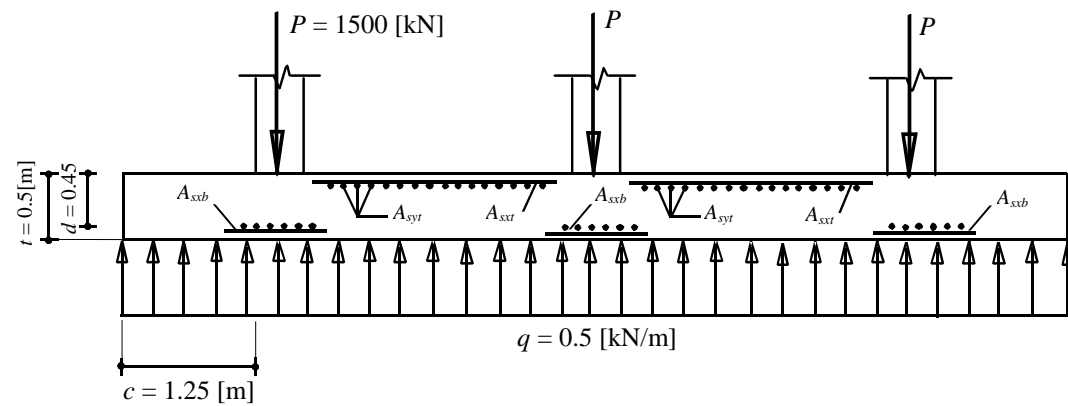
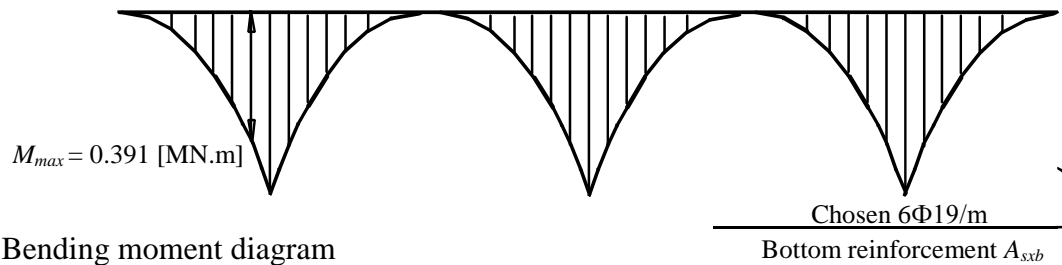


Calculation of Shallow Foundations



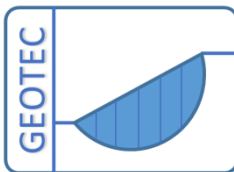
Load diagram



Bending moment diagram

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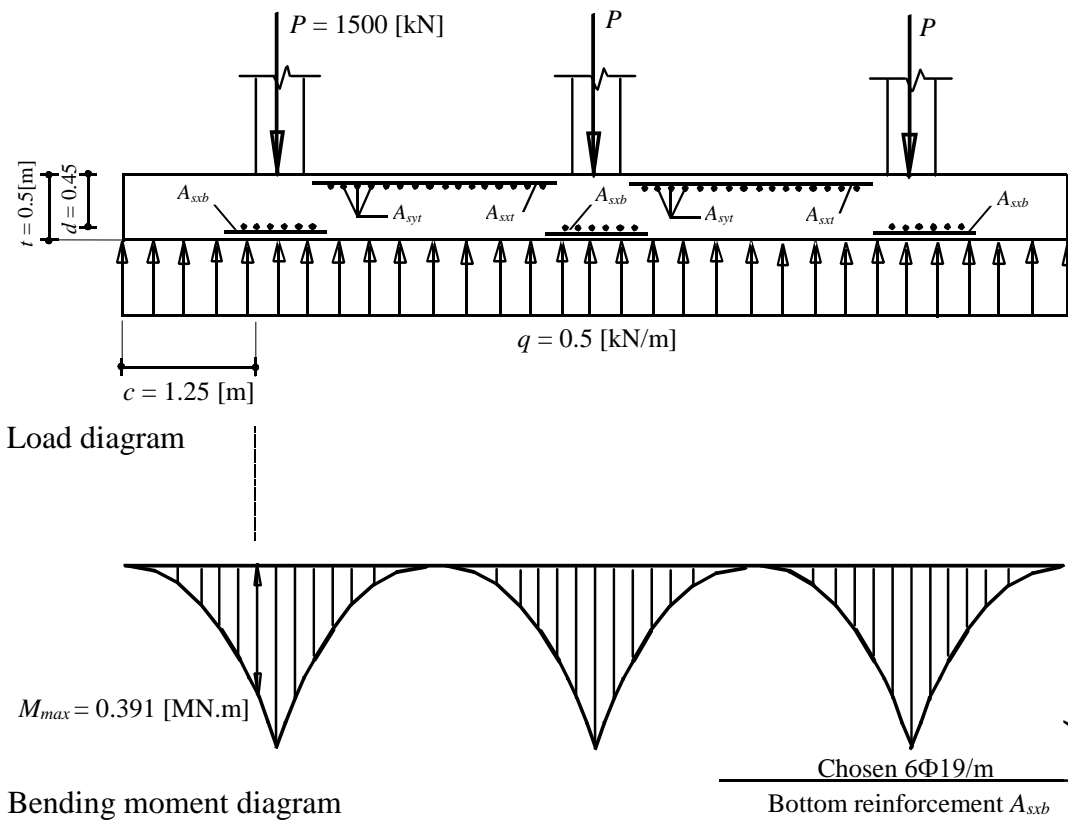
Preface

The purpose of this book is to provide a presentation of the shallow foundation design steps without complication in the form of simplified examples.

For more expansion in the analysis and design of foundations, especially if the previous conditions are not met, refer to our program “ELPLA” for analysis and design of foundations by the finite element method.

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February 2017

Calculation of Shallow Foundations



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2017

Calculation of Shallow Foundations

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تمهيد

تصميم الأساسات هي مادة تطبيقية، معظم المعلومات التي يعتمد عليها التصميم سبق دراستها وتناولها في فروع أخرى من الهندسة المدنية. هذا الكتاب محاولة لترتيب هذه المعلومات وتطويرها لتصميم الأساسات. الكتاب يبدأ بالأمثلة المحلولة لأنها أسرع وسيلة للفهم و ينتهي بالأساسيات النظرية بدون إسهاب حتى لا يشتت تفكير الدارس إذا كان ملماً بتلك المعارف.

