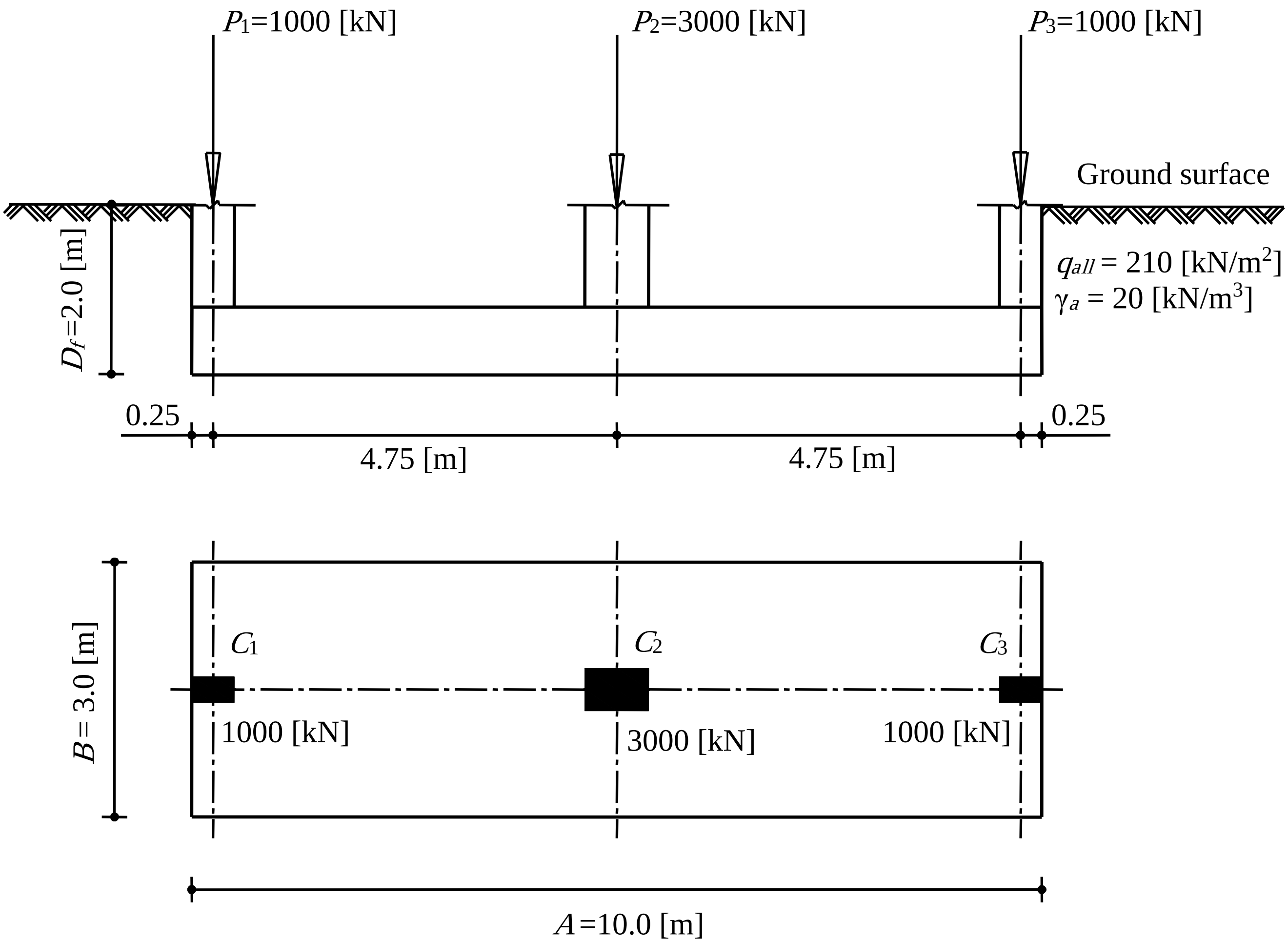
Beam Foundations after *Kany* and *El Gendy* by *GEO Tools* (Analysis and Design)

Part III: Numerical Examples



Mahmoud El Gendy
Mohamed El Gendy

Canada
Tele.:+1(587) 332-3323
geotec@geotecsoftware.com
www.geotecsoftware.com

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Content

Preface

Various problems in Geotechnical Engineering can be investigated by the program *GEO Tools.* The original version of *GEO Tools* in *ELPLA* package was developed by *M. Kany*, *M. El Gendy*, and *A. El Gendy* to determine the contact pressure, settlements, and moments and shear forces of beam foundations. After the death of *Kany*, (*M. & A.*) *El Gendy* further developed the program to meet the needs of the practice.

This book describes the essential methods used in *GEO Tools* to analyze beam foundations with verification examples. *GEO Tools* is a simple user interface program and needs little information to define a problem.

There are three soil models with five methods available in *GEO Tools* for analyzing beam foundations. Many test examples are presented to verify and illustrate the soil models and methods for analyzing beam foundations available in *GEO Tools*.

10 Analysis and Design of Beam Foundations after *Kany* and *El Gendy*

10.1 Introduction

Different calculation methods are known in the literature for the calculation of shallow foundations. The early one is that assumes a uniform contact pressure distribution under shallow foundations. This assumption is too far from the reality, *Winkler* (1867) and *Zimmermann* (1930) developed the Modulus of subgrade method. In the method, the subsoil is simulated by isolated springs. The settlement of the spring is only dependent on the loading at the same point on the subsoil surface at the spring location. This also applies to possible refinements with springs of different stiffness.

However, *Boussinesq* (1885) had already recognized that when the subsoil is loaded at one point, the subsoil also settles outside the load point. Therefore, it does not behave like a spring. Because of this finding, *Ohde* (1942) developed a calculation method for the first time, with which shallow foundations can be analyzed, taking into account the soil structure interaction. This method, which is called Modulus of compressibility method, was later further developed by different authors (*Graßhoff* (1966-1978), *Kany* (1974), *Graßhoff*/*Kany* (1992)). The program *GEO Tools* is based on the Modulus of compressibility method after *Kany* (1974) and the Modulus of subgrade reaction method after *Kany*/ *El Gendy* (1995). However, some refinements are included, some of which are new and have not yet been dealt with in detail in the literature. It is therefore necessary to explain the calculation method in more detail than usual in order to be able to check the results and compare them with other results.

10.2 Numerical Examples

10.2.1 Calculation methods

It is possible by *GEO Tools* to use the same data for analyzing beam foundations by five different conventional and refined calculation methods. The interaction between the beam and the subsoil can be analyzed by:

1 Linear contact pressure method
2 Modulus of subgrade reaction method after *Kany/ El Gendy* (1995)
3 Modulus of compressibility method after *Kany* (1974)
4 Rigid beam foundation
5 Flexible beam foundation

It is also possible to consider irregular soil layers and the thickness of the base beam that varies in each element. Furthermore, the influence of temperature changes and additional settlement on the beam foundation can be taken into account. With the help of *GEO Tools*, an analysis of different examples was carried out to verify and test the methods and the program for analyzing the problems of beam on elastic foundation.

In the analysis, the beam foundation is divided into equal elements according to Figure 10.1. Using the available five calculation methods, the settlement and the contact pressure can be determined in each element.

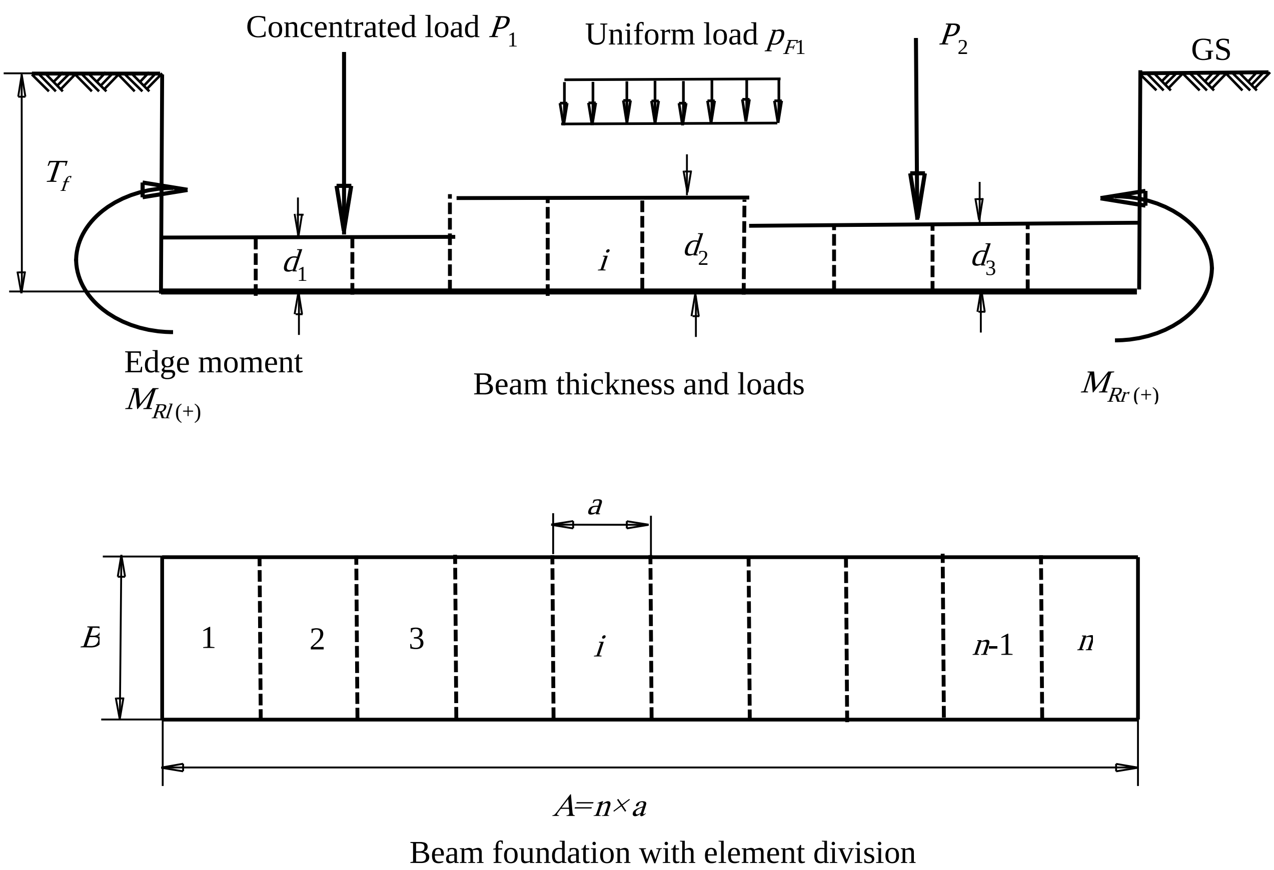


Figure 10.1 Loads, beam thickness und beam foundation with element division

10.2.2 Material and section for concrete design

Concrete design of the beam foundations sections are carried out according to EC 2, DIN 1045, ACI and ECP. The material and section for concrete design are supposed to have the following parameters:

10.2.2.1 Material properties

Concrete grade according to ECP	C 250			
Steel grade according to ECP	S 36/52			
Concrete cube strength	$f_{cu} = 250$	[kg/ cm^2]	= 25	[MN/ m^2]
Concrete cylinder strength	$f'_c = 0.8\ f_{cu}$	[-]	= 20	[MN/ m^2]
Compressive stress of concrete	$f_c = 95$	[kg/ cm^2]	= 9.5	[MN/ m^2]
Tensile stress of steel	$f_s = 2000$	[kg/ cm^2]	= 200	[MN/ m^2]
Reinforcement yield strength	$f_y = 3600$	[kg/ cm^2]	= 360	[MN/ m^2]
Young's modulus of concrete	$E_b = 3\times10^7$	[kN/ m^2]	= 30000	[MN/ m^2]
Poisson's ratio of concrete	$\nu_b = 0.15$	[-]		
Unit weight of concrete	$\gamma_b = 25$	[kN/ m^3]		

In some examples, unit weight of concrete is chosen $\gamma_b = 0.0$ to neglect the own weight of the beam foundation.

10.2.2.2 Section properties

Width of the section to be designed	$b = 1.0$	[m]
Section thickness	t	[m]
Concrete cover + 1/2 bar diameter	$c = 5$	[cm]
Effective depth of the section	$d = t - c = 0.45$	[m]
Steel bar diameter	$\Phi = 16$ to 22	[mm]

10.2.3 Example 1: Design of a combined footing for two unequal columns

10.2.3.1 Description of the problem

It is required to find the contact pressure distribution, settlements, moment and shear force diagrams for a rectangular combined footing for an exterior column *C*1 of a load of 1050 [kN] and an interior column *C*2 of a load of 1800 [kN] as shown in Figure 10.2. *C*1 is 0.50×0.50 [m^2] and reinforced by 6Φ22, while column *C*2 is 0.60×0.50 [m^2] and reinforced by 10Φ22. The allowable soil pressure q_{all} = 185 [kN/m^2] at a depth of D_f = 1.90 [m] and average unit weight of the soil and concrete is γ_a = 20 [kN/m^3]. Modulus of elasticity of the footing concrete is $E_b = 2\times10^7$, while the Modulus of subgrade reaction of the soil is k_s = 25000 [kN/m^3].

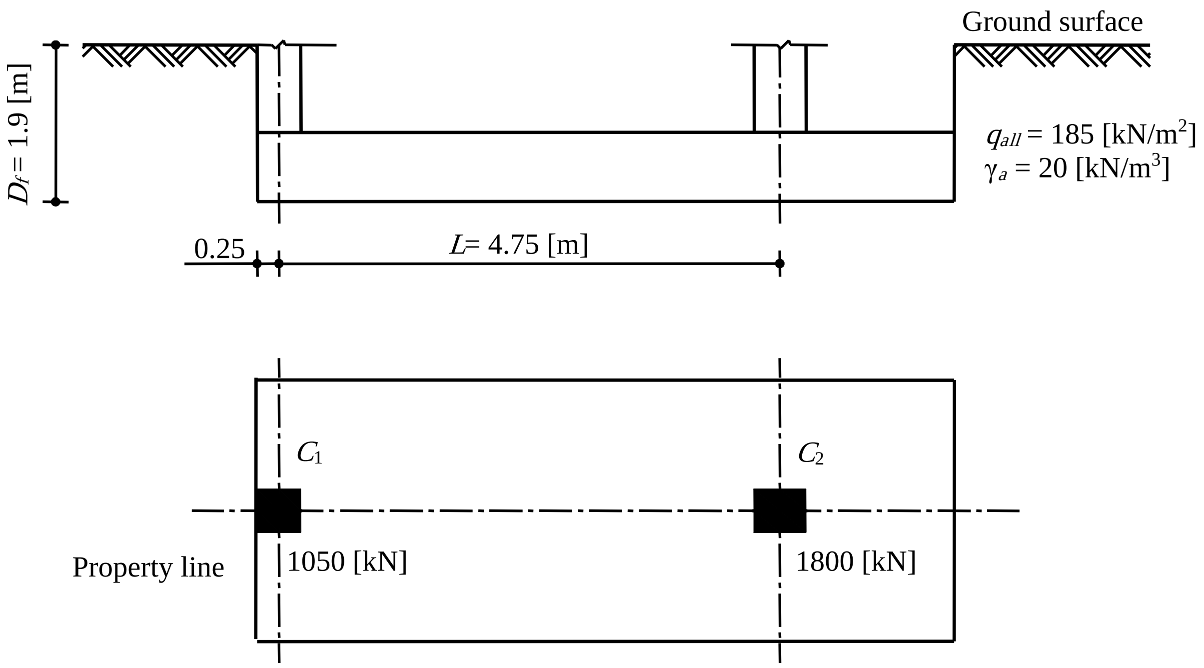


Figure 10.2 Combined rectangular footing

10.2.3.1.1 Determining point of application of the resultant force

Resultant force *R* equals:

$$R = P_1 + P_2 = 1050 + 1800 = 2850 \text{ [kN]}$$

Take the moment about C_1 to get the distance *S*:

$$P_2 A = RS$$

$$1800\times4.75 = 2850\times S$$

$$S = 3.0 \text{ [m]}$$

10.2.3.1.2 Determining footing sides A and B

$$A = 2\,X_p = 2\,(0.25 + S) = 2\,(0.25 + 3.0) = 6.5 \text{ [m]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 185 - 20\times1.9 = 147 \text{ [kN/m}^2\text{]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R}{q_{net}} = \frac{2850}{147} = 19.39 \text{ [m}^2\text{]}$$

Take A_f = 6.5×3.0 = 19.50 [m^2] rectangular combined footing, as shown in Figure 10.3.

