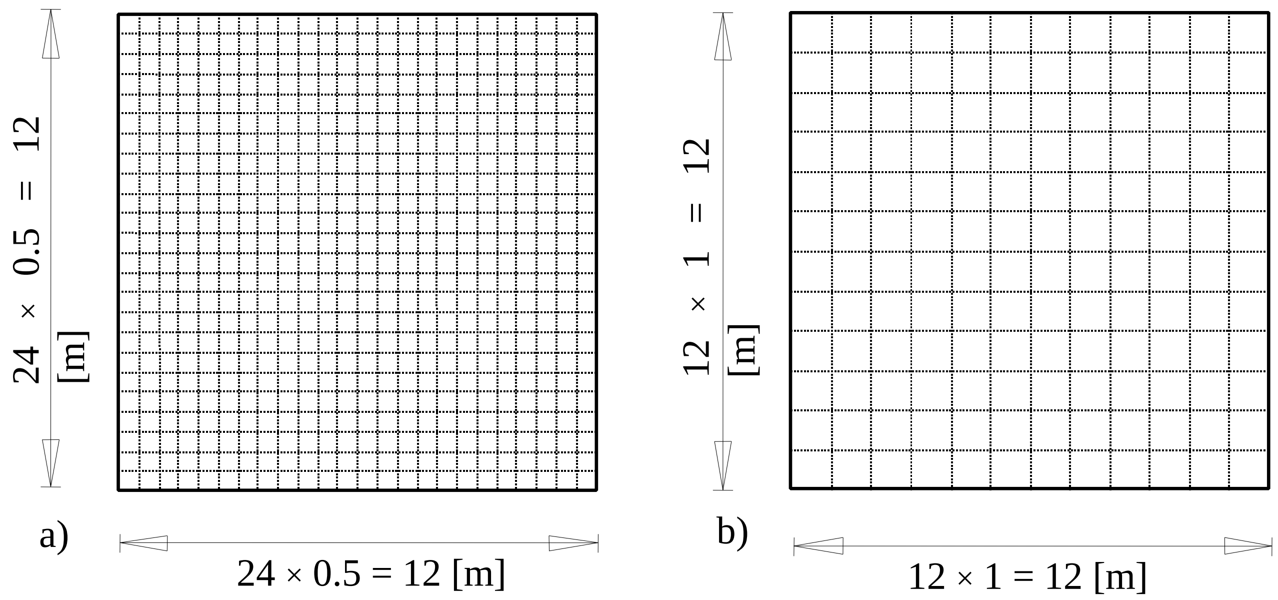

Figure 5.3 Finite element meshes of the raft

To show the convergence of the solution by finite element method and to verify the rigid thickness of the raft, the settlement *s*, at four characteristic points *a*, *b*, *c* and *d* on the raft and the rigid body translation w_o when the raft is perfectly rigid, are plotted against the raft thickness in Figure 5.4 and 5.5. In which

Point *a* Corner point of the raft
Point *b* Middle point of the raft edge
Point *c* Point under the load position
Point *d* Center point of the raft