تم تصميم القطاع الخرساني لجميع أساسات المسائل طبقاً للكود المصري لتصميم و تنفيذ الخرسانة المسلحة [1]، طريقة إجهادات التشغيل، لخرسانة من الرتبة C250، وحديد تسليح رتبته 36/52. بينما أبعاد الأساسات فقد تم تحديدها بناء على قدرة مقاومة التربة طبقاً للكود المصري لميكانيكا التربة و تنفيذ الأساسات [2].

لكي يكون التصميم مقبولاً و آمناً تم مراعاة الشروط التالية في جميع المسائل:

- 1- الأساس متزن إنشائياً و محدد إستاتيكياً، أي أن الفعل و رد الفعل معلومان في المسألة.
- 2- تتطابق محصلة الأحمال على الأساس مع مركز ثقل شكل الأساس لضمان توزيع منتظم لإجهاد التماس.
- 3- القطاع الخرساني يستطيع مقاومة العزم الحاني و القص الثاقب المعرض لهما بدون تسليح إضافي.

الغرض من الكتاب هو تقديم عرض لخطوات تصميم الأساسات الضحلة خالي من التعقيد في صورة مسائل مبسطة. و لمزيد من التوسع في تحليل و تصميم الأساسات خاصة إذا لم تتوفر الشروط السابقة يرجى الرجوع إلى برنامجنا لتحليل و تصميم الأساسات باستخدام العناصر المحددة *إبلا* [3]، وكذلك الاستعانة بكتبتنا: دليل المستخدم لبرنامج *إبلا* [4]، تدريبات برنامج *إبلا* [5]، النظريات المستخدمة في صياغة برنامج *إبلا* [6]، تصميم الخرسانة المسلحة [7]، أمثلة للتحقق و توضيح برنامج *إبلا* [8].

المؤلف

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فبراير 2017م

Summary of general requirements used in the design of shallow foundations

- A thin plain concrete of thickness 0.15 [m] is chosen under all foundations and is not considered in any calculation.
- Design of section for all foundations is carried out only for the moment and punching shear other checks are not required.
- Minimum footing thickness is $t = 0.3$ [m] and must be not less than the smallest side of the column. For more safety, footing thickness may be taken as $t = 0.5$ [m].
- Minimum foundation level from the ground surface is $D_f = 0.8$ [m].
- Minimum area of steel per meter is $A_{min} = 0.15$ [%] \times concrete section = $0.0015 \times t \times 100$ [cm²/m].
- Maximum area of steel per meter is $A_{max} = 1.25$ [%] \times concrete section = $0.0125 \times t \times 100$ [cm²/m].
- Minimum diameter of steel bars used in foundation is $\Phi 13$ [mm]
- Minimum No. of steel bars is 5
- Concrete cover + 1/2 bar diameter $c = 5$ [cm].
- Design of section for moment is carried out according to *ECP*, working stress method using the following equations:

- The depth of singly reinforced section d [m] = $k_1 \sqrt{\frac{M \text{ [MN.m]}}{b \text{ [m]}}}$

- The area of tensile steel reinforcement A_s [m²] = $\frac{M \text{ [MN.m]}}{k_2 \times d \text{ [m]}}$

- where b is section width, M is moment about section. Coefficients k_1 and k_2 are given in Table 1 and Table 2, while A_s can be chosen from Table 3.

- All foundations are supposed to have material of concrete grade C250 and steel grade 36/52, where coefficients $k_1 = 0.766$ and $k_2 = 172$
- Allowable concrete punching strength is $q_{pall} = 0.9$ [MN/m²].

Calculation of Shallow Foundations

Table 1: Coefficients k_1 and k_2 for design of singly reinforced rectangular sections according to *ECP* working stress method ($f_s=140-180$ [MN/m²])

f_c [MN/m ²]	$f_s=140$ [MN/m ²]			$f_s=160$ [MN/m ²]			$f_s=180$ [MN/m ²]		
	k_1	k_2	ξ	k_1	k_2	ξ	k_1	k_2	ξ
2.0	2.454	132	0.176	2.586	152	0.158	2.711	171	0.143
2.5	2.018	130	0.211	2.121	150	0.190	2.219	170	0.172
3.0	1.727	129	0.243	1.810	148	0.220	1.890	168	0.200
3.5	1.518	127	0.273	1.588	147	0.247	1.654	166	0.226
4.0	1.361	126	0.300	1.420	145	0.273	1.477	165	0.250
4.5	1.238	125	0.325	1.289	144	0.297	1.339	164	0.273
5.0	1.139	124	0.349	1.184	143	0.319	1.228	162	0.294
5.5	1.058	123	0.371	1.098	142	0.340	1.137	161	0.314
6.0	0.990	122	0.391	1.026	141	0.360	1.061	160	0.333
6.5	0.932	121	0.411	0.964	140	0.379	0.996	159	0.351
7.0	0.882	120	0.429	0.911	139	0.396	0.940	158	0.368
7.5	0.838	119	0.446	0.865	138	0.413	0.892	157	0.385
8.0	0.800	118	0.462	0.825	137	0.429	0.849	156	0.400
8.5	0.766	118	0.477	0.789	136	0.443	0.811	155	0.415
9.0	0.736	117	0.491	0.757	136	0.458	0.778	154	0.429
9.5	0.708	116	0.504	0.728	135	0.471	0.747	153	0.442
10.0	0.684	116	0.517	0.702	134	0.484	0.720	153	0.455
10.5	0.661	115	0.529	0.678	134	0.496	0.695	152	0.467
11.0	0.640	115	0.541	0.657	133	0.508	0.673	151	0.478
11.5	0.621	114	0.552	0.637	132	0.519	0.652	151	0.489
12.0	0.604	114	0.563	0.618	132	0.529	0.632	150	0.500