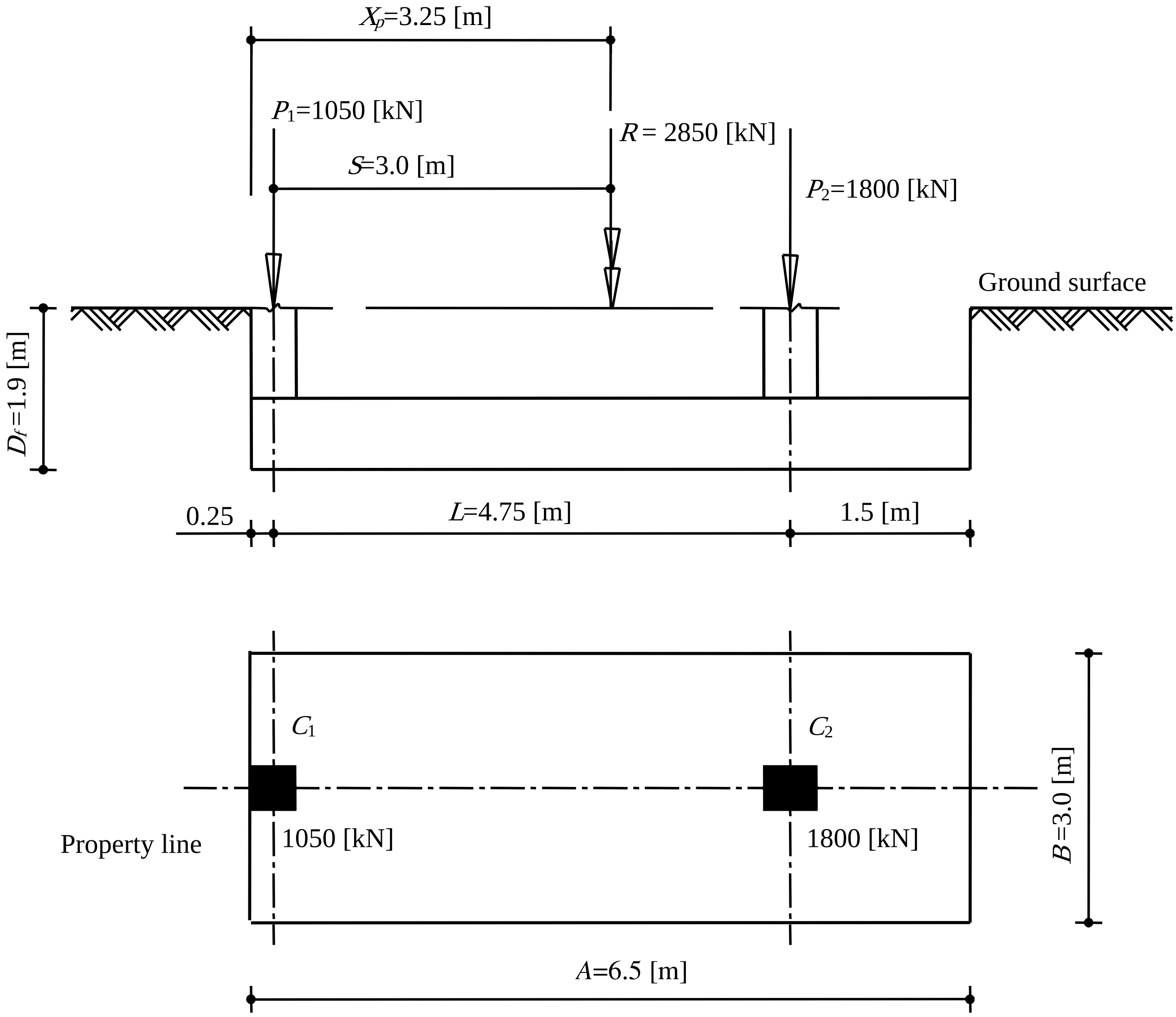


Figure 10.3 The combined rectangular footing with property line and column loads

10.2.3.1.3 Analysis of the combined footing

The footing can be regarded as a beam on elastic foundation. In the analysis, the beam is divided into four equal elements, each 1.625 [m] long (**Error! Reference source not found.**). The self-weight of t he footing is neglected. According to *Kany/ El Gendy* (1995), the analysis of combined footing as a beam on elastic foundation is carried out in the following steps:

10.2.3.1.4 Calculation of u_i, v_i **and** *w_i*:

For a constant beam moment of inertia $I_i = I$

$$u_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i-1}}\right)$$

$$u_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

$$v_i = \frac{1}{4}\left(\frac{I_i}{I_{i-1}} + 14 + \frac{I_i}{I_{i+1}}\right)$$

$$v_i = \frac{1}{4}\left(\frac{I}{I} + 14 + \frac{I}{I}\right) = \frac{1}{4} \times 16 = 4$$

$$w_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i+1}}\right)$$

$$w_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

Moment of inertia $I_i = I$:

$$I_i = I = \frac{B {d_i}^3}{12} = \frac{3 \times 0.55^3}{12} = 0.0416 \text{ [m}^4\text{]}$$

$$\alpha = \frac{a^4 B}{E_p I} = \frac{1.625^4 \times 3}{20000000 \times 0.0416} = 2.514 \times 10^{-5} \text{ [m}^3\text{/kN]}$$

10.2.3.1.5 Determining external moments $M_i^{(l)}$

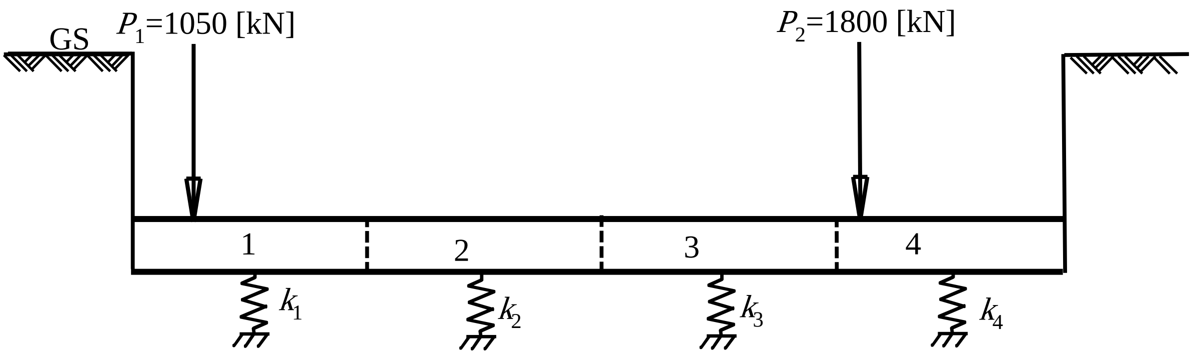
The external moments $M_i^{(l)}$ at points 1, 2, 3 and 4 are:

$$M_1^{(l)} = 0 \text{ [kN.m]}$$

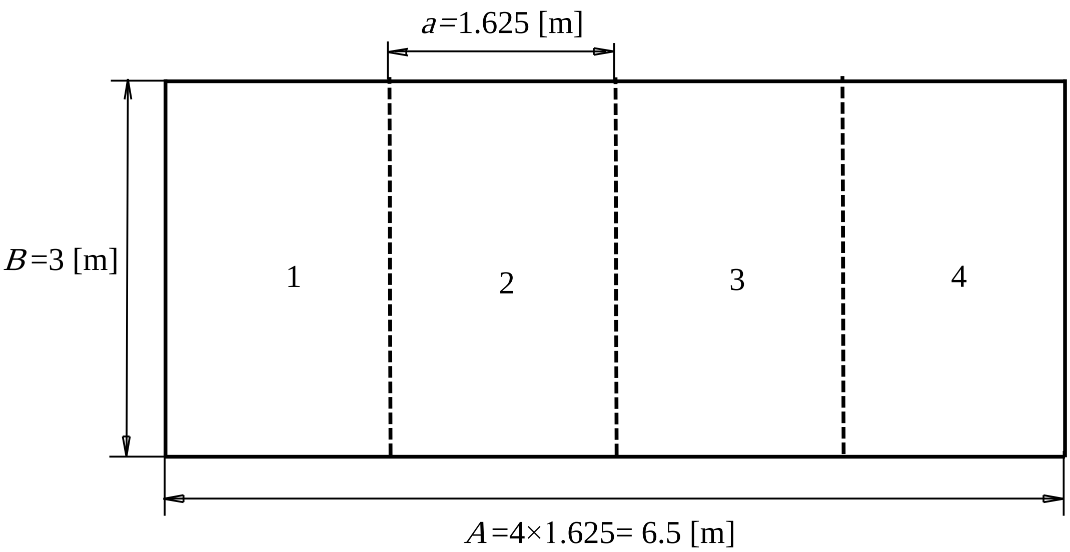
$$M_2^{(l)} = 1050 \times (1.5 \times 1.625 - 0.25) = 2296.875 \text{ [kN.m]}$$

$$M_3^{(l)} = 1050 \times (2.5 \times 1.625 - 0.25) = 4003.125 \text{ [kN.m]}$$

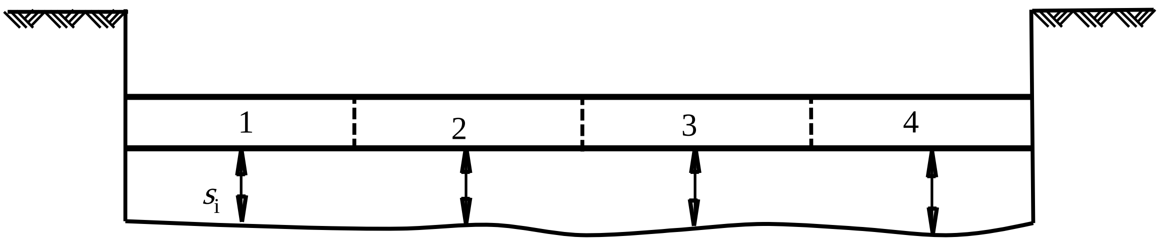
$$M_4^{(l)} = 1050 \times (3.5 \times 1.625 - 0.25) + 1800(1.5 - \frac{1.625}{2}) = 6946.875 \text{ [kN.m]}$$



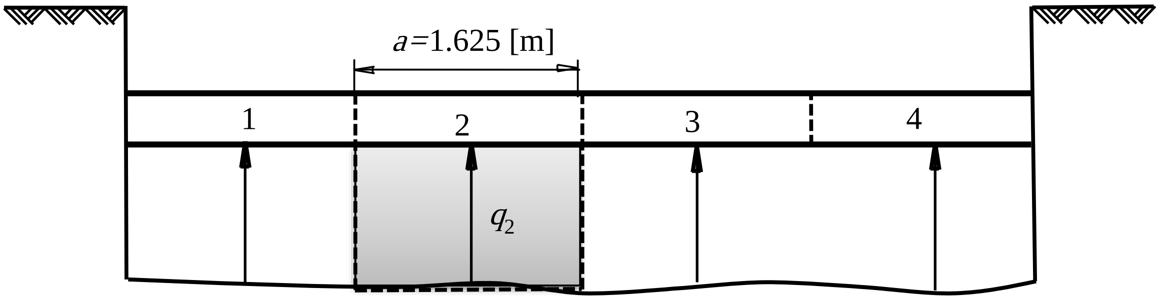
Beam foundation with loads and springs



Plan of beam foundation with elements



Settlement



Contact pressure

Figure 10.4 Combined footing as a beam on elastic foundation

10.2.3.1.6 Determining the right hand side R_i

The right hand side R_i of the contact pressure equation is:

$$R_i = \left(u_i M_{i-1}^{(l)} + v_i M_i^{(l)} + w_i M_{i+1}^{(l)}\right) \frac{a^2}{6EI_i}$$

$$R_i = \left(1 \times M_{i-1}^{(l)} + 4 \times M_i^{(l)} + 1 \times M_{i+1}^{(l)}\right) \frac{1.625^2}{6 \times 20000000 \times 0.0416}$$

$$R_i = 5.29 \times 10^{-7}\left(M_{i-1}^{(l)} + 4M_i^{(l)} + M_{i+1}^{(l)}\right)$$

Apply the above equation at points 2 and 3:

At point 2:

$$R_2 = 5.29 \times 10^{-7}\left(M_1^{(l)} + 4M_2^{(l)} + M_3^{(l)}\right)$$

$$R_2 = 5.29 \times 10^{-7}(0 + 4 \times 2296.875 + 4003.125) = 0.00698$$

At point 3:

$$R_3 = 5.29 \times 10^{-7}\left(M_2^{(l)} + 4M_3^{(l)} + M_4^{(l)}\right)$$

$$R_3 = 5.29 \times 10^{-7}(2296.875 + 4 \times 4003.125 + 6946.875\,) = 0.0134$$

10.2.3.1.7 Determining contact pressures

The contact pressure equation is:

$$\left(\frac{1}{k_{i+1}}\right) q_{i+1} - \left(\frac{2}{k_i} - \frac{\alpha_i}{6} w_i\right) q_i + \left(\frac{1}{k_{i-1}} + \frac{\alpha_i}{6}(v_i + 2w_i)\right) q_{i-1}$$

$$+ \frac{\alpha_i}{6}\left(\sum_{j=1}^{i-2}[(i-j-1)u_i + (i-j)v_i + (i-j+1)w_i]\, q_j\right) = R_i$$

$$\left(\frac{1}{25000}\right) q_{i+1} - \left(\frac{2}{25000} - \frac{2.514 \times 10^{-5}}{6}\right) q_i + \left(\frac{1}{25000} + \frac{2.514 \times 10^{-5}}{6}(4+2)\right) q_{i-1}$$

$$+ \frac{2.514 \times 10^{-5}}{6}\left(\sum_{j=1}^{i-2}[(i-j-1) + 4(i-j) + (i-j+1)]\, q_j\right) = R_i$$

or

$$q_{i+1} - 1.895\, q_i + 1.629\, q_{i-1} + 0.629\left(\sum_{j=1}^{i-2}(i-j)\, q_j\right) = 25000R_i$$

Apply the above equation at points 2 and 3:

At point 2:

$$q_3 - 1.895\, q_2 + 1.629\, q_1 = 25000 \times 0.00698$$

$$q_3 - 1.895\, q_2 + 1.629\, q_1 = 174.5$$

At point 3:

$$q_4 - 1.895\, q_3 + 1.629\, q_2 + 0.629 \times 2\, q_1 = 25000 \times 0.0134$$

$$q_4 - 1.895\, q_3 + 1.629\, q_2 + 1.258\, q_1 = 334.014$$

There are four unknown q_1, q_2, q_3, and q_4, so a farther two equations are required. This can be obtained by considering the overall equilibrium of vertical forces and moments.

Overall equilibrium of vertical forces

$$\sum V = 0$$

$$aB(\, q_1 + \; q_2 + \; q_3 + \; q_4) = P_1 + P_2$$

$$1.625 \times 3(\, q_1 + \; q_2 + \; q_3 + \; q_4) = 1050 + 1800$$

$$q_1 + \; q_2 + \; q_3 + \; q_4 = 584.615$$

Overall equilibrium of vertical forces

$$\sum M = 0$$

$$aB(\, 3.5aq_1 + \; 2.5aq_2 + \; 1.5aq_3 + \; 0.5aq_4) = P_1(A - 0.25) + P_2 1.5$$

$$1.625 \times 3\big(\, q_1(3.5 \times 1.625) + \; q_2(2.5 \times 1.625) + \; q_3(1.5 \times 1.625) + \; q_4(0.5 \times 1.625)\big) = 1050(6.5 - 0.25) + 1800 \times 1.5$$

$$3.5q_1 + \; 2.5q_2 + \; 1.5q_3 + \; 0.5q_4 = 1169.231$$

Contact pressure equations in matrix form:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 3.5 & 2.5 & 1.5 & 0.5 \\ 1.629 & -1.895 & 1 & 0 \\ 1.258 & 1.629 & -1.895 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 584.615 \\ 1169.231 \\ 174.5 \\ 334.014 \end{bmatrix}$$

Solving the above system of linear equations to obtain the contact pressures q_1, q_2, q_3, and q_4.

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 169.188 \\ 120.775 \\ 127.808 \\ 166.844 \end{bmatrix} \text{ [kN/m}^2\text{]}$$

10.2.3.1.8 Determining settlements s_i

The settlement s_i can be given by:

$$s_i = \frac{q_i}{k_i} = \frac{q_i}{25000} \text{ [m]}$$

s_1 = 0.68 [cm]

s_2 = 0.48 [cm]

s_3 = 0.51 [cm]

s_4 = 0.67 [cm]

The contact pressure distribution, settlement, moment and shear force diagrams for the raft are shown in **Error! Reference source not found.** to **Error! Reference source not found.**. Once the internal forces are obtained at various sections, the design of the raft can be completed in the normal manner.

10.2.3.1.9 Computer calculation

The input data and results of *GEO Tools* are presented on the next pages. By comparison, one can see an agreement with the hand calculation.