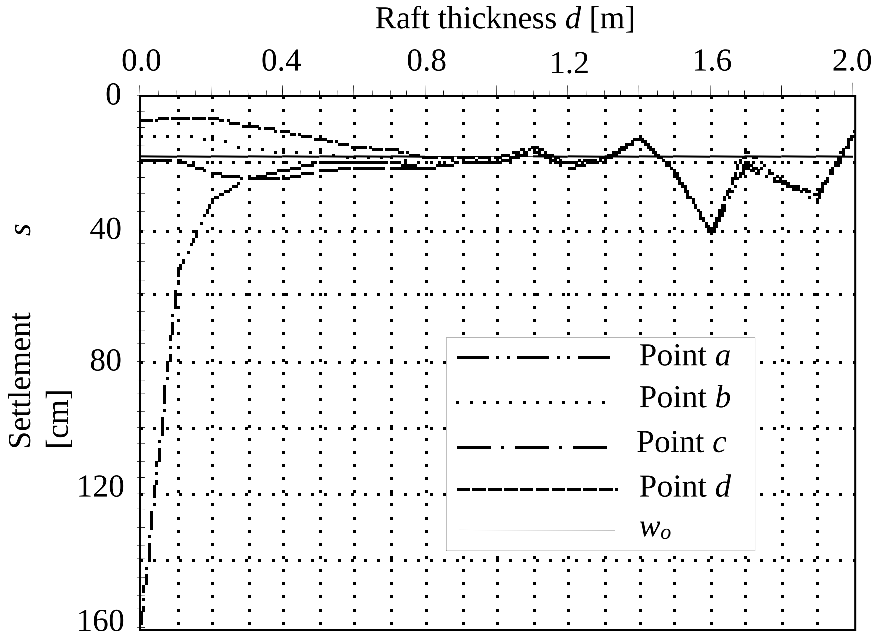

Figure 5.4 Settlement at the four characteristic points using a mesh of 24 × 24 elements

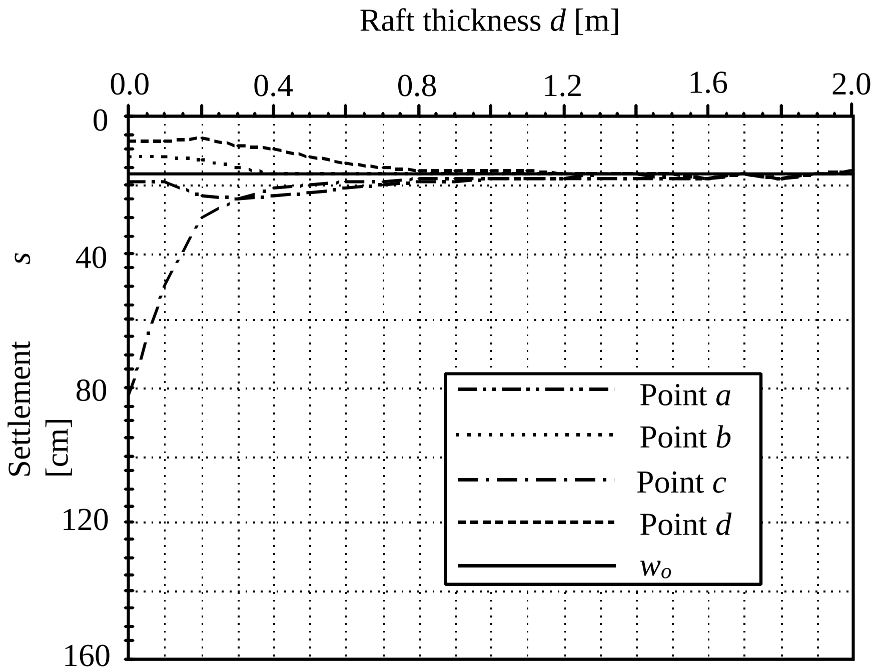

Figure 5.5 Settlement at the four characteristic points using a mesh of 12 × 12 elements

Figure 5.6 shows the moment m_x at point *c* under the concentrated load position using finite element mesh of 24 × 24 elements and 12 × 12 [m] elements, respectively.