Units in [MN] and [m]. To convert from [MN/m²] to [kg/cm²], multiply by 10

Calculation of Shallow Foundations

Table 2: Coefficients k_1 and k_2 for design of singly reinforced rectangular sections according to *ECP* working stress method ($f_s=200-240$ [MN/m²])

f_c [MN/m ²]	$f_s=200$ [MN/m ²]			$f_s=220$ [MN/m ²]			$f_s=240$ [MN/m ²]		
	k_1	k_2	ξ	k_1	k_2	ξ	k_1	k_2	ξ
2.0	2.831	191	0.130	2.946	211	0.120	3.057	231	0.111
2.5	2.313	189	0.158	2.403	209	0.146	2.490	229	0.135
3.0	1.966	188	0.184	2.040	208	0.170	2.111	227	0.158
3.5	1.718	186	0.208	1.780	206	0.193	1.840	226	0.179
4.0	1.532	185	0.231	1.585	204	0.214	1.637	224	0.200
4.5	1.387	183	0.252	1.433	203	0.235	1.478	222	0.220
5.0	1.270	182	0.273	1.311	201	0.254	1.351	221	0.238
5.5	1.175	181	0.292	1.211	200	0.273	1.247	220	0.256
6.0	1.095	179	0.310	1.127	199	0.290	1.160	218	0.273
6.5	1.027	178	0.328	1.057	197	0.307	1.086	217	0.289
7.0	0.968	177	0.344	0.996	196	0.323	1.022	216	0.304
7.5	0.917	176	0.360	0.943	195	0.338	0.967	214	0.319
8.0	0.873	175	0.375	0.896	194	0.353	0.919	213	0.333
8.5	0.833	174	0.389	0.855	193	0.367	0.876	212	0.347
9.0	0.798	173	0.403	0.818	192	0.380	0.838	211	0.360
9.5	0.766	172	0.416	0.785	191	0.393	0.803	210	0.373
10.0	0.738	171	0.429	0.755	190	0.405	0.772	209	0.385
10.5	0.712	171	0.441	0.728	189	0.417	0.744	208	0.396
11.0	0.688	170	0.452	0.704	189	0.429	0.719	207	0.407
11.5	0.666	169	0.463	0.681	188	0.439	0.695	207	0.418
12.0	0.646	168	0.474	0.660	187	0.450	0.674	206	0.429

Units in [MN] and [m]. To convert from [MN/m²] to [kg/cm²], multiply by 10

Calculation of Shallow Foundations

Table 3: Steel bars used in the foundation design

Φ [mm]	Area of cross-section in [cm ²]									
	1	2	3	4	5	6	7	8	9	10
16	2.01	4.02	6.03	8.04	10.10	12.10	14.10	16.10	18.10	20.10
19	2.84	5.67	8.50	11.30	14.20	17.00	19.90	22.70	25.50	28.40
22	3.80	7.60	11.40	15.20	19.00	22.80	26.60	30.40	34.20	38.00

Part A
Solved Examples

1- Bearing Capacity of the Soil

Solved Examples

Example 1: Determining the allowable bearing capacity of a strip footing

A long strip footing of $B = 2.8$ [m] wide is placed at a depth of $D_f = 1.2$ [m] in a soil of unit weight $\gamma = 18.5$ [kN/m³], cohesion $c = 5$ [kN/m²] and angle of shearing resistance $\phi = 28$ [°]. Determine the allowable bearing capacity of the footing for a factor of safety $F_b = 2.5$ [-], Figure 1.

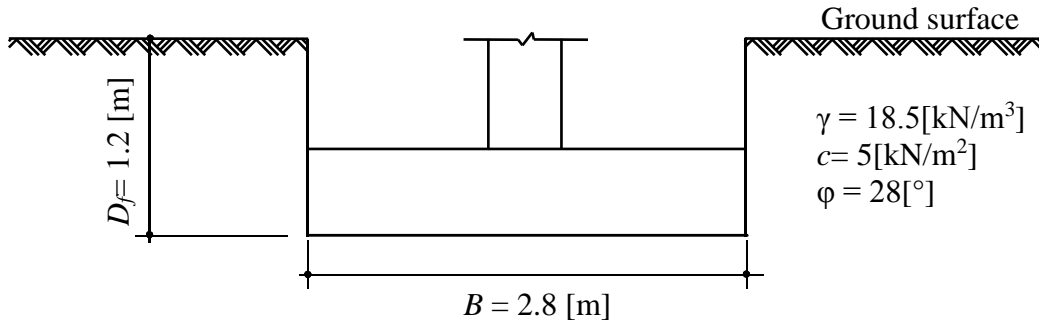


Figure 1: Strip footing

Bearing capacity factors N_q , N_c and N_γ

Bearing capacity factors can be determined form:

$$N_q = e^{\pi \tan \phi} \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$N_q = e^{\pi \tan 28} \tan^2 \left(45 + \frac{28}{2} \right) = 14.72$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_c = (14.72 - 1) \cot 28 = 25.8$$

$$N_\gamma = (N_q - 1) \tan \phi$$

$$N_\gamma = (14.72 - 1) \tan 28 = 7.3$$

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity of the soil for a strip footing is given by:

$$q_{ult} = c N_c + \gamma_1 D_f N_q + \gamma_2 B N_\gamma$$

$$q_{ult} = 5 \times 25.8 + 18.5 \times 1.2 \times 14.72 + 18.5 \times 2.8 \times 7.3$$

$$q_{ult} = 833.92 \text{ [kN/m}^2\text{]}$$

Solved Examples

Allowable bearing capacity q_{all}

The allowable bearing capacity of the soil is:

$$q_{all} = \frac{q_{ult}}{F_b} = \frac{833.92}{2.5} = 333.57 \text{ [kN/m}^2\text{]}$$

Solved Examples

Example 2: Determining the allowable bearing capacity of a square footing

A square footing of $L = 2.5$ [m] side is placed at a depth of $D_f = 1.25$ [m] in a sand of unit weight $\gamma = 18$ [kN/m³] and angle of shearing resistance $\phi = 35$ [°]. Determine the allowable bearing capacity of the footing for a factor of safety $F_b = 2.5$ [-], Figure 2.