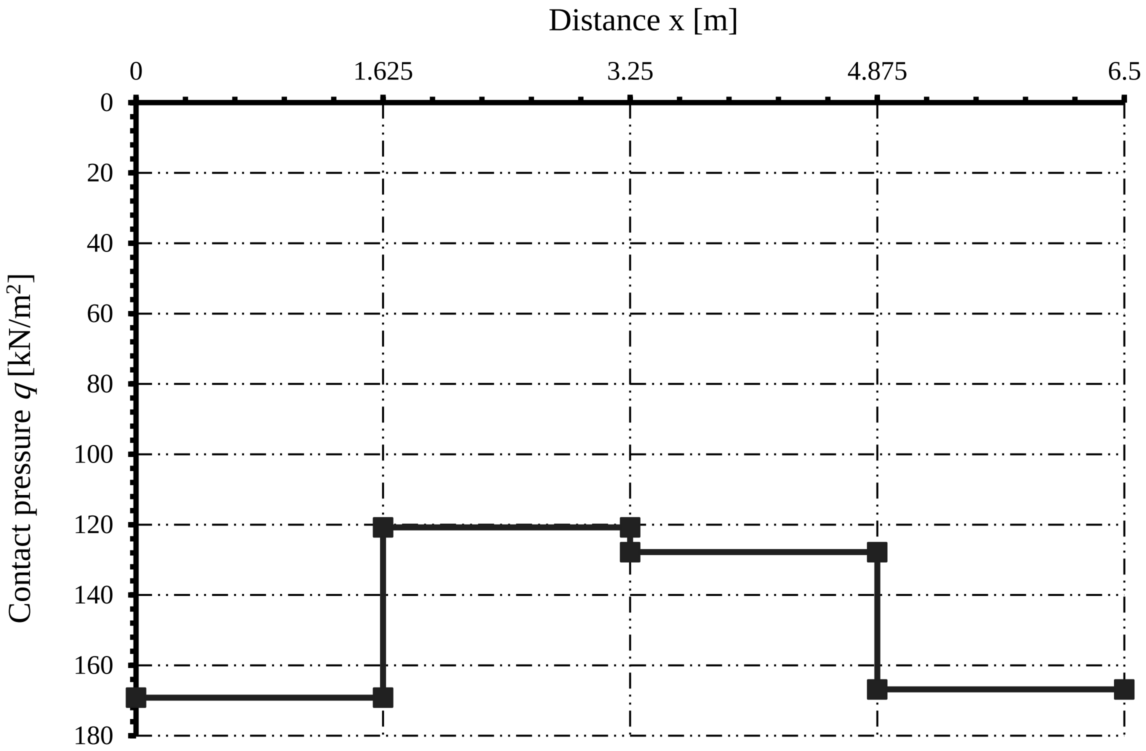


Figure 10.5 Contact pressures

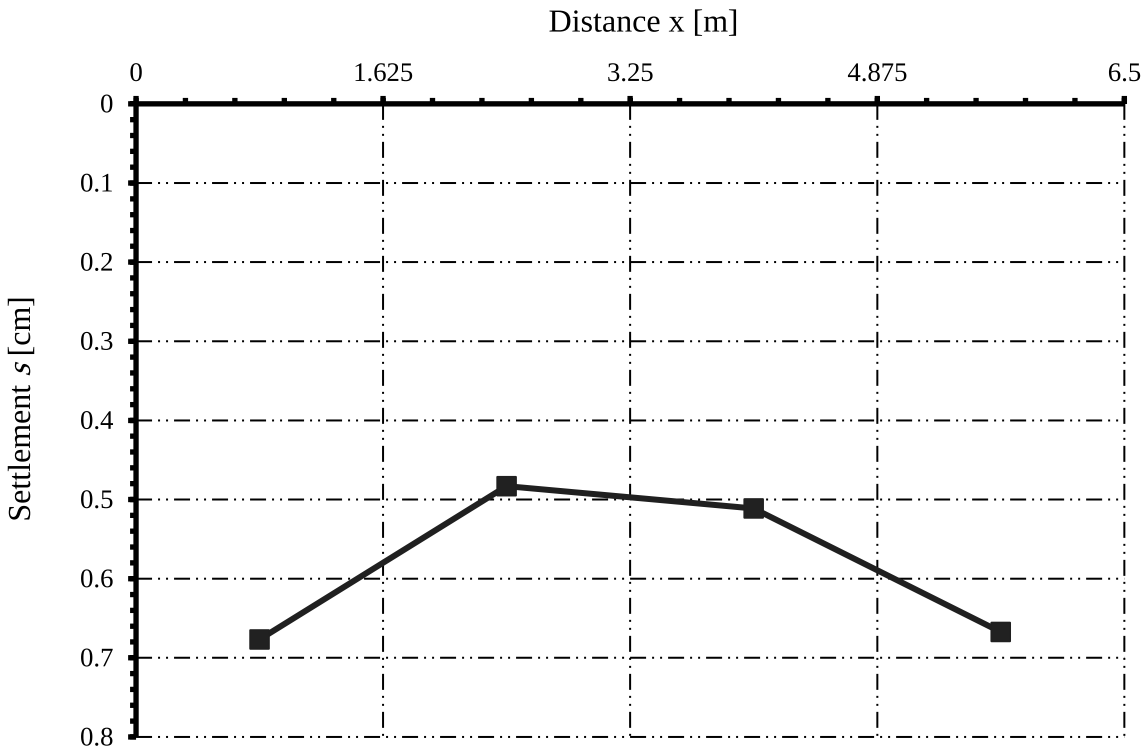


Figure 10.6 Settlements

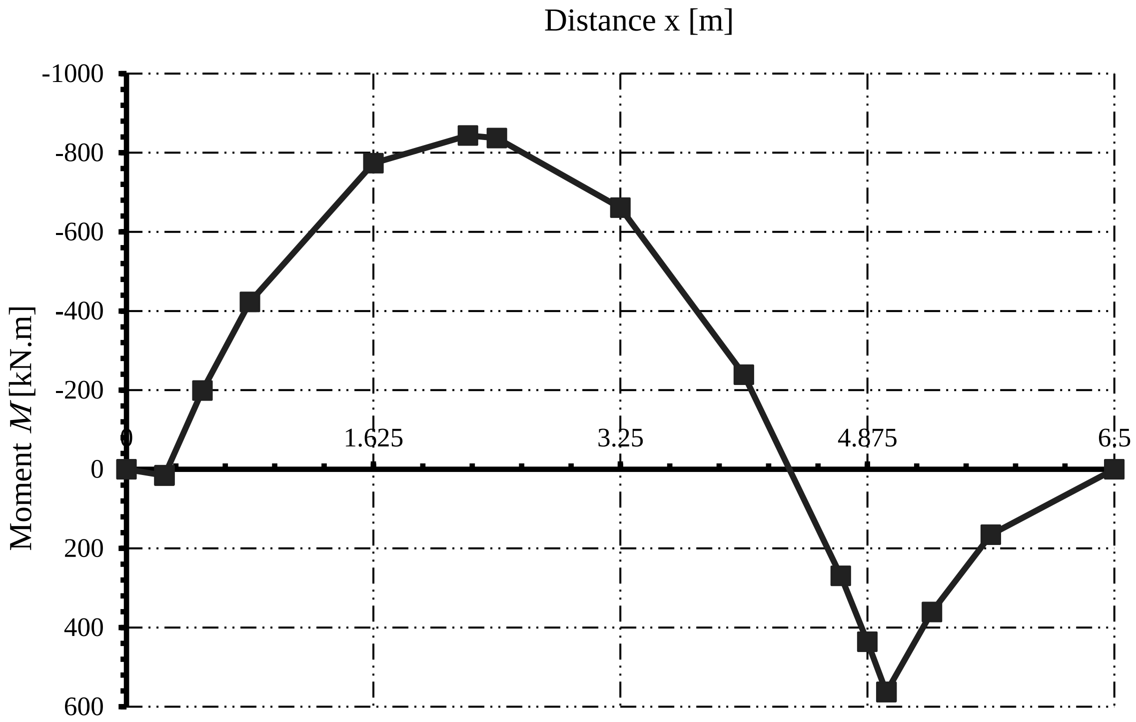


Figure 10.7 Moments

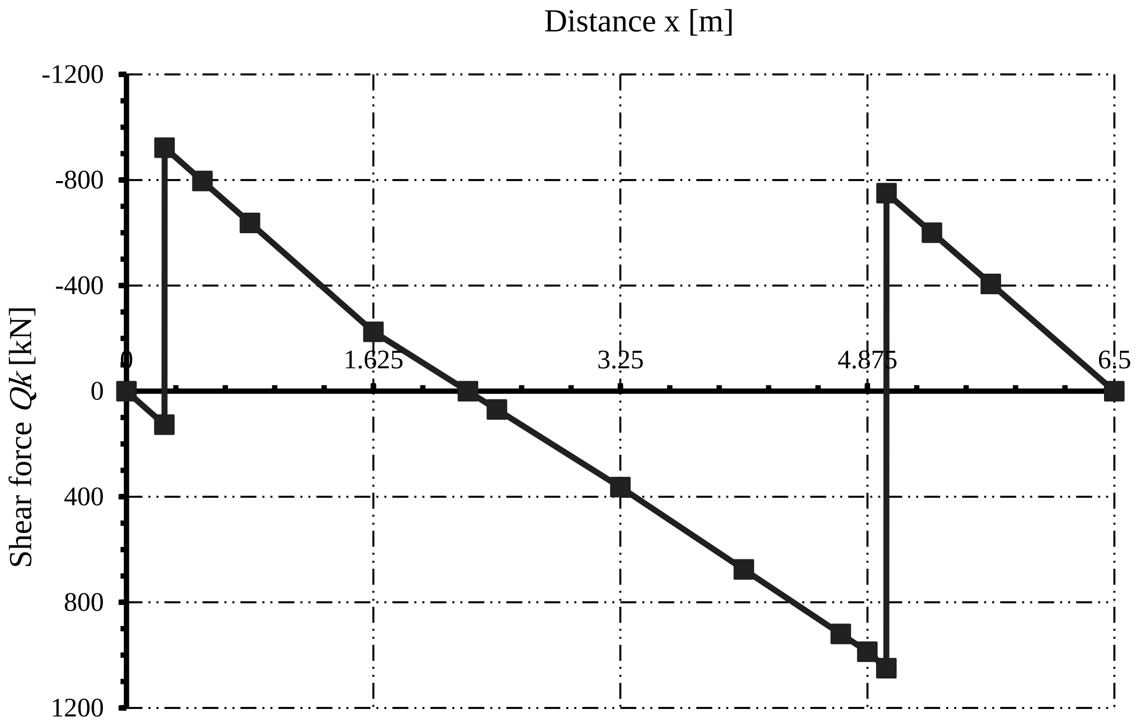


Figure 10.8 Shear forces

10.2.4 Example 2: Design of a combined footing for two equal columns

10.2.4.1 Description of the problem

It is required to find the contact pressure distribution, settlements, moment and shear force diagrams for a rectangular combined footing for for two equal edge columns, each carrying a load of P=1125 [kN] as shown in Figure 10.9. Column sides are 0.50×0.50 [m^2] while column reinforcement is 4Φ19. The allowable soil pressure is q_{all}=210 [kN/m^2] at a depth of D_f=1.5 [m] and average unit weight of the soil and concrete is γ_a=20 [kN/m^3]. Modulus of elasticity of the footing concrete is E_b=3×10^7, while the Modulus of subgrade reaction of the soil is k_s=35000 [kN/m^3].

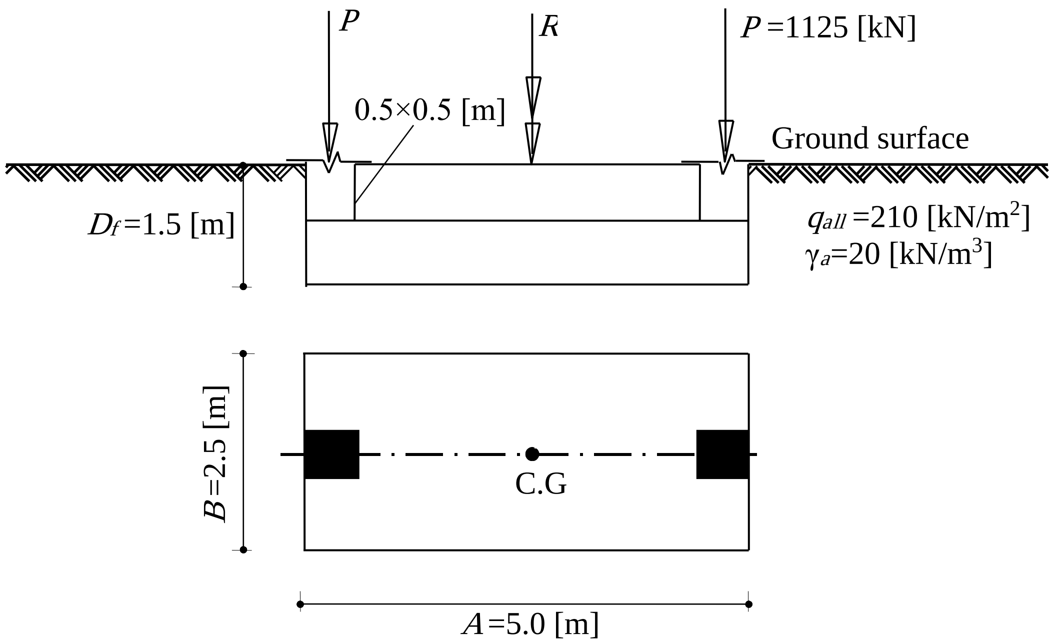


Figure 10.9 Combined rectangular footing

1. *Determining footing sides A and B*

Resultant of loads R at the ground surface level is given by:

$$R = 2\,P = 2\times1125 = 2250 \text{ [kN]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a\, D_f = 210 - 20\times1.5 = 180 \text{ [kN/m}^2\text{]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R}{q_{net}} = \frac{2250}{180} = 12.5 \text{ [m}^2\text{]}$$

Take A_f = 2.5×5.0 = 12.5 [m^2] rectangular combined footing

10.2.4.1.1 Analysis of the combined footing

The footing can be regarded as a beam on elastic foundation. In the analysis, the beam is divided into four equal elements, each 1.625 [m] long (**Error! Reference source not found.**). The self-weight of t he footing is neglected. According to *Kany/ El Gendy* (1995), the analysis of combined footing as a beam on elastic foundation is carried out in the following steps:

10.2.4.1.2 Calculation of u_i, v_i and w_i:

For a constant beam moment of inertia $I_i = I$

$$u_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i-1}}\right)$$

$$u_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

$$v_i = \frac{1}{4}\left(\frac{I_i}{I_{i-1}} + 14 + \frac{I_i}{I_{i+1}}\right)$$

$$v_i = \frac{1}{4}\left(\frac{I}{I} + 14 + \frac{I}{I}\right) = \frac{1}{4} \times 16 = 4$$

$$w_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i+1}}\right)$$

$$w_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

Moment of inertia $I_i = I$:

$$I_i = I = \frac{B {d_i}^3}{12} = \frac{2.5 \times 0.6^3}{12} = 0.045 \text{ [m}^4\text{]}$$

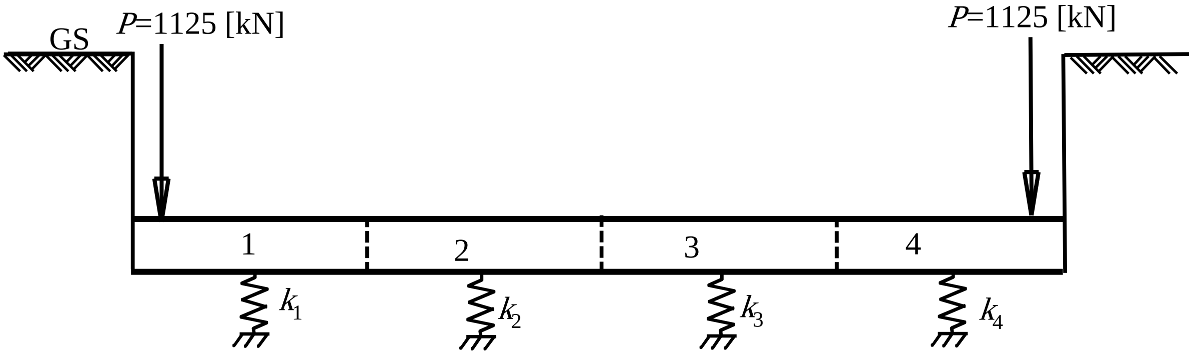
$$\alpha = \frac{a^4 B}{E_b I} = \frac{1.25^4 \times 2.5}{30000000 \times 0.045} = 4.52 \times 10^{-6} \text{ [m}^3\text{/kN]}$$

10.2.4.1.3 Determining external moments $M_i^{(l)}$

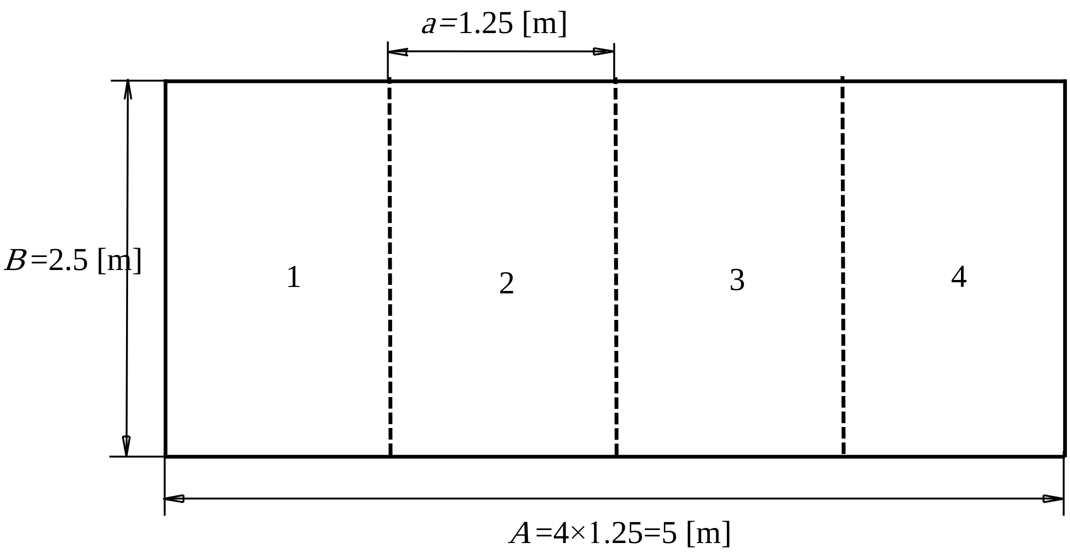
The external moments $M_i^{(l)}$ at points 2 and 3 are:

$$M_2^{(l)} = 1125 \times (1.875 - 0.25) = 1828.125 \text{ [kN.m]}$$

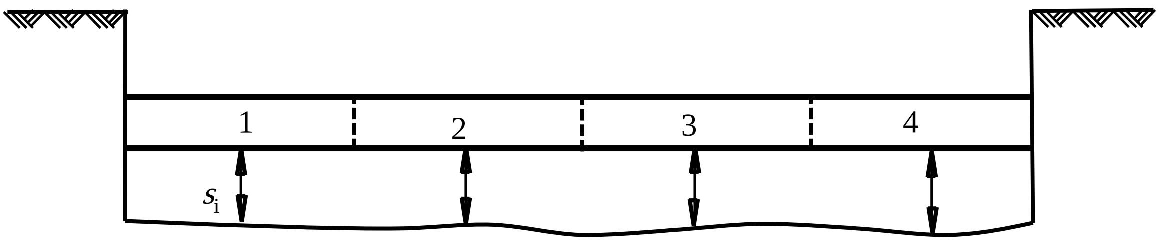
$$M_3^{(l)} = 1125 \times (3.125 - 0.25) = 3234.375 \text{ [kN.m]}$$