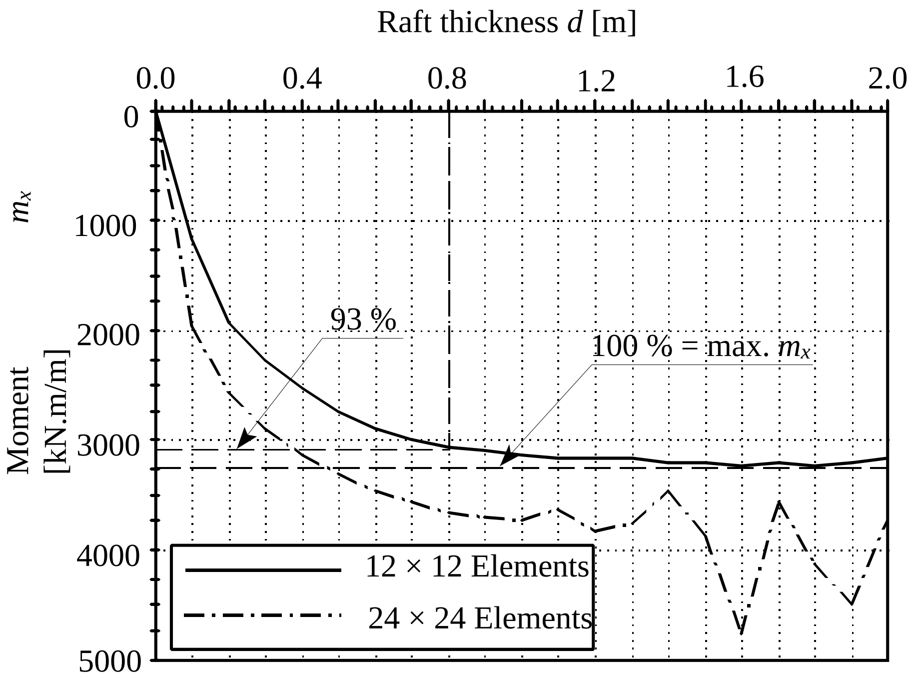

Figure 5.6 Moment m_x at characteristic point *c*

Figure 5.4 indicates that, if a fine mesh of 24 × 24 elements is used, the solution for the raft thickness far 0.8 [m] will become divergence. In which no stability in the overall matrix occurs. As a result, if the foundation is rigid enough, the raft rotations will approach to zero and the raft will settle the same value of a displacement w_o. Therefore, the number of equations becomes greater than the number of unknowns. Another problem may be found that the relation between the plate element thickness and element size is limited by application of the finite element method using plate-bending elements.

Figure 5.5 shows that using a mesh of 12 × 12 elements gives good results. A comparison between Figure 5.4 and 5.5 indicates that, although the solution by using a fine mesh of 12 × 12 elements is divergence, the rigid thickness of the raft can be determined because the limit of rigid translation is known from the rigid solution.

Figure 5.6 shows that *Deninger*'s analysis cannot be used in case of using a fine mesh of 24 × 24 elements, to find the rigid thickness of the raft where the position of maximum moment at point *c* is not clear in the figure. Further, for a raft with complex load geometry or types, using this analysis is not practical, which represents the rigidity of the foundation only at the selected point.

Figure 5.7 shows the parameter k_r for the four characteristic points *a*, *b*, *c* and *d* of the raft. Figure 5.8 shows the parameter k_r for the same characteristic points if a uniform load of 250 [kN/m^2] replaces the external concentrated loads on the raft, which is equal to the average contact pressure, using also mesh of 12 × 12 elements.

The raft may be considered as rigid at thickness gives k_r more than 90 [%] for all characteristic points.

From Figure 5.7, the raft is considered rigid for thickness more than 0.80 [m]. The moment for this thickness m_x is 93 [%] from maximum moment at point *c*. This thickness also is different from that of *Deninger* (1964) by 5.6 [%] and makes the raft very stiff according to *Graßhoff* (1987).

According to this analysis, Figure 5.8 shows that the raft is considered rigid for thickness more than 0.7 [m] when it carries a uniform load of 250 [kN/m^2]. This means that the type of loading has influence on the raft rigidity. Although the solution in this example is reported for a square raft, the approach can be also considered applicable for general problems.