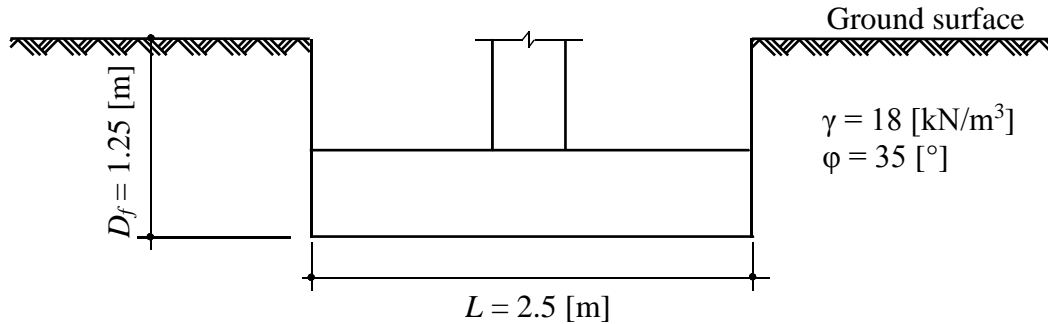


Figure 2: Square footing

Bearing capacity factors N_q and N_γ

Bearing capacity factors for the square footing are:

$$N_q = e^{\pi \tan \phi} \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$N_q = e^{\pi \tan 35} \tan^2 \left(45 + \frac{35}{2} \right) = 33.3$$

$$N_\gamma = (N_q - 1) \tan \phi$$

$$N_\gamma = (33.3 - 1) \tan 35 = 22.61$$

Shape factors λ_q and λ_γ

Shape factors for the square footing are:

$$\lambda_q = 1.3$$

$$\lambda_\gamma = 0.7$$

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity for sand is given by:

$$q_{ult} = \gamma_1 D_f N_q \lambda_q + \gamma_2 B N_\gamma \lambda_\gamma$$

$$q_{ult} = 18 \times 1.25 \times 33.3 \times 1.3 + 18 \times 2.5 \times 22.61 \times 0.7$$

$$q_{ult} = 1686 \text{ [kN/m}^2\text{]}$$

Solved Examples

Allowable bearing capacity q_{all}

The allowable bearing capacity of the soil is:

$$q_{all} = \frac{q_{ult}}{F_b} = \frac{1686}{2.5} = 674 \text{ [kN/m}^2\text{]}$$

Solved Examples

Example 3: Determining the allowable bearing capacity of a rectangular footing

Find the allowable bearing capacity of a rectangular footing having sides $L = 2.8$ [m] and $B = 2.1$ [m], Figure 3. The footing is placed at a depth of $D_f = 1.2$ [m] in a soil of cohesion $c = 2$ [kN/m²] and angle of shearing resistance $\phi = 25$ [°]. Unit weights of the soil above and under the foundation level are $\gamma_1 = 18.5$ [kN/m³] and $\gamma_2 = 8.69$ [kN/m³], respectively. The factor of safety is $F_b = 2.5$ [-].

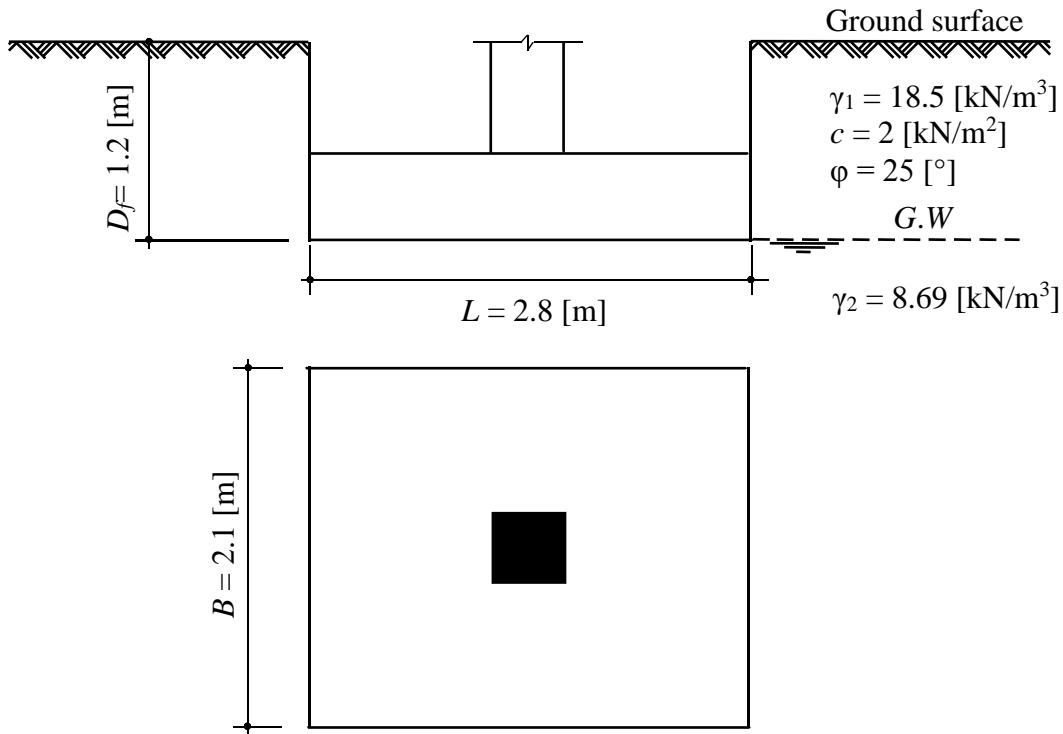


Figure 3: Rectangular footing

Shape factors λ_c , λ_q and λ_γ

Shape factors for the rectangular footing are:

$$\lambda_c = \lambda_q = 1 + 0.3 \frac{B}{L} = 1 + 0.3 \frac{2.1}{2.8} = 1.225$$

$$\lambda_\gamma = 1 - 0.3 \frac{B}{L} = 1 - 0.3 \frac{2.1}{2.8} = 0.775$$

Bearing capacity factors N_q , N_c and N_γ

Bearing capacity factors for the rectangular footing are:

$$N_q = e^{\pi \tan \phi} \tan^2 \left(45 + \frac{\phi}{2} \right)$$

Solved Examples

$$N_q = e^{\pi \tan 25} \tan^2 \left(45 + \frac{25}{2} \right) = 10.66$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_c = (10.66 - 1) \cot 25 = 20.72$$

$$N_\gamma = (N_q - 1) \tan \phi$$

$$N_\gamma = (10.66 - 1) \tan 25 = 4.5$$

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity of the soil is given by:

$$q_{ult} = c N_c \lambda_c + \gamma_1 D_f N_q \lambda_q + \gamma_2 B N_\gamma \lambda_\gamma$$

$$q_{ult} = 2 \times 20.72 \times 1.225 + 18.5 \times 1.2 \times 10.66 \times 1.225 + 8.69 \times 2.1 \times 4.5 \times 0.775$$

$$q_{ult} = 404.31 \text{ [kN/m}^2\text{]}$$

Allowable bearing capacity q_{all}

The allowable bearing capacity of the soil is:

$$q_{all} = \frac{q_{ult}}{F_b} = \frac{404.31}{2.5} = 161.72 \text{ [kN/m}^2\text{]}$$