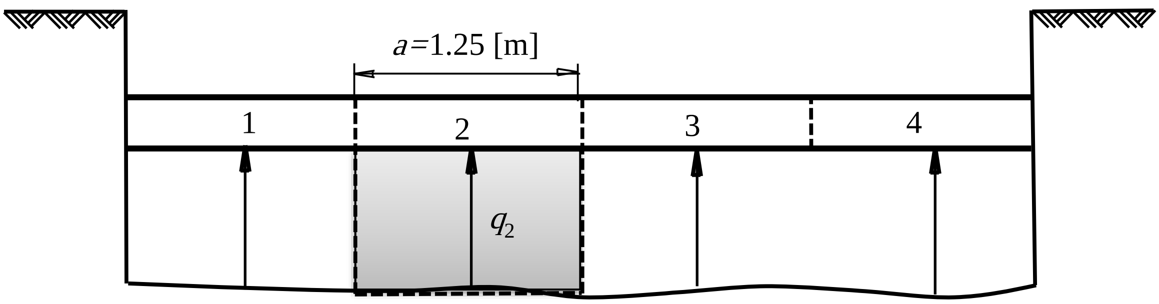
Beam foundation with loads and springs



Plan of beam foundation with elements



Settlement



Contact pressure

Figure 10.10 Combined footing as a beam on elastic foundation

10.2.4.1.4 Determining the right hand side R_i

The right hand side R_i of the contact pressure equation is:

$$R_i = \left(u_i M_{i-1}^{(l)} + v_i M_i^{(l)} + w_i M_{i+1}^{(l)}\right)\frac{a^2}{6EI_i}$$

$$R_i = \left(1 \times M_{i-1}^{(l)} + 4 \times M_i^{(l)} + 1 \times M_{i+1}^{(l)}\right)\frac{1.25^2}{6 \times 30000000 \times 0.045}$$

$$R_i = 1.93 \times 10^{-7}\left(M_{i-1}^{(l)} + 4M_i^{(l)} + M_{i+1}^{(l)}\right)$$

Apply the above equation at point 2:

$$R_2 = 1.93 \times 10^{-7}\left(M_1^{(l)} + 4M_2^{(l)} + M_3^{(l)}\right)$$

$$R_2 = 1.93 \times 10^{-7}(0 + 4 \times 1828.125 + 3234.375) = 2.036 \times 10^{-3}$$

10.2.4.1.5 Determining contact pressures

The contact pressure equation is:

$$\left(\frac{1}{k_{i+1}}\right) q_{i+1} - \left(\frac{2}{k_i} - \frac{\alpha_i}{6} w_i\right) q_i + \left(\frac{1}{k_{i-1}} + \frac{\alpha_i}{6}(v_i + 2w_i)\right) q_{i-1}$$

$$+ \frac{\alpha_i}{6}\left(\sum_{j=1}^{i-2}[(i-j-1)u_i + (i-j)v_i + (i-j+1)w_i]\, q_j\right) = R_i$$

$$\left(\frac{1}{35000}\right) q_{i+1} - \left(\frac{2}{35000} - \frac{4.52 \times 10^{-6}}{6}\right) q_i + \left(\frac{1}{35000} + \frac{4.52 \times 10^{-6}}{6}(4+2)\right) q_{i-1}$$

$$+ \frac{4.52 \times 10^{-6}}{6}\left(\sum_{j=1}^{i-2}[(i-j-1) + 4 \times (i-j) + (i-j+1)]\, q_j\right) = R_i$$

or

$$q_{i+1} - 1.974\, q_i + 1.158\, q_{i-1} + 0.026\left(\sum_{j=1}^{i-2}(i-j)\, q_j\right) = 35000R_i$$

Apply the above equation at points 2:

$$q_3 - 1.974\, q_2 + 1.158\, q_1 = 35000 \times 2.036 \times 10^{-3}$$

$$-0.974\, q_2 + 1.158\, q_1 = 71.244$$

$$-\, q_2 + 1.189\, q_1 = 73.146$$

There are four unknown q_1, q_2, q_3, and q_4, so a farther equation is required. This can be obtained by considering the overall equilibrium of vertical forces.

$$\sum V = 0$$

$$aB(\, q_1 +\, q_2 +\, q_3 +\, q_4) = P_1 + P_2$$

or

$$q_1 +\, q_2 = 360$$

Contact pressure equations in matrix form:

$$\begin{bmatrix} 1.189 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 73.146 \\ 360 \end{bmatrix}$$

Solving the above system of linear equations to obtain the contact pressures q_1, q_2, q_3, and q_4.

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 197.874 \\ 162.126 \end{bmatrix} \text{ [kN/m}^2\text{]}$$

10.2.4.1.6 Determining settlements s_i

The settlement s_i can be given by:

$$s_i = \frac{q_i}{k_i} = \frac{q_i}{35000} \text{ [m]}$$

$$s_1 = 0.57 \text{ [cm]}$$

$$s_2 = 0.46 \text{ [cm]}$$

The contact pressure distribution, settlement, moment and shear force diagrams for the raft are shown in Figure 10.11 to Figure 10.14. Once the internal forces are obtained at various sections, the design of the raft can be completed in the normal manner.

10.2.4.1.7 Computer calculation

The input data and results of *GEO Tools* are presented on the next pages. By comparison, one can see an agreement with the hand calculation.

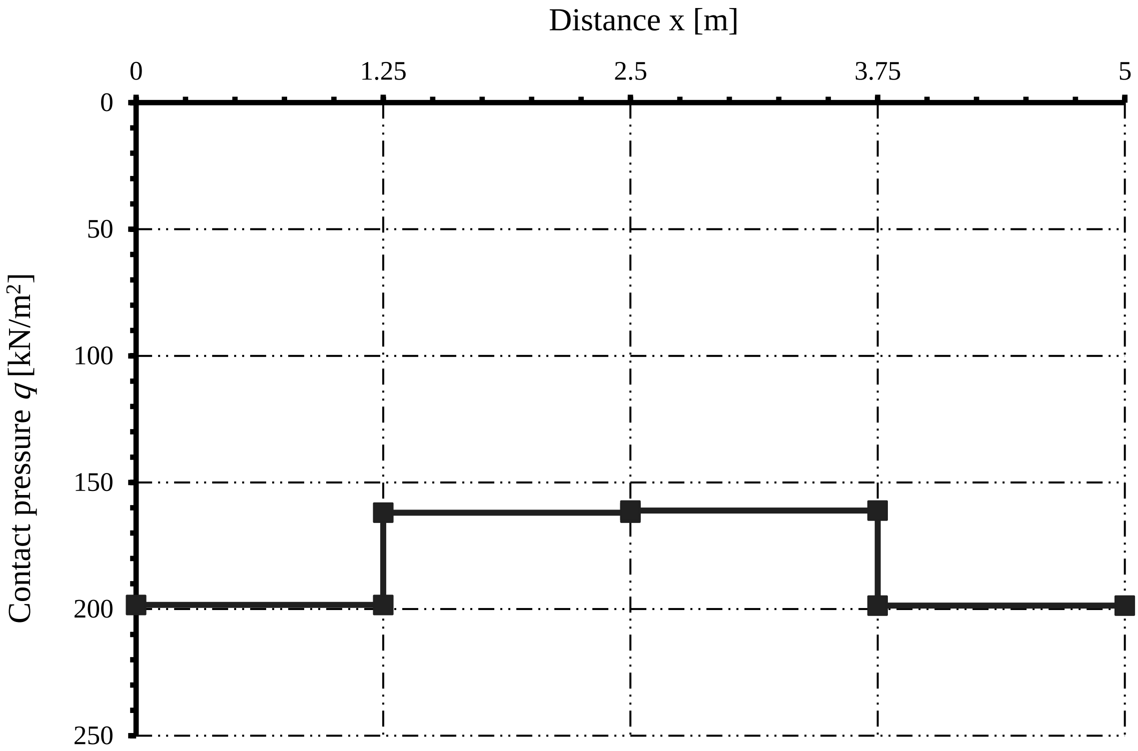


Figure 10.11 Contact pressures

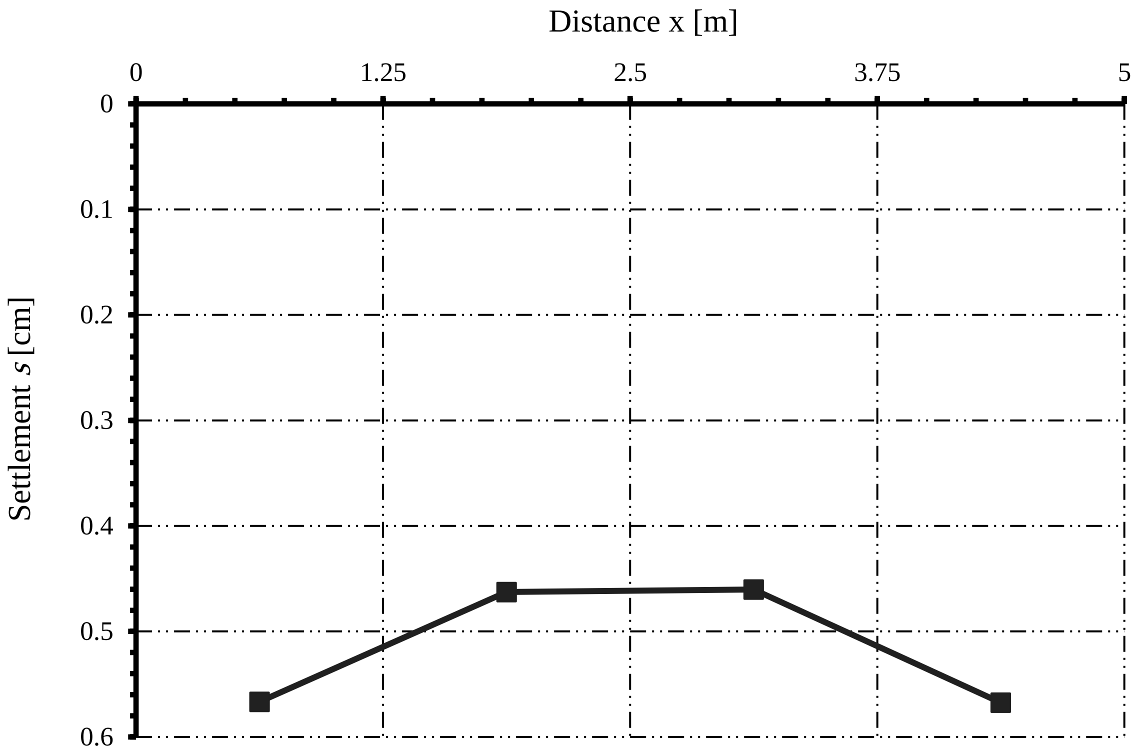


Figure 10.12 Settlements

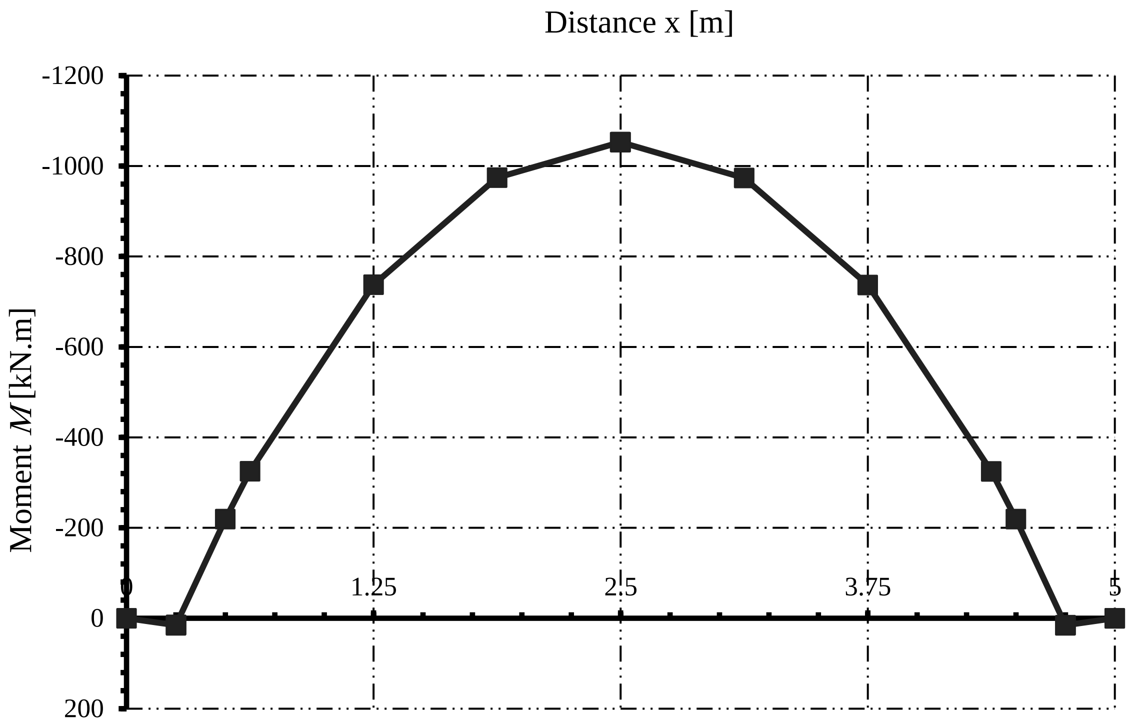


Figure 10.13 Moments

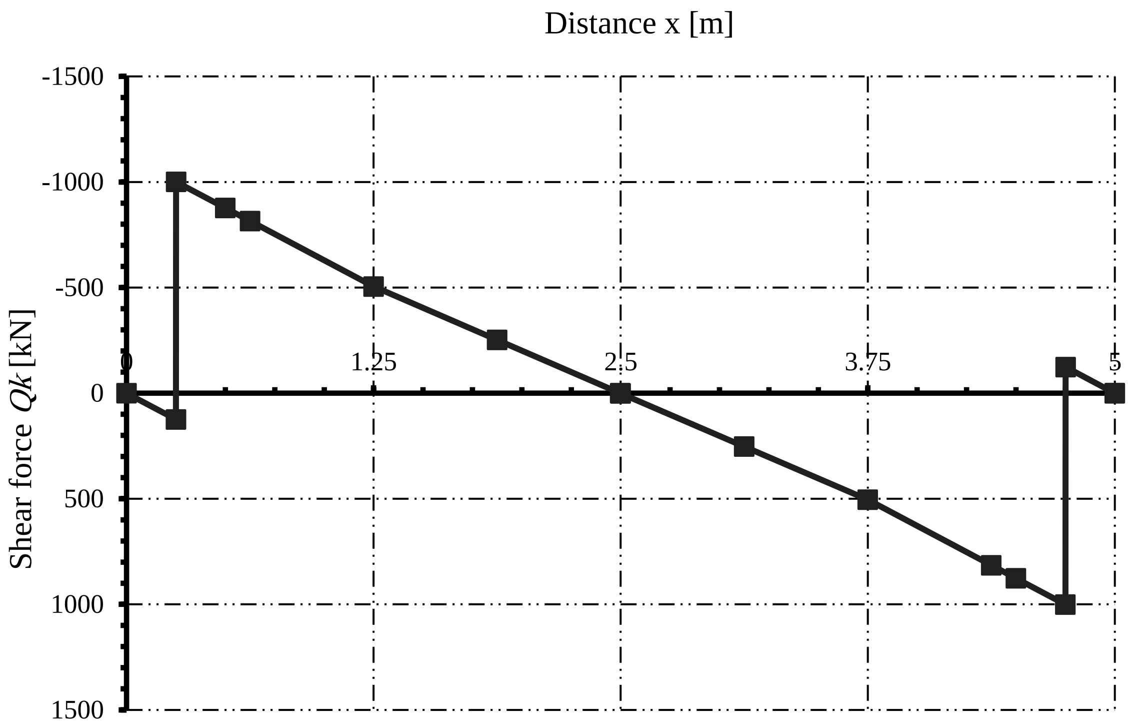


Figure 10.14 Shear forces

10.2.5 Example 3: Design of a combined footing for three equal columns

10.2.5.1 Description of the problem

An example is carried out to design a combined footing for three equal columns according to EC 2, DIN 1045, ACI and ECP.

A rectangular combined footing of 0.5 [m] thickness with dimensions of 7.8 [m]×2.6 [m] is chosen (Figure 10.15). The footing is supported to three equal columns, each of dimension 0.4 [m]×0.4 [m], reinforced by 8Φ16 and carries a load of 1276 [kN]. The footing rests on *Winkler* springs that have Modulus of Subgrade Reaction of k_s=40000 [kN/ m^3]. A thin plain concrete of thickness 0.15 [m] is chosen under the footing and is not considered in any calculations.

10.2.5.2 Footing material and section

The footing material and section are supposed to have the following parameters:

10.2.5.3 Material properties

Concrete grade according to ECP	C 250			
Steel grade according to ECP	S 36/52			
Concrete cube strength	f_{cu} = 250	[kg/ cm^2]	= 25	[MN/ m^2]
Concrete cylinder strength	f'_c = 0.8 f_{cu}	[-]	= 20	[MN/ m^2]
Compressive stress of concrete	f_c = 95	[kg/ cm^2]	= 9.5	[MN/ m^2]
Tensile stress of steel	f_s = 2000	[kg/ cm^2]	= 200	[MN/ m^2]
Reinforcement yield strength	f_y = 3600	[kg/ cm^2]	= 360	[MN/ m^2]
Young's modulus of concrete	E_b = 3×10^7	[kN/ m^2]	= 30000	[MN/ m^2]
Poisson's ratio of concrete	ν_b = 0.15	[-]		
Unit weight of concrete	γ_b = 0.0	[kN/ m^3]		

Unit weight of concrete is chosen γ_b = 0.0 to neglect the own weight of the footing.