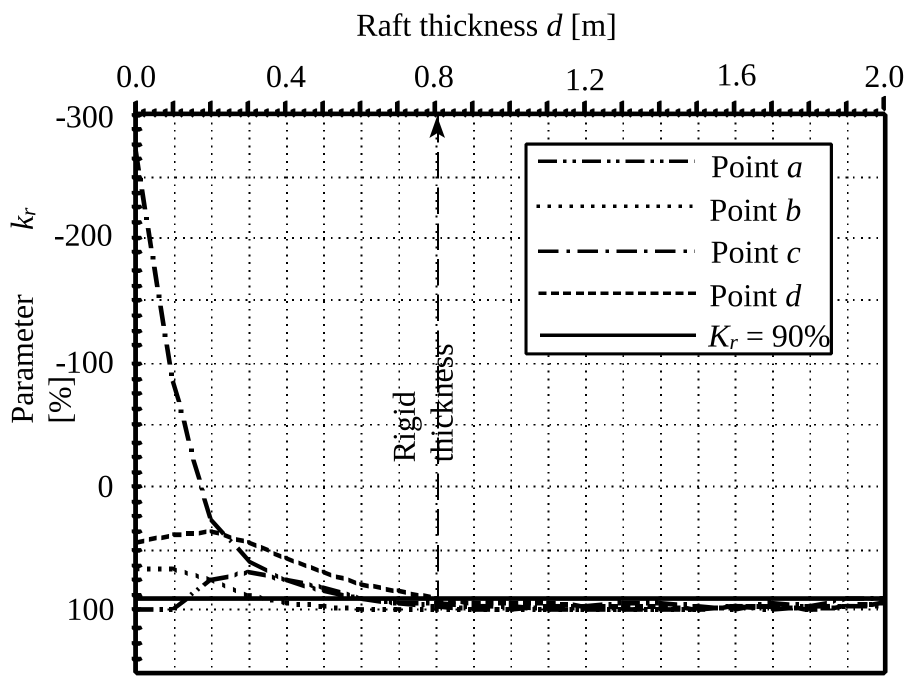

Figure 5.7 Parameter k_r for the characteristic points (raft carries concentrated loads)

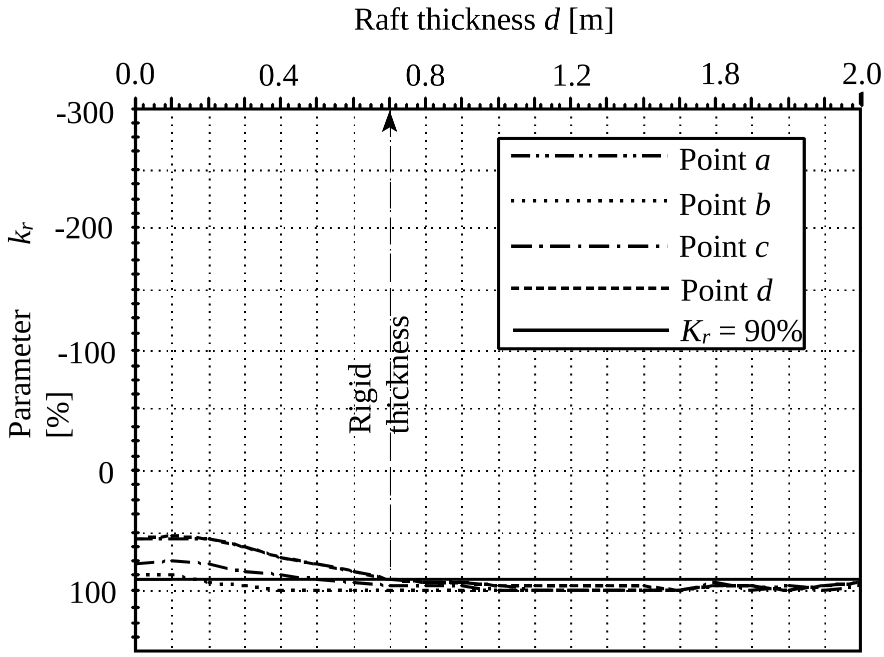

Figure 5.8 Parameter k_r for the characteristic points (raft carries a uniform load)