Solved Examples

Example 4: Determining the allowable bearing capacity for rectangular footing

A rectangular footing of $L = 2.5$ [m] and $B = 2.0$ [m] is placed at depth $D_f = 2.0$ [m] in a soil of cohesion $c = 7$ [kN/m²] and angle of shearing resistance $\phi = 20$ [°]. The ground water level lies at depth $h = 1.75$ [m] under the ground surface as shown in Figure 4. Unit weight of the soil above the water table is $\gamma_1 = 18.5$ [kN/m³], while that under the water table is $\gamma_2 = 8.69$ [kN/m³]. Find the allowable bearing capacity. The factor of safety is $F_b = 2.5$ [-].

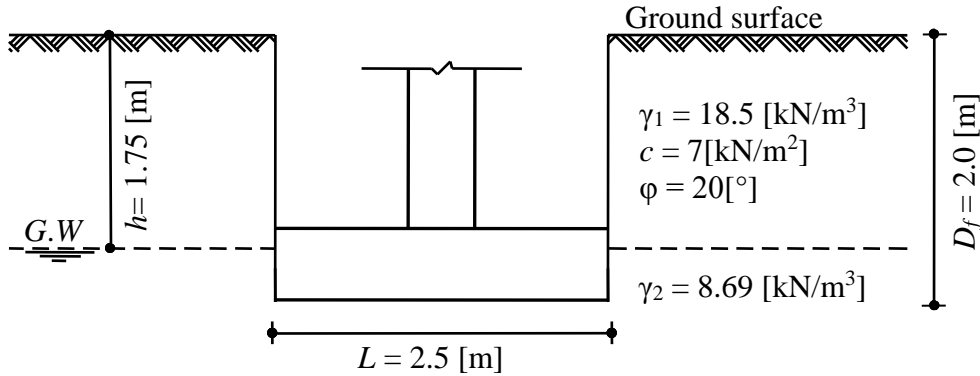


Figure 4: Bearing capacity for medium sand layer

Shape factors λ_c , λ_q and λ_γ

Shape factors for the rectangular footing are:

$$\lambda_c = \lambda_q = 1 + 0.3 \frac{B}{L} = 1 + 0.3 \frac{2.0}{2.5} = 1.24$$

$$\lambda_\gamma = 1 - 0.3 \frac{B}{L} = 1 - 0.3 \frac{2.0}{2.5} = 0.76$$

Soil weight until the foundation level is:

$$\gamma_1 D_f = \sum_{i=1}^n \gamma_{si} h_i = \gamma_{s1} h_1 + \gamma_{s2} h_2$$

$$\gamma_1 D_f = 18.5 \times 1.75 + 8.69 \times 0.25$$

$$\gamma_1 D_f = 34.55 \text{ [kN/m}^2\text{]}$$

Bearing capacity factors N_q , N_c and N_γ

Bearing capacity factors can be determined from:

$$N_q = e^{\pi \tan \phi} \tan^2 \left(45 + \frac{\phi}{2} \right)$$

Solved Examples

$$N_q = e^{\pi \tan 20} \tan^2 \left(45 + \frac{20}{2} \right) = 6.4$$

$$N_c = (N_q - 1) \cot \varphi$$

$$N_c = (6.4 - 1) \cot 20 = 14.84$$

$$N_\gamma = (N_q - 1) \tan \varphi$$

$$N_\gamma = (6.4 - 1) \tan 20 = 1.97$$

Ultimate bearing capacity

The ultimate bearing capacity of the soil is given by:

$$q_{ult} = c N_c \lambda_c + \gamma_1 D_f N_q \lambda_q + \gamma_2 B N_\gamma \lambda_\gamma$$

$$q_{ult} = 7 \times 14.84 \times 1.24 + 34.55 \times 6.4 \times 1.24 + 8.69 \times 2.0 \times 1.97 \times 0.76$$

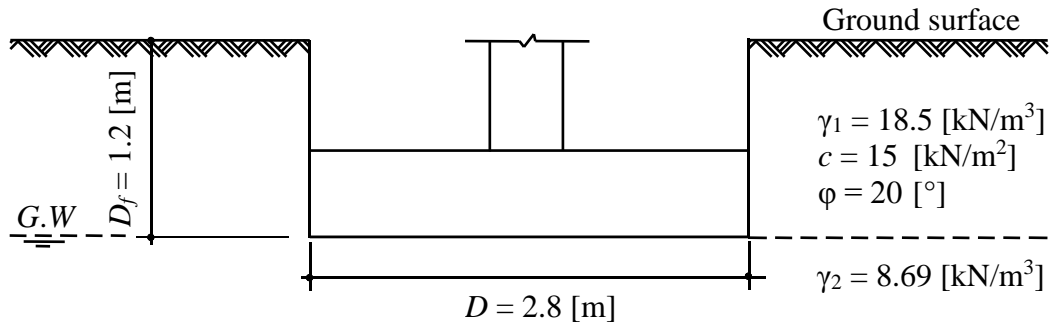
$$q_{ult} = 429.02 \text{ [kN/m}^2\text{]}$$

$$q_{all} = \frac{q_{ult}}{F_b} = \frac{429.02}{2.5} = 171.61 \text{ [kN/m}^2\text{]}$$

Solved Examples

Example 5: Determining the allowable bearing capacity of a circular footing

A circular footing of $D = 2.8$ [m] diameter is placed at a depth of $D_f = 1.2$ [m] in a soil of unit weight $\gamma_1 = 18.5$ [kN/m³], cohesion $c = 15$ [kN/m²] and angle of shearing resistance $\phi = 20$ [°]. The ground water level lies at the foundation level as shown in Figure 5. Unit weight of the soil under the ground level is $\gamma_2 = 8.69$ [kN/m³]. Determine the allowable bearing capacity of the footing for a factor of safety $F_b = 2.5$ [-].