10.2.5.4 Section properties

Width of the section to be designed	b = 1.0	[m]
Section thickness	t = 0.50	[m]
Concrete cover + 1/2 bar diameter	c = 5	[cm]
Effective depth of the section	$d = t - c$ = 0.45	[m]
Steel bar diameter	Φ = 18	[mm]

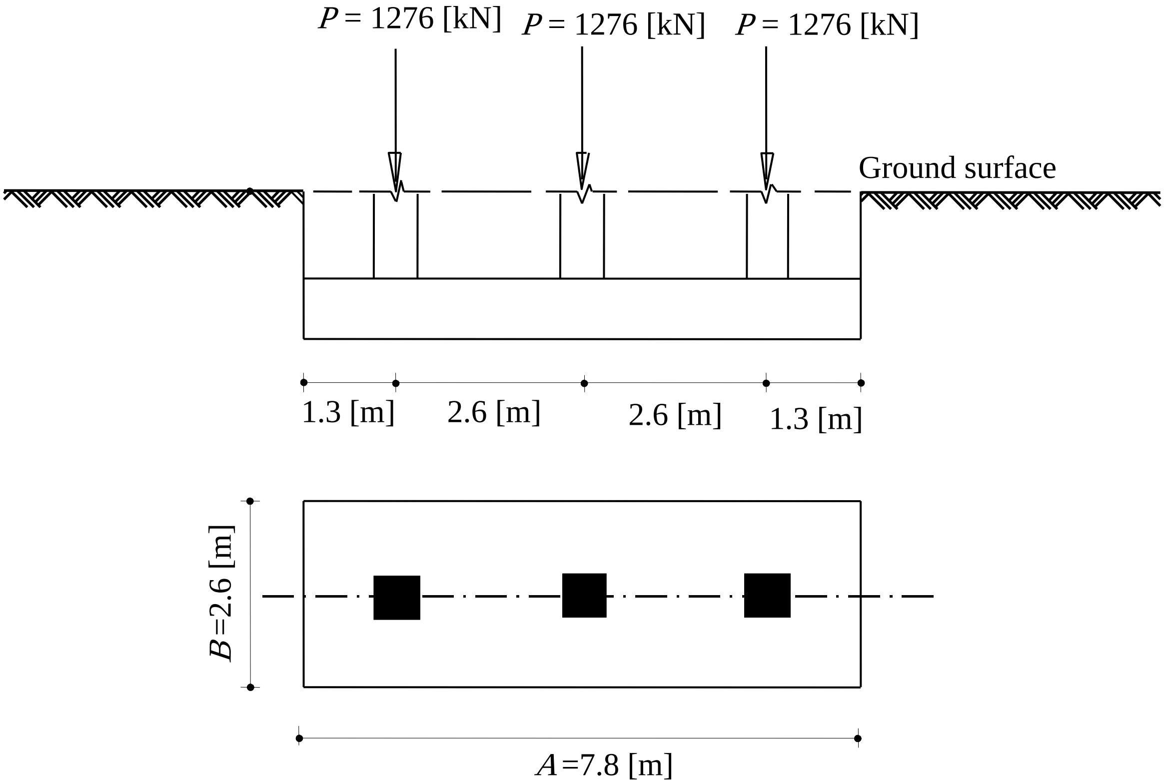


Figure 10.15 Combined rectangular footing for three columns

10.2.5.5 Analysis of the combined footing

The footing can be regarded as a beam on elastic foundation. In the analysis, the beam is divided into four equal elements, each 1.95 [m] long (**Error! Reference source not found.**Figure 10.16). The self-weight of the footing is neglected. According to *Kany/ El Gendy* (1995), the analysis of combined footing as a beam on elastic foundation is carried out in the following steps:

10.2.5.5.1 Calculation of u_i, v_i and w_i:

For a constant beam moment of inertia $I_i = I$

$$u_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i-1}}\right)$$

$$u_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

$$v_i = \frac{1}{4}\left(\frac{I_i}{I_{i-1}} + 14 + \frac{I_i}{I_{i+1}}\right)$$

$$v_i = \frac{1}{4}\left(\frac{I}{I} + 14 + \frac{I}{I}\right) = \frac{1}{4} \times 16 = 4$$

$$w_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i+1}}\right)$$

$$w_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

Moment of inertia $I_i = I$:

$$I_i = I = \frac{B{d_i}^3}{12} = \frac{2.6 \times 0.5^3}{12} = 0.027 \text{ [m}^4\text{]}$$

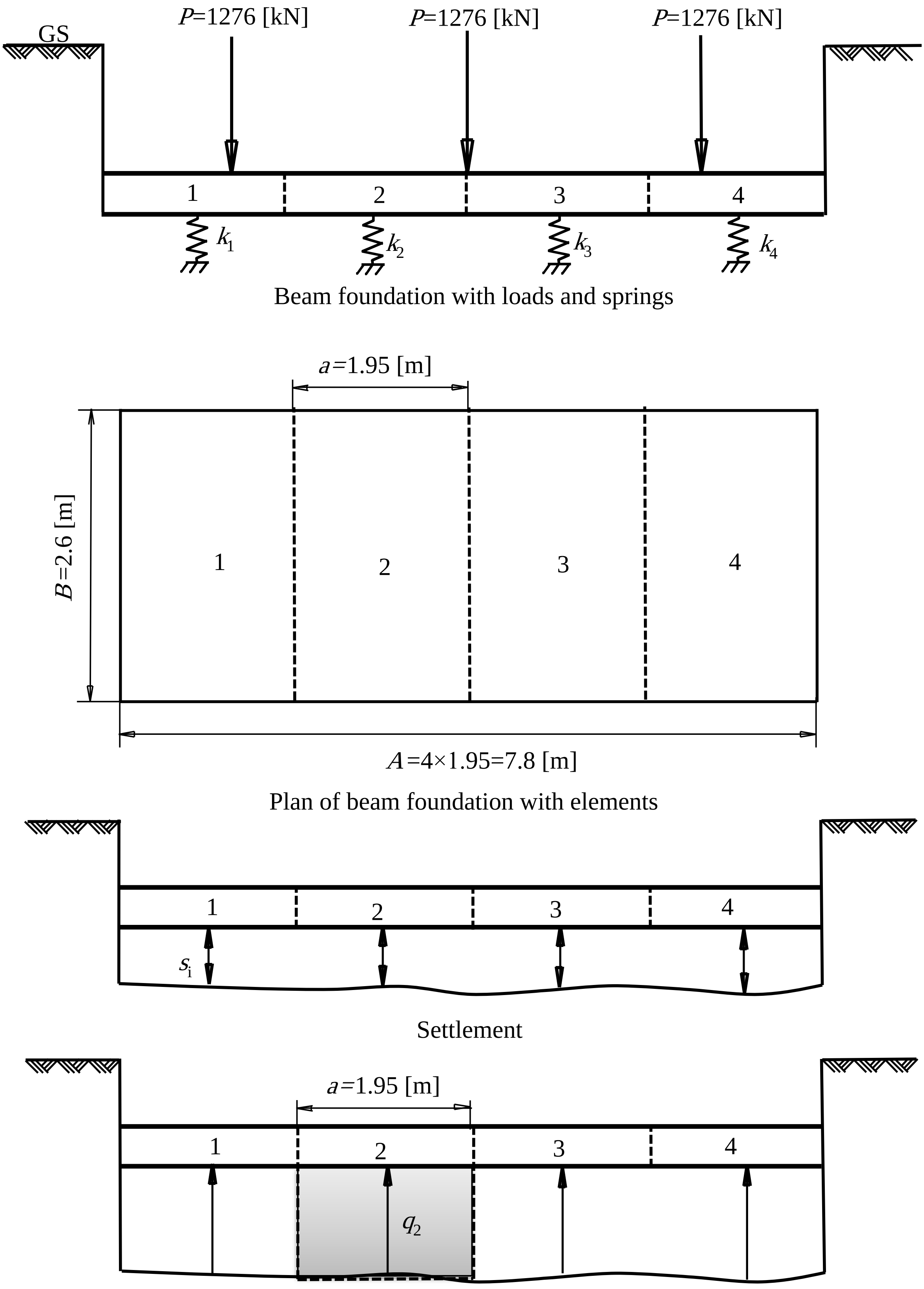
$$\alpha = \frac{a^4 B}{E_p I} = \frac{1.95^4 \times 2.6}{30000000 \times 0.027} = 4.64 \times 10^{-5} \text{ [m}^3\text{/kN]}$$

10.2.5.5.2 Determining external moments $M_i^{(l)}$

The external moments $M_i^{(l)}$ at points 2 and 3 are:

$$M_2^{(l)} = 1276(2.925 - 1.3) = 2073.5 \text{ [kN.m]}$$

$$M_3^{(l)} = 1276(4.875 - 1.3) + 1276\,\frac{1.95}{2} = 5805.8 \text{ [kN.m]}$$



Beam foundation with loads and springs

Plan of beam foundation with elements

Settlement

Contact pressure

Figure 10.16 Combined footing as a beam on elastic foundation

10.2.5.5.3 Determining the right hand side R_i

The right hand side R_i of the contact pressure equation is:

$$R_i = \left(u_i M_{i-1}^{(l)} + v_i M_i^{(l)} + w_i M_{i+1}^{(l)}\right)\frac{a^2}{6EI_i}$$

$$R_i = \left(1 \times M_{i-1}^{(l)} + 4 \times M_i^{(l)} + 1 \times M_{i+1}^{(l)}\right)\frac{1.95^2}{6 \times 30000000 \times 0.027}$$

$$R_i = 7.82 \times 10^{-7}\left(M_{i-1}^{(l)} + 4M_i^{(l)} + M_{i+1}^{(l)}\right)$$

Apply the above equation at point 2:

$$R_2 = 7.82 \times 10^{-7}\left(M_1^{(l)} + 4M_2^{(l)} + M_3^{(l)}\right)$$

$$R_2 = 7.82 \times 10^{-7}(0 + 4 \times 2073.5 + 5805.8) = 0.011$$

10.2.5.5.4 Determining contact pressures

The contact pressure equation is:

$$\left(\frac{1}{k_{i+1}}\right)q_{i+1} - \left(\frac{2}{k_i} - \frac{\alpha_i}{6}w_i\right)q_i + \left(\frac{1}{k_{i-1}} + \frac{\alpha_i}{6}(v_i + 2w_i)\right)q_{i-1}$$

$$+\frac{\alpha_i}{6}\left(\sum_{j=1}^{i-2}[(i-j-1)u_i + (i-j)v_i + (i-j+1)w_i]\,q_j\right) = R_i$$

$$\left(\frac{1}{40000}\right)q_{i+1} - \left(\frac{2}{40000} - \frac{4.64 \times 10^{-5}}{6}\right)q_i + \left(\frac{1}{40000} + \frac{4.64 \times 10^{-5}}{6}(4+2)\right)q_{i-1}$$

$$+\frac{4.64 \times 10^{-5}}{6}\left(\sum_{j=1}^{i-2}[(i-j-1) + 4 \times (i-j) + (i-j+1)]\,q_j\right) = R_i$$

or

$$q_{i+1} - 1.691\,q_i + 2.856\,q_{i-1} + 1.856\left(\sum_{j=1}^{i-2}(i-j)\,q_j\right) = 40000R_i$$

Apply the above equation at points 2:

$$q_3 - 1.691\,q_2 + 2.856\,q_1 = 40000 \times 0.011$$

$$-0.691\,q_2 + 2.856\,q_1 = 440$$

$$-q_2 + 4.133\, q_1 = 636.758$$

There are four unknown q_1, q_2, q_3, and q_4, so a farther equation is required. This can be obtained by considering the overall equilibrium of vertical forces.

$$\sum V = 0$$

$$aB(\, q_1 + \; q_2 + \; q_3 + \; q_4) = P_1 + P_2 + P_3$$

or

$$q_1 + \; q_2 = 377.515$$

Contact pressure equations in matrix form:

$$\begin{bmatrix} 4.133 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 636.758 \\ 377.515 \end{bmatrix}$$

Solving the above system of linear equations to obtain the contact pressures q_1, q_2, q_3, and q_4.

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 197.600 \\ 179.917 \end{bmatrix} \text{ [kN/m}^2\text{]}$$

10.2.5.5.5 Determining settlements s_i

The settlement s_i can be given by:

$$s_i = \frac{q_i}{k_i} = \frac{q_i}{40000} \text{ [m]}$$

$$s_1 = 0.49 \text{ [cm]}$$

$$s_2 = 0.45 \text{ [cm]}$$

The contact pressure distribution, settlement, moment and shear force diagrams for the raft are shown in Figure 10.17 to Figure 10.20. Once the internal forces are obtained at various sections, the design of the raft can be completed in the normal manner.

10.2.5.5.6 Computer calculation

The input data and results of *GEO Tools* are presented on the next pages. By comparison, one can see an agreement with the hand calculation.

For the different codes, the footing is designed to resist the bending moment and punching shear. Then, the required reinforcement is obtained. Finally, a comparison among the results of the four codes is presented.