Bearing capacity factors N_q , N_c and N_γ

Bearing capacity factors for the circular footing are:

$$N_q = e^{\pi \tan \phi} \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$N_q = e^{\pi \tan 20} \tan^2 \left(45 + \frac{20}{2} \right) = 6.4$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_c = (6.4 - 1) \cot 20 = 14.84$$

$$N_\gamma = (N_q - 1) \tan \phi$$

$$N_\gamma = (6.4 - 1) \tan 20 = 1.97$$

Shape factors λ_γ , λ_c and λ_q

Shape factors for the rectangular footing are:

$$\lambda_\gamma = 0.7$$

$$\lambda_c = \lambda_q = 1.3$$

Solved Examples

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity of the soil is given by:

$$q_{ult} = c N_c \lambda_c + \gamma_1 D_f N_q \lambda_q + \gamma_2 B N_\gamma \lambda_\gamma$$
$$q_{ult} = 15 \times 14.84 \times 1.3 + 18.5 \times 1.2 \times 6.4 \times 1.3 + 8.69 \times 2.8 \times 1.97 \times 0.7$$
$$q_{ult} = 507.64 \text{ [kN/m}^2\text{]}$$

Allowable bearing capacity q_{all}

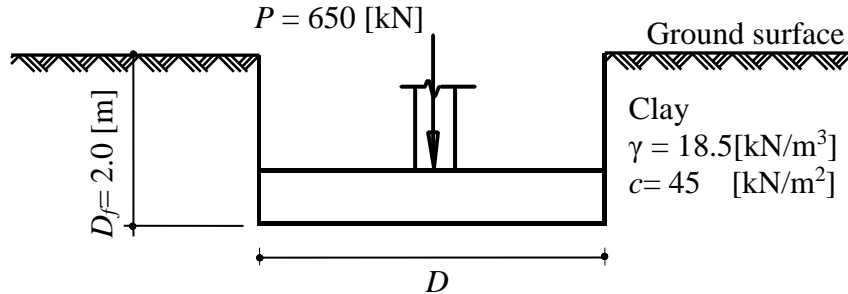
The allowable bearing capacity of the soil is:

$$q_{all} = \frac{q_{ult}}{F_b} = \frac{507.64}{2.5} = 203.06 \text{ [kN/m}^2\text{]}$$

Solved Examples

Example 6: Determining the diameter of a circular footing

Find the diameter of a circular footing D having a load of $P = 650$ [kN]. The footing is placed at a depth of $D_f = 2.0$ [m] in a clay layer of cohesion $c = 45$ [kN/m²] and unit weight $\gamma = 18.5$ [kN/m³], Figure 6. The factor of safety is $F_b = 2.5$ [-].



Bearing capacity factors N_c , N_q and N_γ

Bearing capacity factors can for the clay are:

$$N_c = 5.0, N_q = 1.0 \text{ and } N_\gamma = 0.0$$

Shape factors λ_c and λ_q

Shape factors for the circular footing are:

$$\lambda_c = \lambda_q = 1.3$$

Allowable bearing capacity of the soil q_{all}

The allowable bearing capacity of the soil is given by:

$$q_{all} = \frac{P}{A_f} = \frac{650}{\frac{\pi D^2}{4}} = \frac{827.61}{D^2}$$

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity may be determined from:

$$q_{ult} = q_{all} F_b = \frac{827.61}{D^2} 2.5 = \frac{2069.01}{D^2} \quad (1)$$

But the ultimate bearing capacity equation for the clay is given by:

$$q_{ult} = c N_c \lambda_c + \gamma_1 D_f N_q \lambda_q$$

$$q_{ult} = 45 \times 5.0 \times 1.3 + 18.5 \times 2.0 \times 1.0 \times 1.3$$

Solved Examples

$$q_{ult} = 340.6 \text{ [kN/m}^2\text{]} \quad (2)$$

Equating the above two equations 1 and 2 gives:

$$\frac{2069.01}{D^2} = 340.6$$

Then, the diameter of the footing is:

$$D = 2.47 \text{ [m]}$$

Solved Examples

Example 7: Determining the allowable load of a square footing due to eccentric load

Find the allowable load P_{all} of a square footing having a side $L = 2.5$ [m] with eccentricity of $e_x = 0.25$ [m]. The footing is placed at a depth of $D_f = 2.0$ [m] in a layer of cohesion $c = 10$ [kN/m²], angle of shearing resistance $\phi = 22$ [°] and unit weight of $\gamma = 17.5$ [kN/m³], Figure 7. The factor of safety is $F_b = 2.5$ [-].