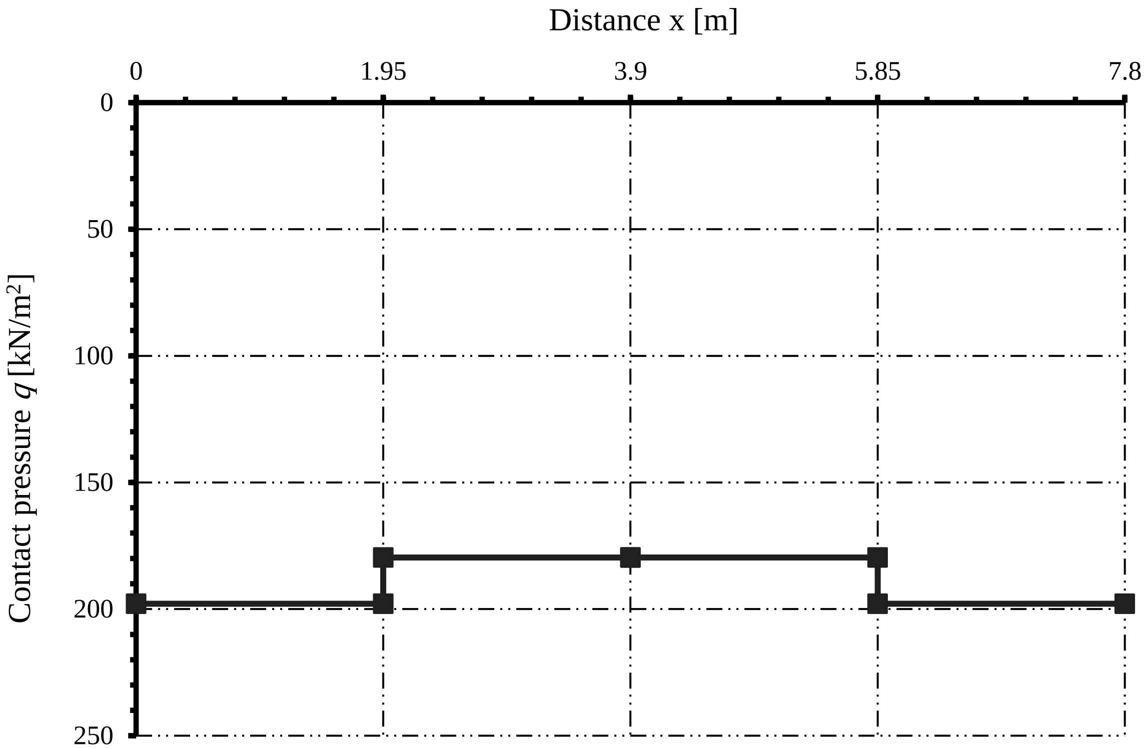


Figure 10.17 Contact pressures

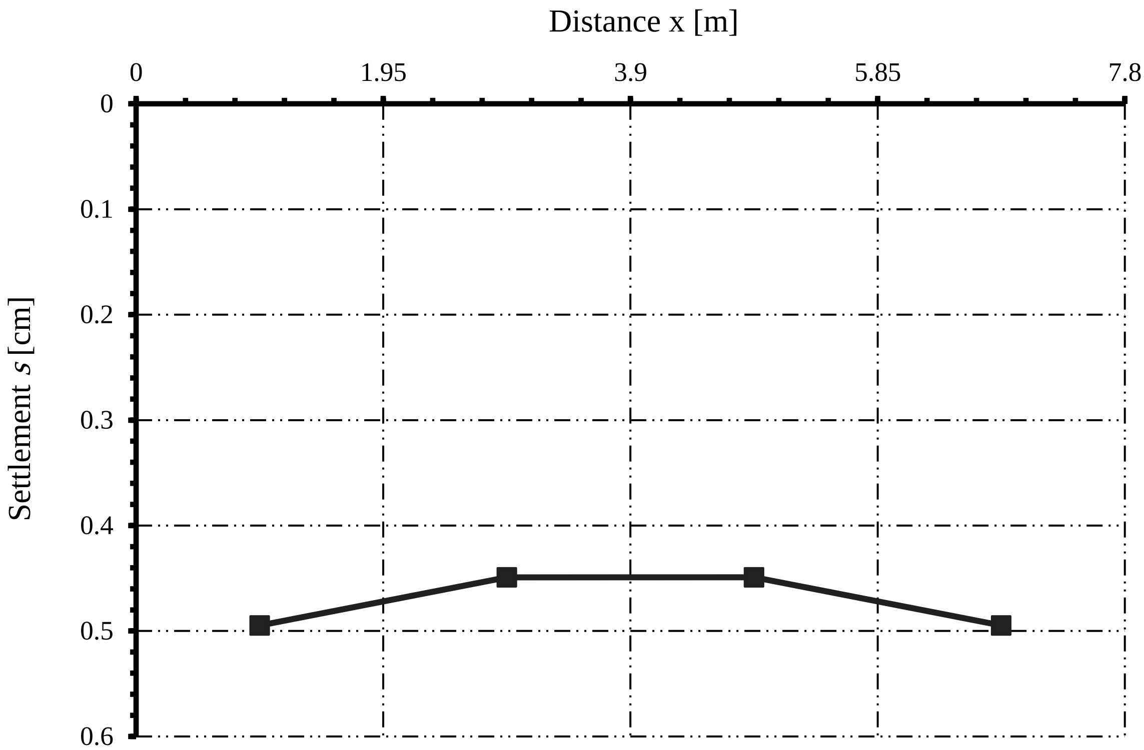


Figure 10.18 Settlements

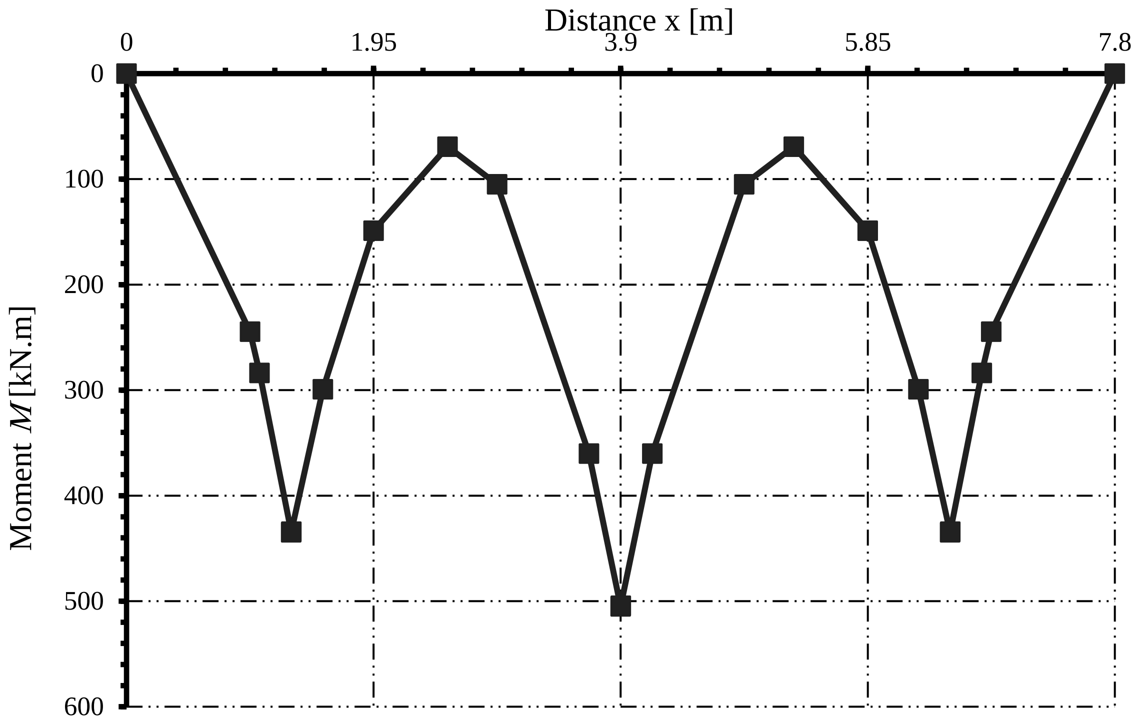


Figure 10.19 Moments

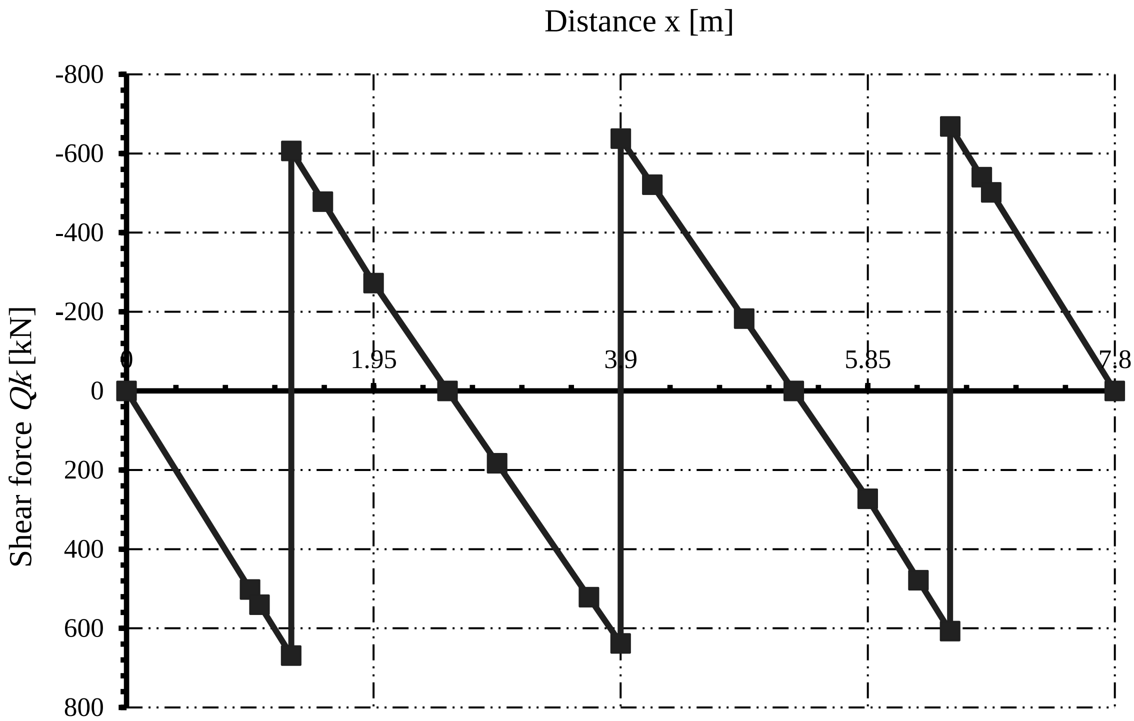


Figure 10.20 Shear forces

10.2.6 Example 4: Design of a combined footing for three unequal columns

10.2.6.1 Description of the problem

Design a combined rectangular footing for three unequal columns. Edge columns C_1 and C_3 have dimensions of 0.3×0.5 [m^2], reinforcement 6Φ19 and a load of 1000 [kN], while the central column C_2 has dimensions of 0.5×0.75 [m^2], reinforcement 10Φ25 and a load of 3000 [kN] as shown in Figure 10.1Figure 10.1. The allowable soil pressure q_{all} = 210 [kN/m^2] at a depth of D_f = 2.0 [m] and the average unit weight of the soil and concrete is γ_a = 20 [kN/m^3]. The footing rests on *Winkler* springs that have Modulus of Subgrade Reaction of k_s=30000 [kN/ m^3]. A thin plain concrete of thickness 0.15 [m] is chosen under the footing and is not considered in any calculations. The loading and the footing are symmetrical.

10.2.6.2 Footing material and section

The footing material and section are supposed to have the following parameters:

10.2.6.3 Material properties

Concrete grade according to ECP	C 250			
Steel grade according to ECP	S 36/52			
Concrete cube strength	f_{cu} = 250	[kg/ cm^2]	= 25	[MN/ m^2]
Concrete cylinder strength	$f0_c$ = 0.8 f_{cu}	[-]	= 20	[MN/ m^2]
Compressive stress of concrete	f_c = 95	[kg/ cm^2]	= 9.5	[MN/ m^2]
Tensile stress of steel	f_s = 2000	[kg/ cm^2]	= 200	[MN/ m^2]
Reinforcement yield strength	f_y = 3600	[kg/ cm^2]	= 360	[MN/ m^2]
Young's modulus of concrete	E_b = 3×10^7	[kN/ m^2]	= 30000	[MN/ m^2]
Poisson's ratio of concrete	ν_b = 0.15	[-]		
Unit weight of concrete	γ_b = 0.0	[kN/ m^3]		

Unit weight of concrete is chosen γ_b = 0.0 to neglect the own weight of the footing.

10.2.6.4 Section properties

Width of the section to be designed	b = 1.0	[m]
Section thickness	t = 0.8	[m]
Concrete cover + 1/2 bar diameter	c = 5	[cm]
Effective depth of the section	$d = t - c$ = 0.75	[m]
Steel bar diameter	Φ = 18	[mm]

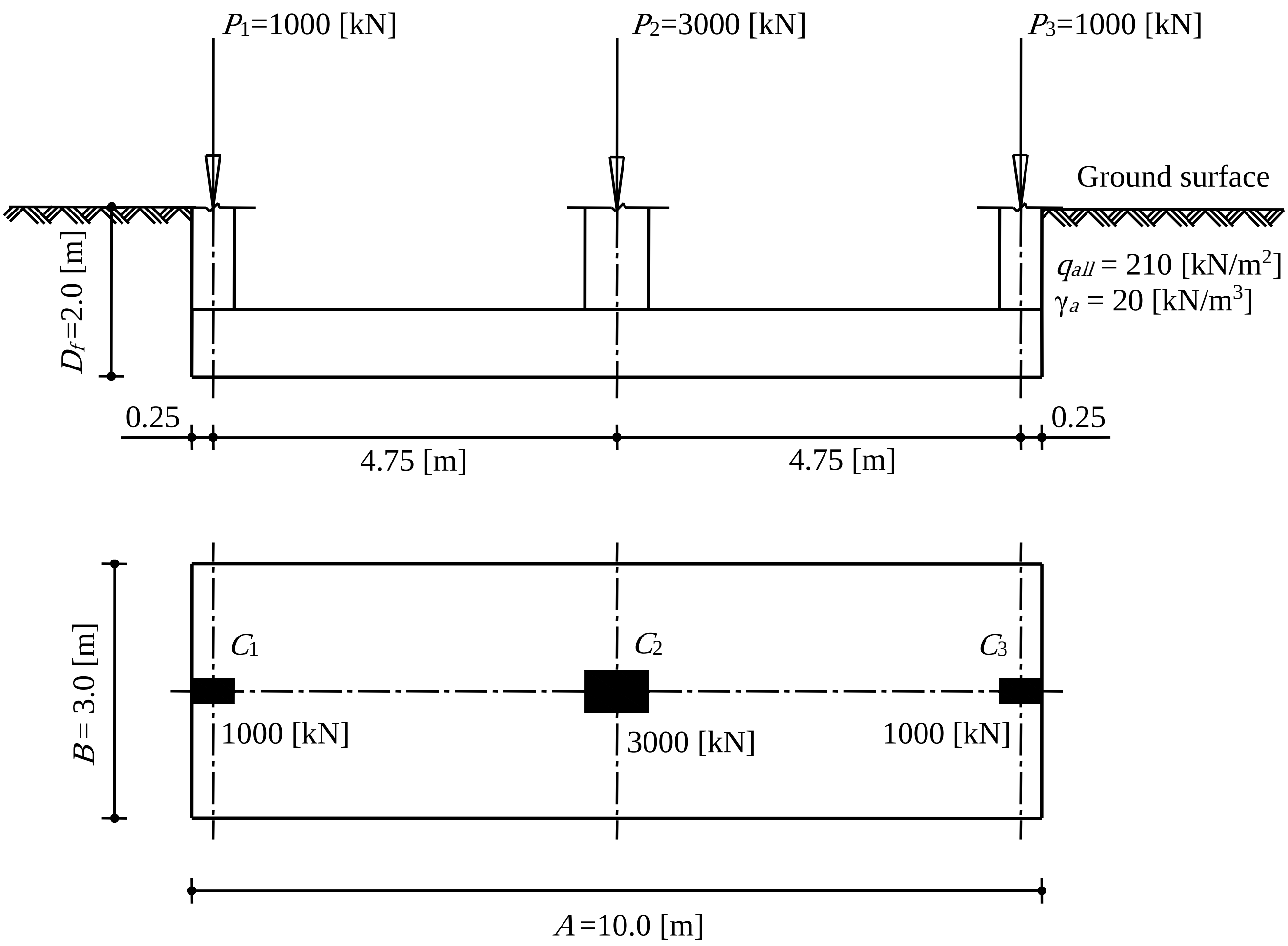


Figure 10.1 Combined rectangular footing for three unequal columns

1. Determining footing sides A and B

Resultant of loads R at the ground surface level is given by:

$$R = 2\,P_1 + P_2 = 2 \times 1000 + 3000 = 5000 \text{ [kN]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 210 - 20 \times 2.0 = 170 \text{ [kN/m}^2\text{]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R}{q_{net}} = \frac{5000}{170} = 29.41 \text{ [m}^2\text{]}$$

take $A_f = 10.0 \times 3.0 = 30.0$ [m^2] rectangular combined footing

10.2.6.5 Hand calculation

The footing can be regarded as a beam on elastic foundation subjected to three unequal concentrated forces. The beam is divided into eight equal elements, each 1.25 [m] long (Figure 10.21). Because of the symmetry of the system, the analysis can be carried out by considering only half of the beam. Hence, the total number of equations is reduced to four.

According to *Kany/ El Gendy* (1995), the analysis of beam on elastic foundation is carried out in the following steps:

10.2.6.5.1 Calculation of u_i, v_i **and** *w_i***:**

For a constant beam moment of inertia $I_i = I$

$$u_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i-1}}\right)$$

$$u_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

$$v_i = \frac{1}{4}\left(\frac{I_i}{I_{i-1}} + 14 + \frac{I_i}{I_{i+1}}\right)$$

$$v_i = \frac{1}{4}\left(\frac{I}{I} + 14 + \frac{I}{I}\right) = \frac{1}{4} \times 16 = 4$$

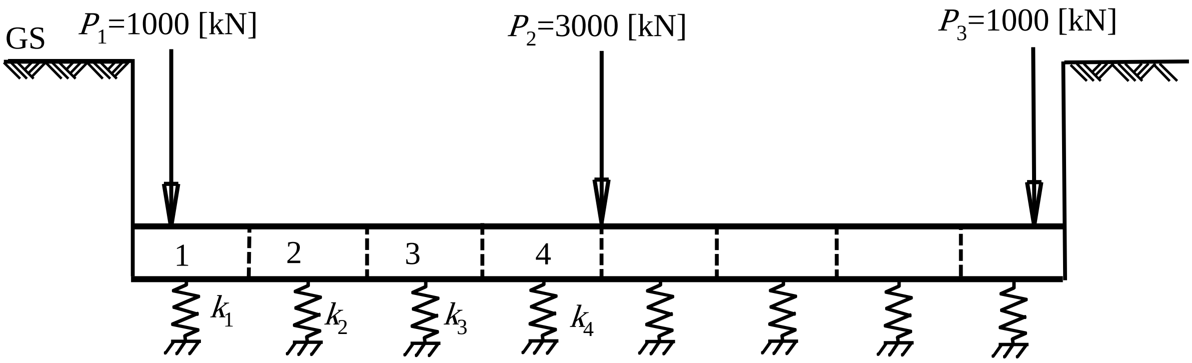
$$w_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i+1}}\right)$$

$$w_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

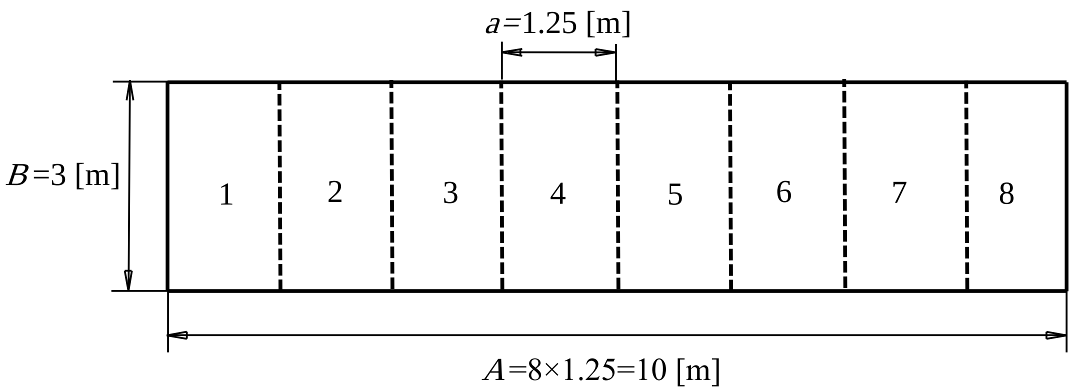
Moment of inertia $I_i = I$:

$$I_i = I = \frac{B{d_i}^3}{12} = \frac{3 \times 0.8^3}{12} = 0.128\ [m^4]$$

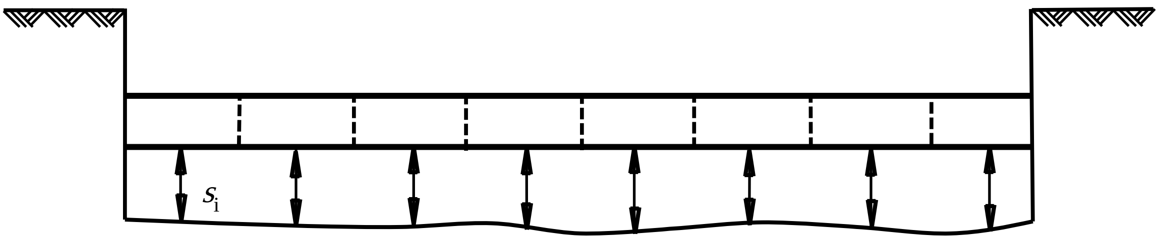
$$\alpha = \frac{a^4 B}{E_p I} = \frac{1.25^4 \times 3}{30000000 \times 0.128} = 1.91 \times 10^{-6}\ [\mathrm{m^3/kN}]$$



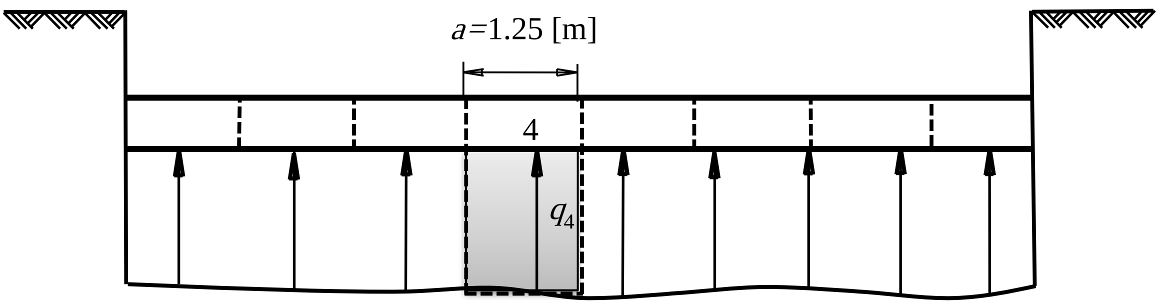
Beam foundation with loads and springs



Plan of beam foundation with elements



Settlement



Contact pressure

Figure 10.21 Combined footing as a beam on elastic foundation

10.2.6.5.2 Determining external moments $M_i^{(l)}$

$$M_1^{(l)} = zero$$

$$M_2^{(l)} = 1000(1.5 \times 1.25 - 0.25) = 1625 \text{ [kN.m]}$$

$$M_3^{(l)} = 1000(2.5 \times 1.25 - 0.25) = 2875 \text{ [kN.m]}$$

$$M_4^{(l)} = 1000(3.5 \times 1.25 - 0.25) = 4125 \text{ [kN.m]}$$

$$M_5^{(l)} = 1000(4.5 \times 1.25 - 0.25) + 3000 \times 0.5 \times 1.25 = 7250 \text{ [kN.m]}$$