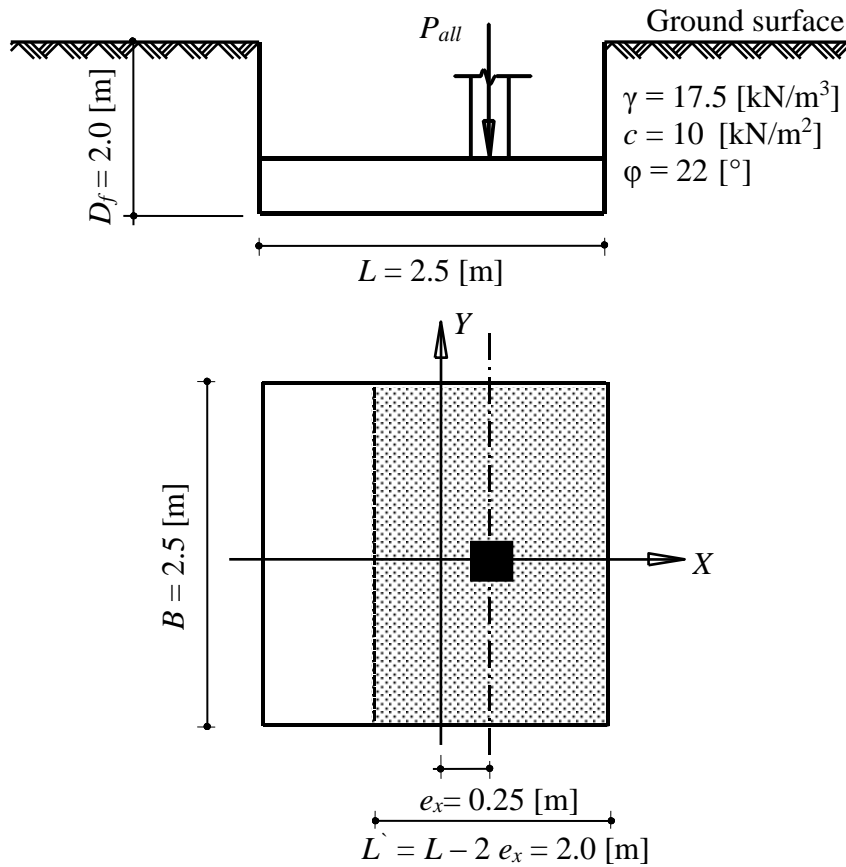


Figure 7: Square footing subjected to an eccentric load

Bearing capacity factors N_c , N_q and N_γ

Bearing capacity factors for the soil can be determined from:

$$N_q = e^{\pi \tan \phi} \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$N_q = e^{\pi \tan 22} \tan^2 \left(45 + \frac{22}{2} \right) = 7.82$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_c = (7.82 - 1) \cot 22 = 16.88$$

Solved Examples

$$N_\gamma = (N_q - 1) \tan \phi$$

$$N_\gamma = (7.82 - 1) \tan 22 = 2.76$$

Shape factors λ_c , λ_q and λ_γ

Shape factors for the square footing subjected to an eccentric load are:

$$\lambda_c = \lambda_q = 1 + 0.3 \frac{L}{B}$$

$$\lambda_c = \lambda_q = 1 + 0.3 \frac{2.0}{2.5} = 1.24$$

$$\lambda_\gamma = 1 - 0.3 \frac{L}{B} = 1 - 0.3 \frac{2.0}{2.5} = 0.76$$

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity of the soil is given by:

$$q_{ult} = c N_c \lambda_c + \gamma_1 D_f N_q \lambda_q + \gamma_2 L N_\gamma \lambda_\gamma$$

$$q_{ult} = 10 \times 16.88 \times 1.24 + 17.5 \times 2.0 \times 7.82 \times 1.24 + 17.5 \times 2.0 \times 2.76 \times 0.76$$

$$q_{ult} = 622.12 \text{ [kN/m}^2\text{]}$$

Allowable bearing capacity of the soil q_{all}

The allowable bearing capacity of the soil is given by:

$$q_{all} = \frac{q_{ult}}{F_b} = \frac{622.12}{2.5} = 248.85 \text{ [kN/m}^2\text{]}$$

Allowable load P_{all} of the footing

The load P_{all} of the footing is given by:

$$P_{all} = q_{all} \times A_f = q_{all} \times B \times L = 248.85 \times 2.5 \times 2.0 = 1244.24 \text{ [kN]}$$

Solved Examples

Example 8: Check the allowable bearing capacity due to a horizontal load

Check the allowable bearing capacity of a square footing having a side $L=2.0$ [m] and carrying a horizontal load of $H = 100$ [kN] and vertical load of $V = 500$ [kN]. The footing is placed at a depth of $D_f = 1.6$ [m] in a soil layer of cohesion $c = 30$ [kN/m²], angle of shearing resistance $\phi = 10$ [°] and unit weight of $\gamma = 17.5$ [kN/m³], Figure 8. The factor of safety is $F_b = 2.5$ [-].

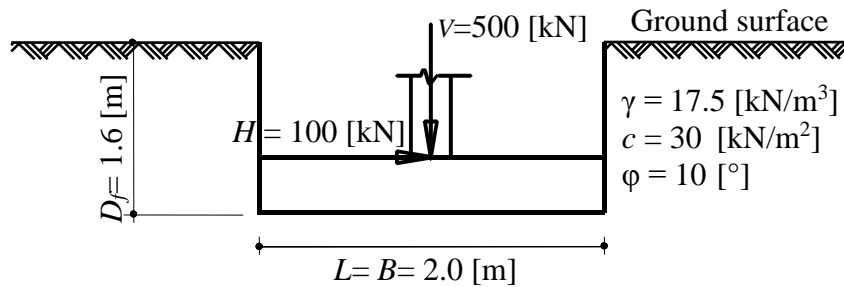


Figure 8: Square footing subjected to horizontal load

Horizontal and vertical ultimate bearing capacities H_b , V_b :

The horizontal and vertical ultimate bearing capacities are given by:

$$H_b = F_b H = 2.5 \times 100 = 250 \text{ [kN]}$$

$$V_b = F_b V = 2.5 \times 500 = 1250 \text{ [kN]}$$

Allowable bearing capacity q_{all}

The allowable bearing capacity of the soil is given by:

$$q_{all} = \frac{V}{A_f} = \frac{500}{2.0^2} = 125 \text{ [kN/m}^2\text{]} \quad (1)$$

Bearing capacity factors N_q , N_c and N_γ

Bearing capacity factors for the soil can be determined form:

$$N_q = e^{\pi \tan \phi} \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$N_q = e^{\pi \tan 10} \tan^2 \left(45 + \frac{10}{2} \right) = 2.47$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_c = (2.47 - 1) \cot 10 = 8.34$$

$$N_\gamma = (N_q - 1) \tan \phi$$

Solved Examples

$$N_\gamma = (2.47 - 1) \tan 10 = 0.26$$

Inclination factors i_q and i_c

The inclination factors for the soil can be determined from:

$$i_q = \left[1 - 0.7 \frac{H_b}{V_b + A_f c \cot \phi} \right]^3$$

$$i_q = \left[1 - 0.7 \frac{250}{1250 + 4.0 \times 30 \times \cot 10} \right]^3 = 0.75$$