10.2.6.5.3 Determining the right hand side R_i

The right hand side R_i of the contact pressure equation is:

$$R_i = \left(u_i M_{i-1}^{(l)} + v_i M_i^{(l)} + w_i M_{i+1}^{(l)}\right)\frac{a^2}{6EI_i}$$

$$R_i = \left(1 \times M_{i-1}^{(l)} + 4 \times M_i^{(l)} + 1 \times M_{i+1}^{(l)}\right)\frac{1.25^2}{6 \times 30000000 \times 0.128}$$

$$R_i = 6.8 \times 10^{-8}\left(M_{i-1}^{(l)} + 4M_i^{(l)} + M_{i+1}^{(l)}\right)$$

Apply the above equation at points 2, 3 and 4:

At point 2:

$$R_2 = 6.8 \times 10^{-8}\left(M_1^{(l)} + 4M_2^{(l)} + M_3^{(l)}\right)$$

$$R_2 = 6.782 \times 10^{-8}(0 + 4 \times 1625 + 2875) = 6.358 \times 10^{-4}$$

At point 3:

$$R_3 = 6.782 \times 10^{-8}\left(M_2^{(l)} + 4M_3^{(l)} + M_4^{(l)}\right)$$

$$R_3 = 6.782 \times 10^{-8}(1625 + 4 \times 2875 + 4125) = 1.17 \times 10^{-3}$$

At point 4:

$$R_4 = 6.782 \times 10^{-8}\left(M_3^{(l)} + 4M_4^{(l)} + M_5^{(l)}\right)$$

$$R_4 = 6.782 \times 10^{-8}(2875 + 4 \times 4125 + 7250) = 1.806 \times 10^{-3}$$

10.2.6.5.4 Determining contact pressures

The contact pressure equation is:

$$\left(\frac{1}{k}\right) q_{i+1} - \left(\frac{2}{k} - \frac{\alpha}{6}\right) q_i + \left(\frac{1}{k} + \alpha\right) q_{i-1} + \alpha \left(\sum_{j=1}^{i-2} (i-j)\, q_j\right) = R_i$$

$$\left(\frac{1}{30000}\right) q_{i+1} - \left(\frac{2}{30000} - \frac{1.91 \times 10^{-6}}{6}\right) q_i + \left(\frac{1}{30000} + 1.91 \times 10^{-6}\right) q_{i-1}$$

$$+ 1.91 \times 10^{-6} \left(\sum_{j=1}^{i-2} (i-j)\, q_j\right) = R_i$$

or

$$q_{i+1} - 1.991\, q_i + 1.0573\, q_{i-1} + 0.0573 \left(\sum_{j=1}^{i-2} (i-j)\, q_j\right) = 30000 R_i$$

Apply the above equation at points 2, 3 and 4:

$$q_3 - 1.991\, q_2 + 1.0573\, q_1 = 19.074$$

$$q_4 - 1.991\, q_3 + 1.0573\, q_2 + 0.115\, q_1 = 35.1$$

$$-0.991\, q_4 + 1.0573\, q_3 + 0.115\, q_2 + 0.172\, q_1 = 54.18$$

There are four unknown q_1, q_2, q_3, and q_4, so a farther equation is required. This can be obtained by considering the overall equilibrium of vertical forces.

$$\sum V = 0$$

$$aB(q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8) = P_1 + P_2 + P_3$$

or

$$q_1 + q_2 + q_3 + q_4 = 666.667$$

Contact pressure equations in matrix form:

$$\begin{bmatrix} 1.0573 & -1.991 & 1 & 0 \\ 0.115 & 1.0573 & -1.991 & 1 \\ 0.172 & 0.115 & 1.0573 & -0.991 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 19.074 \\ 35.1 \\ 54.18 \\ 666.667 \end{bmatrix}$$

Solving the above system of linear equations to obtain the contact pressures q_1, q_2, q_3, and q_4.

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 171.021 \\ 163.505 \\ 163.719 \\ 168.539 \end{bmatrix} \text{[kN/m}^2\text{]}$$

10.2.6.5.5 Determining settlements s_i

The settlement s_i can be given by:

$$s_i = \frac{q_i}{k_i} = \frac{q_i}{30000} \text{ [m]}$$

$$s_1 = 0.57 \text{ [cm]}$$

$$s_2 = 0.55 \text{ [cm]}$$

$$s_3 = 0.55 \text{ [cm]}$$

$$s_4 = 0.56 \text{ [cm]}$$

The contact pressure distribution, settlement, moment and shear force diagrams for the raft are shown in Figure 10.22 to Figure 10.25. Once the internal forces are obtained at various sections, the design of the raft can be completed in the normal manner.

10.2.6.5.6 Computer calculation

The input data and results of *GEO Tools* are presented on the next pages. By comparison, one can see an agreement with the hand calculation.

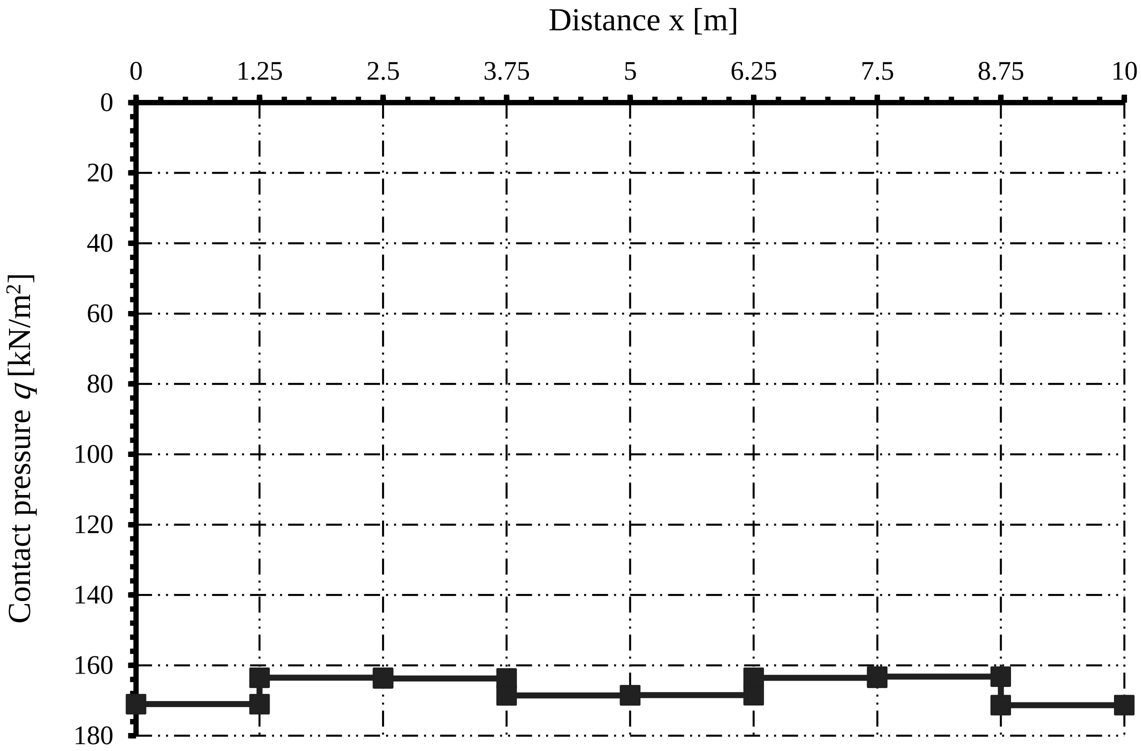


Figure 10.22 Contact pressures

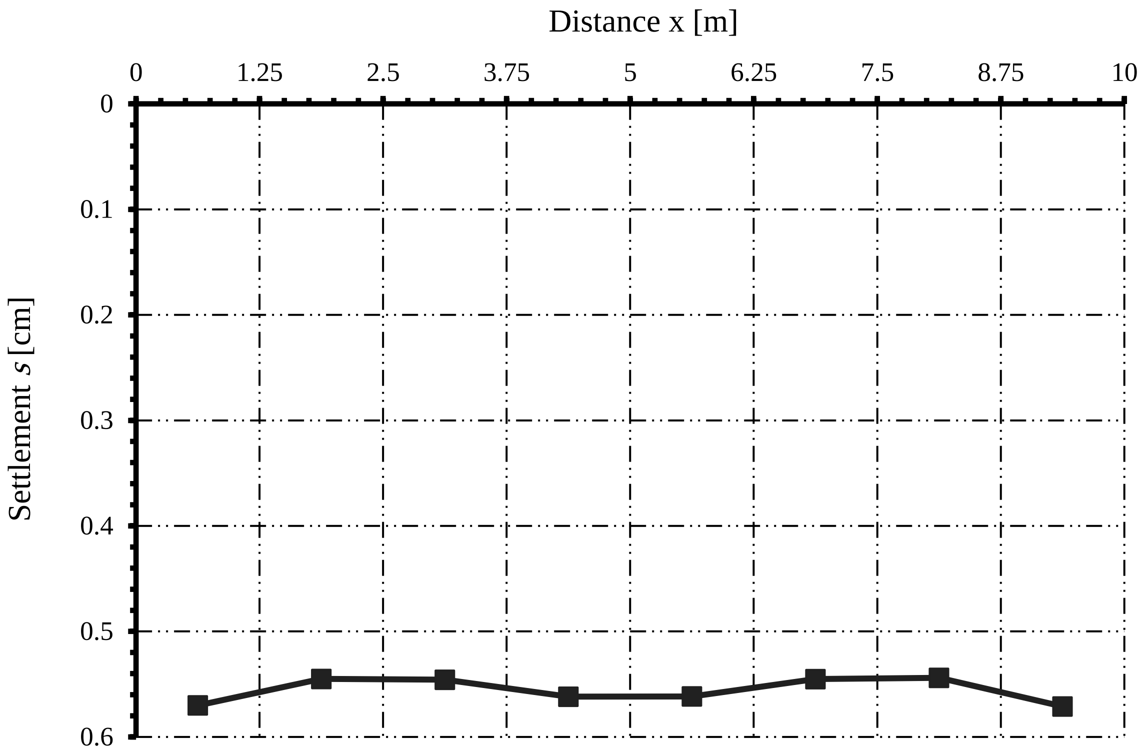


Figure 10.23 Settlements

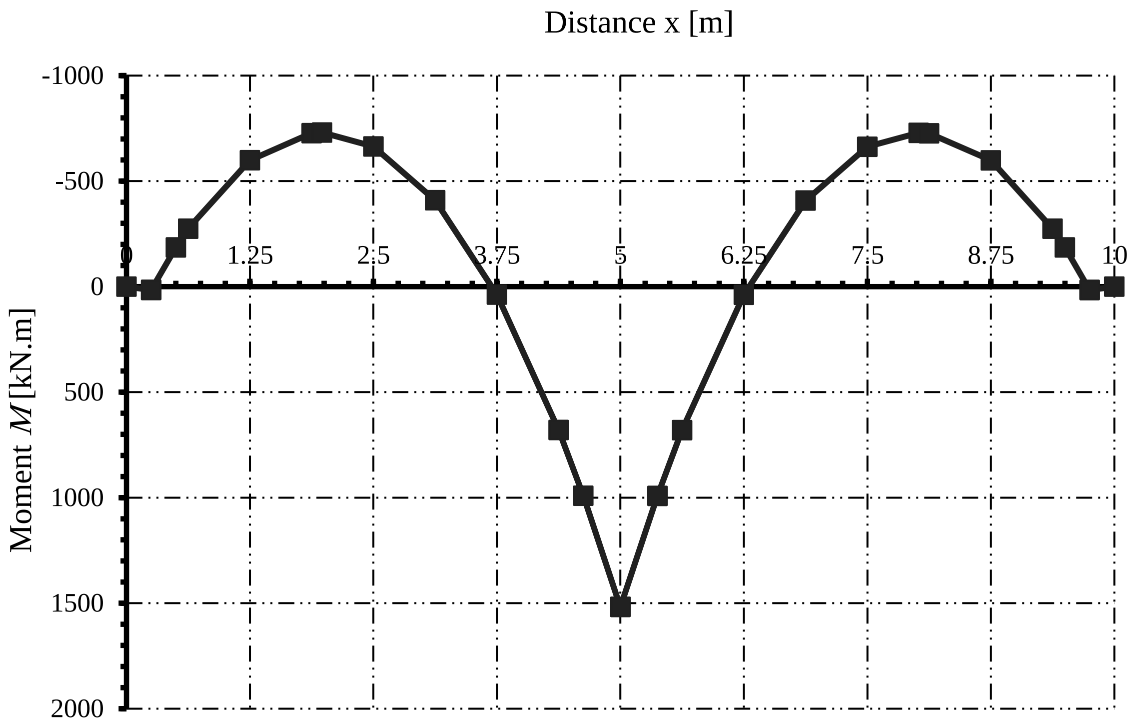


Figure 10.24 Moments

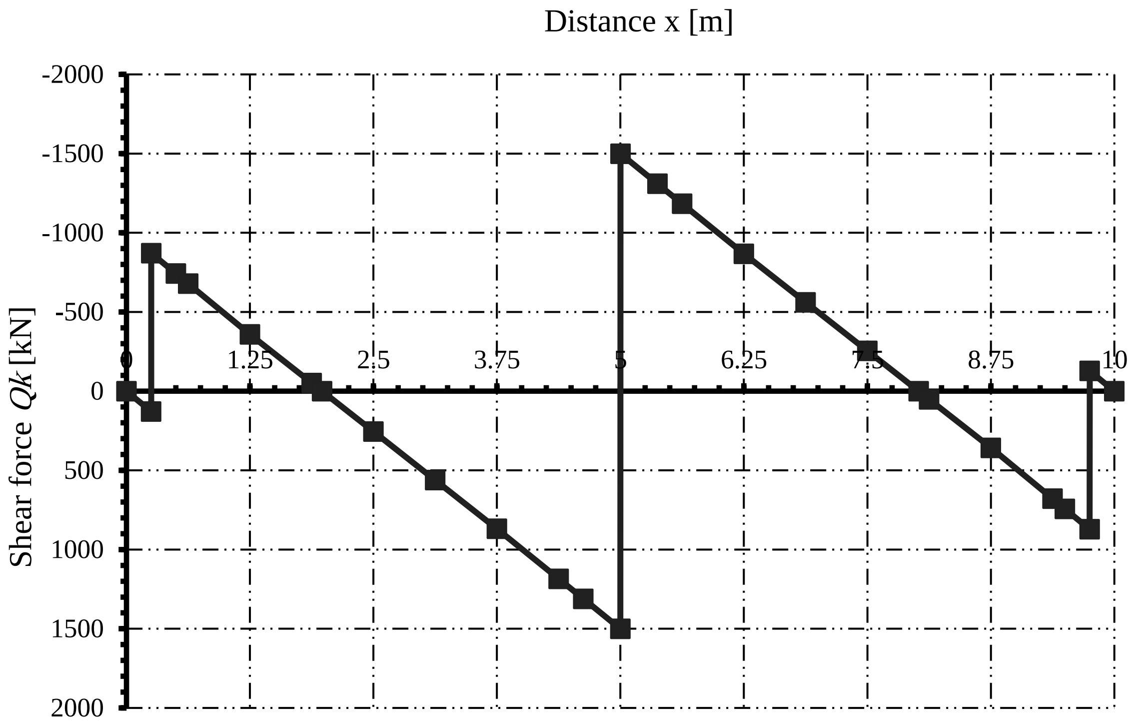


Figure 10.25 Shear forces

10.2.7 Example 5: Design of a combined footing for three equal columns

Design a combined footing for three equal columns, each of 0.50×0.50 [m], reinforced by 4Φ19, carrying a load of P = 1500 [kN] as shown in Figure 10.26. The distance center to center of columns is 3.0 [m]. The allowable soil pressure is q_{all} = 210 [kN/m^2] at a depth of D_f= 2.0 [m] and average unit weight of the soil and concrete is γ_a = 20 [kN/m^3].

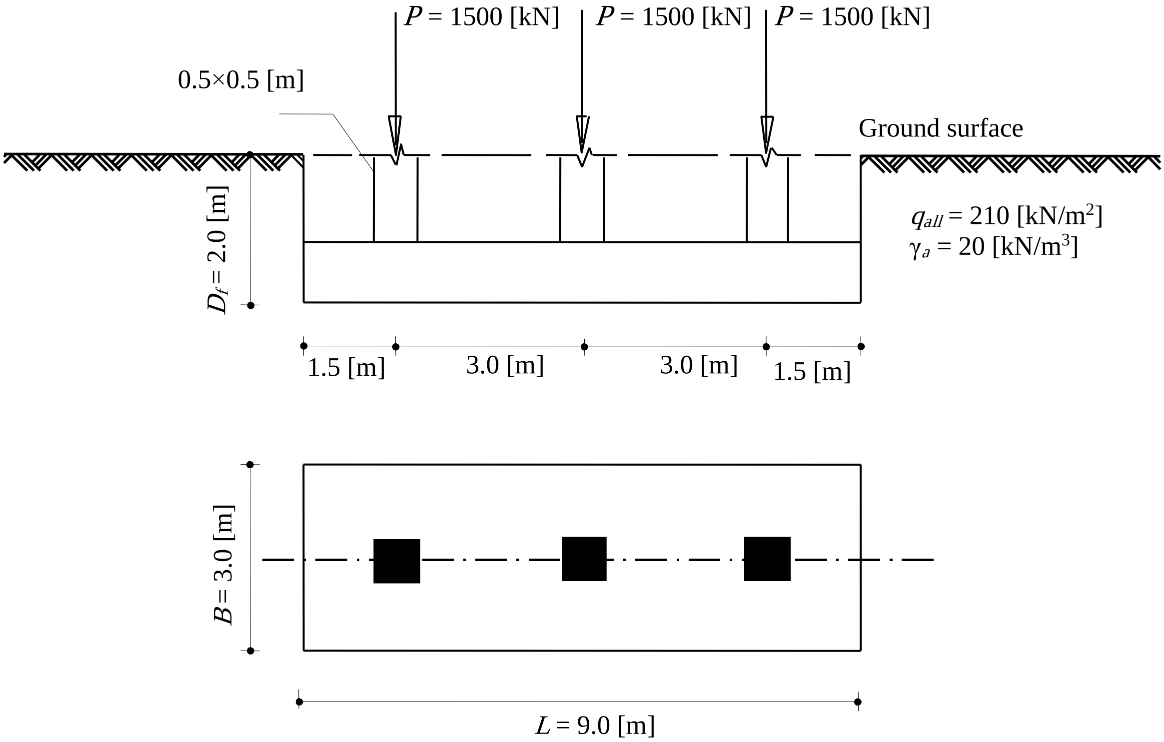


Figure 10.26 Combined rectangular footing for three columns

10.2.7.1.1 Determining footing sides L and B

Resultant of loads R at the ground surface level is given by:

$$R = 3\,P = 3\times1500 = 4500 \text{ [kN]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 210 - 20\times2.0 = 170 \text{ [kN/m}^2\text{]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R}{q_{net}} = \frac{4500}{170} = 26.47 \text{ [m}^2\text{]}$$

take A_f = 3.0×9.0 = 27.0 [m^2] rectangular combined footing

10.2.7.2 Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{4500}{27} = 166.67 \text{ [kN/m}^2\text{]} = 0.167 \text{ [MN/m}^2\text{]}$$

Now, the rectangular footing is treated as a beam footing. The contact pressure per meter is given by:

$$q = q_o\, B = 0.167 \times 3.0 = 0.5 \text{ [MN/m]}$$

Figure 10.27 shows the load diagram and moment diagram for the beam footing.

10.2.7.3 Determining the maximum moment M_{max}

From Figure 10.27, the maximum bending moment in the longitudinal direction occurs at the middle of the footing, which is:

$$M_{\max} = P\,\frac{c^2}{2} = 0.5\,\frac{1.25^2}{2} = 0.391 \text{ [MN.m]}$$

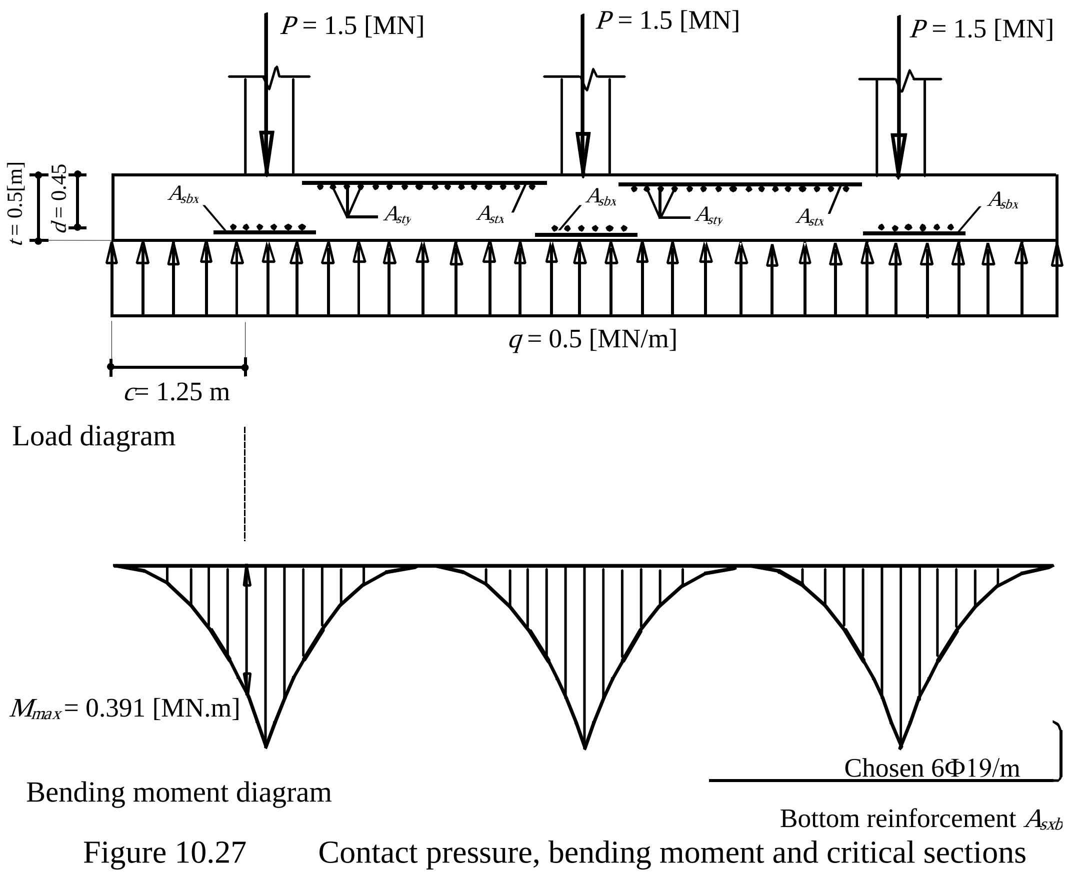


Figure 10.27 Contact pressure, bending moment and critical sections

10.2.7.4 Determining the depth required to resist the moment d_m

From Table 2 for f_c = 9.5 [MN/m^2] and f_s = 200 [MN/m^2], the coefficient k_1 to obtain the section depth at balanced condition is k_1 = 0.766, while the coefficient k_2 [MN/m^2] to obtain the tensile reinforcement for singly reinforced section is k_2 = 172 [MN/m^2].

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.391}{3.0}} = 0.28 \text{ [m]}$$

Take d = 0.45 [m] >d_m = 0.28 [m], then the section is designed as singly reinforced section.

The corresponding k_1 for d = 0.45 [m] is given by:

$$k_1 = 0.45 \sqrt{\frac{3.0}{0.391}} = 1.25$$

From Table 2 at k_1 = 1.27, f_c = 5.0 [MN/m^2] and k_2 = 182

10.2.7.5 Check for punching shear

The critical punching shear section lies on a perimeter at a distance d/2 = 0.225 [m] from the face of the column as shown in Figure 10.28.

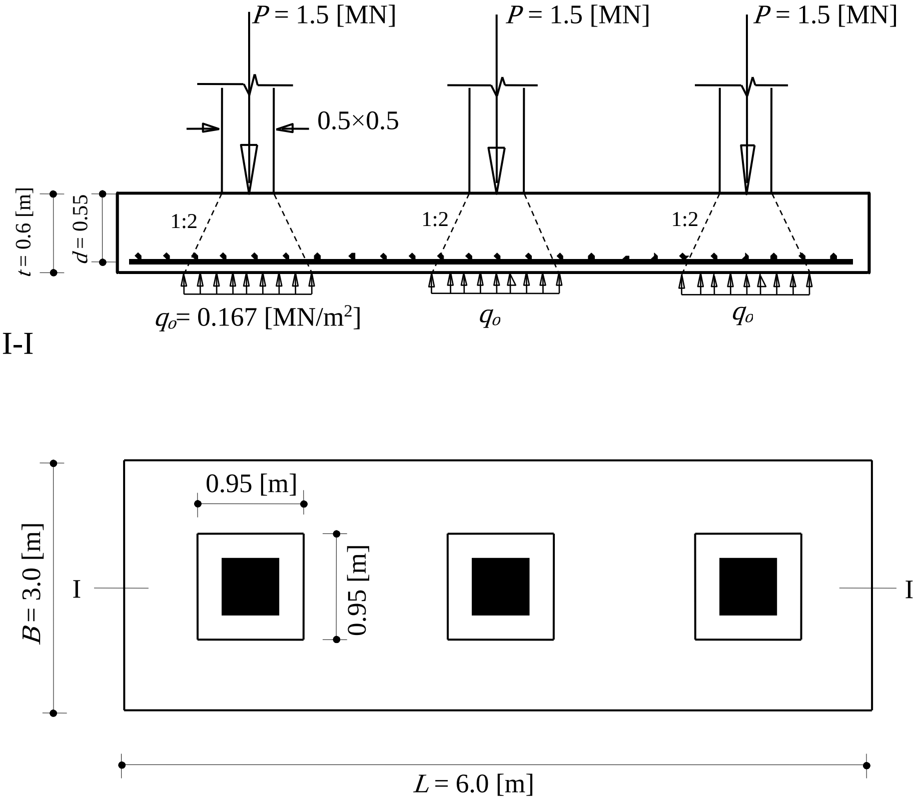


Figure 10.28 Critical section for punching shear

10.2.7.5.1 Geometry (Figure 10.28)

Effective depth of the section	d = 0.45 [m]
Column side	$a = b$ = 0.5 [m]
Area of critical punching shear section	$A_p = (a + d)^2$ = 0.9025 [m^2]
Perimeter of critical punching shear section	$b_o = 4\,(a + d)$ = 3.8 [m]

10.2.7.5.2 Loads and stresses

Column load $P = 1.5$ [MN]
Soil pressure under the column $q_o = 0.167$ [MN/m^2]
Main value of shear strength for concrete C 250 $q_{cp} = 0.9$ [MN/m^2]

10.2.7.5.3 Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.5 - 0.167 \times 0.9025 = 1.35 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o\, d}$$

$$q_p = \frac{1.35}{3.8 \times 0.45} = 0.79 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m^2] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9$ [MN/m^2] $> q_p = 0.79$ [MN/m^2], the section is safe for punching shear.

10.2.7.6 Computing the area of steel reinforcement

Minimum area of steel reinforcement $A_{smin} = 0.15\%\ A_c = 0.0015 \times 50 \times 100 = 7.5$ [cm^2/ m]
Take $A_{s\,min} = 5\Phi16 = 10.1$ [cm^2/ m].

The required area of steel reinforcement A_s is:

$$A_{sxb} = \frac{M_{\max}}{k_2\, d}$$

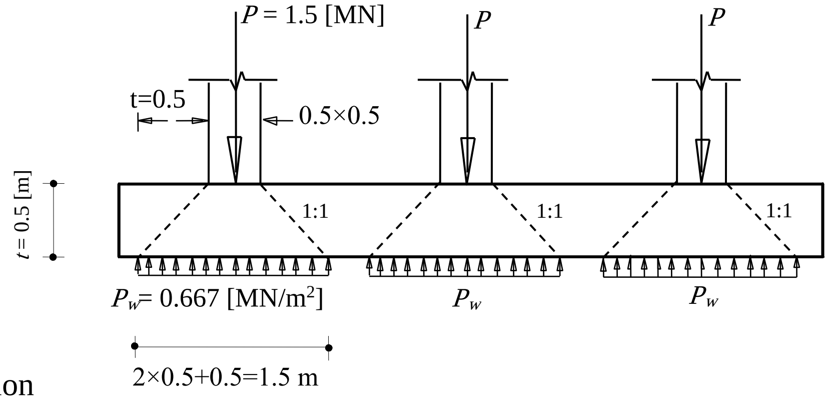
$$A_{sxb} = \frac{0.391}{182 \times 0.45} = 0.004774 \text{ [m}^2\text{/3.0 m]}$$

$$A_{sxt} = 47.74 \text{ [cm}^2\text{/3.0 m]} = 15.91 \text{[cm}^2\text{/m]}$$

Chosen steel $6\Phi19$/m = 17.0 [cm^2/m]

10.2.7.7 Computing the steel reinforcement in transverse direction

The transverse bending moment may be approximately determined at each column by assuming that the column load is distributed outward at 45 [°] from the face of the column over an appropriate area as shown in Figure 10.29. Both sides of this area should not be greater than the breadth of the footing.



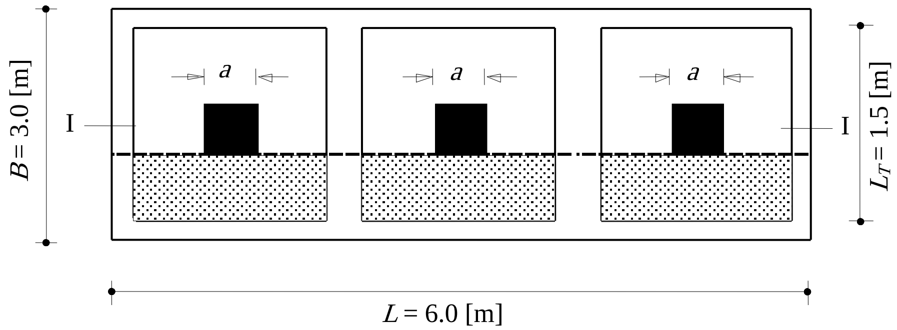


Figure 10.29 Sections for determining transverse moments

Side length of the loaded area L_T is given by:

$$L_T = a + 2\,t = 0.5 + 2\times0.5 = 1.5 \text{ [m]}$$

The transverse bending moment M_T under the column is given by:

$$M_T = \frac{P}{8}\frac{(L_T - a)^2}{L_T} = \frac{1.5}{8}\frac{(1.5 - 0.5)^2}{1.5} = 0.125 \text{ [MN.m]}$$

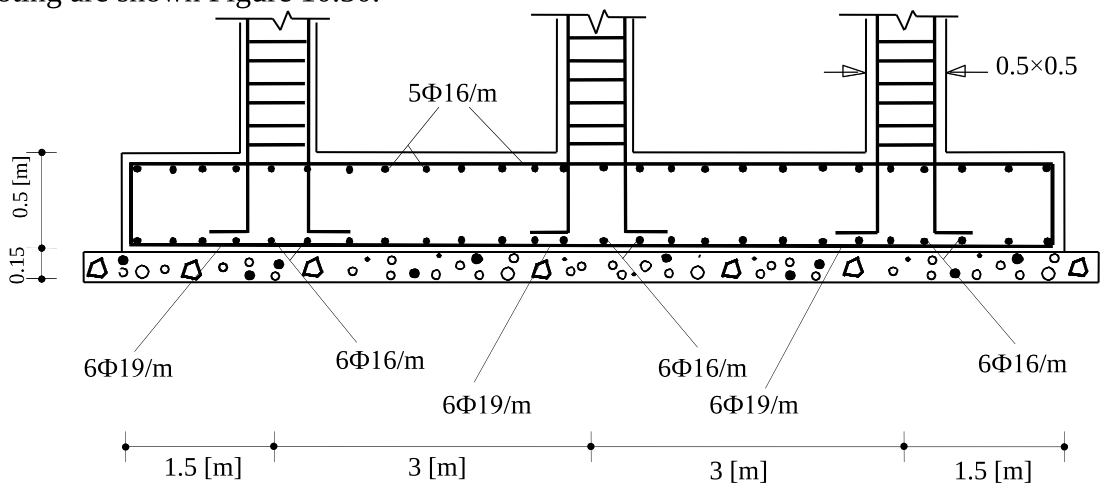
The required area of steel reinforcement in transverse direction under the column A_{sT} is:

$$A_{sT} = \frac{M_T}{k_2\,d}$$

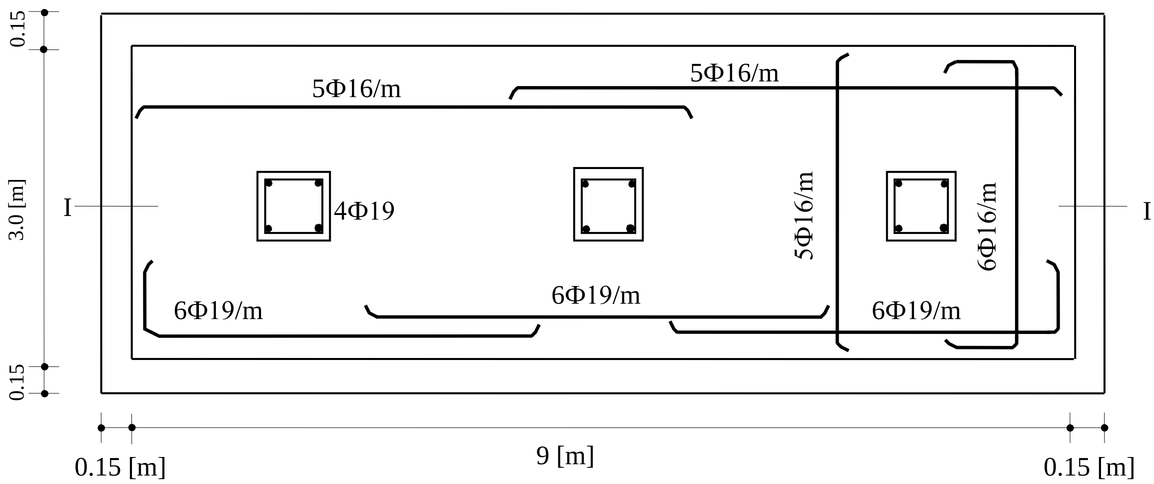
$$A_{sT} = \frac{0.125}{182 \times 0.45} = 0.001526 \text{ [m}^2\text{/1.5m]}$$

$$A_{sT} = 15.26 \text{ [cm}^2\text{/1.5m]} = 10.17 \text{ [cm}^2\text{/m]}$$

Chosen steel 6Φ16/m = 12.1 [cm^2/m]. The details of reinforcement in plan and section *a-a* through the footing are shown Figure 10.30.



a) Section I-I



b) Plan

Figure 10.30 Details of reinforcement in plan and section *a-a* through the footing