$$i_\gamma = \left[1 - \frac{H_b}{V_b + A_f c \cot \phi} \right]^3$$

$$i_\gamma = \left[1 - \frac{250}{1250 + 4.0 \times 30 \times \cot 10} \right]^3 = 0.66$$

$$i_c = i_q - \frac{1 - i_q}{N_q - 1}$$

$$i_c = 0.75 - \frac{1 - 0.75}{2.47 - 1} = 0.58$$

Shape factors λ_c , λ_q and λ_γ

Shape factors for the square footing are:

$$\lambda_c = \lambda_q = 1.3$$

$$\lambda_\gamma = 0.7$$

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity of the soil is given by:

$$q_{ult} = c N_c \lambda_c i_c + \gamma_1 D_f N_q \lambda_q i_q + \gamma_2 B N_\gamma \lambda_\gamma i_\gamma$$

$$q_{ult} = 30 \times 8.34 \times 1.3 \times 0.58 + 17.5 \times 1.6 \times 2.47 \times 1.3 \times 0.75 + 17.5 \times 2.0 \times 0.26 \times 0.7 \times 0.66$$

$$q_{ult} = 260.29 \text{ [kN/m}^2\text{]}$$

Solved Examples

Allowable bearing capacity q_{all}

The allowable bearing capacity of the soil is:

$$q_{all} = \frac{q_{ult}}{F_b} = \frac{260.29}{2.5} = 104.11 \text{ [kN/m}^2\text{]} \quad (2)$$

Comparing the bearing capacities obtained from Equations 1 and 2 shows that the footing dimensions are unsafe. The footing size must be increased.

Solved Examples

Example 9: Determining the side of a square footing having an eccentric load

Find the side of a square footing L having an allowable load of $P_{all} = 620$ [kN] with eccentricity of $e_x = 0.25$ [m]. The footing is placed at a depth of $D_f = 2.0$ [m] in a clay layer of cohesion $c = 43$ [kN/m²] and unit weight $\gamma = 17.5$ [kN/m³], Figure 9. The factor of safety is $F_b = 2.5$ [-].

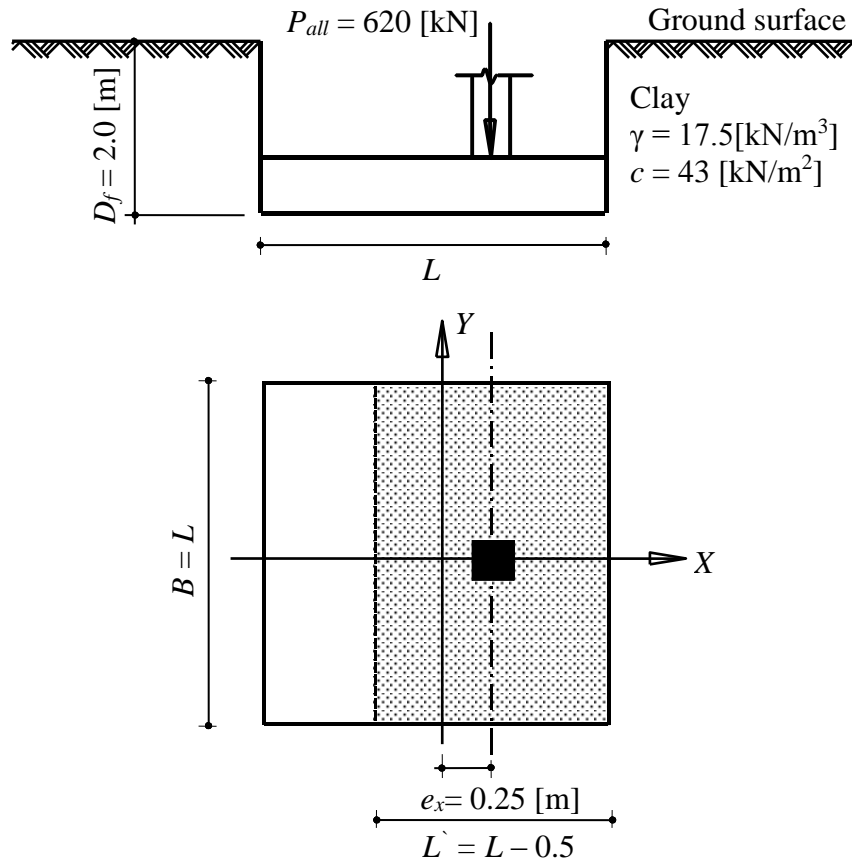


Figure 9: Square footing subjected to an eccentric load

Bearing capacity factors N_c , N_q and N_γ

Bearing capacity factors for the clay can be determined from:

$$N_c = 5.0, N_q = 1.0 \text{ and } N_\gamma = 0.0$$

Shape factors λ_c and λ_q

Shape factors for the square footing subjected to an eccentric load are:

$$\lambda_c = \lambda_q = 1 + 0.3 \frac{L'}{B}$$

$$\lambda_c = \lambda_q = 1 + 0.3 \frac{L - 0.5}{L} = 1.3 - \frac{0.15}{L}$$

Solved Examples

Allowable bearing capacity of the soil q_{all}

The allowable bearing capacity of the soil is given by:

$$q_{all} = \frac{P_{all}}{A_f} = \frac{620}{B L} = \frac{620}{L(L-0.5)} = \frac{620}{L^2 - 0.5 L}$$

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity may be determined from:

$$q_{ult} = q_{all} F_b = \frac{620}{L^2 - 0.5 L} \cdot 2.5 = \frac{1550}{L^2 - 0.5 L}$$

But the ultimate bearing capacity equation for the clay is given by:

$$q_{ult} = c N_c \lambda_c + \gamma_1 D_f N_q \lambda_q$$

$$q_{ult} = 43 \times 5.0 \times \left(1.3 - \frac{0.15}{L}\right) + 17.5 \times 2.0 \times 1.0 \times \left(1.3 - \frac{0.15}{L}\right)$$

$$q_{ult} = 250 \times \left(1.3 - \frac{0.15}{L}\right)$$

Equating the above two equations gives:

$$\frac{1550}{L^2 - 0.5 L} = 250 \times \left(1.3 - \frac{0.15}{L}\right)$$

or

$$1.3 L^2 - 0.8 L - 6.125 = 0.0$$

Solving the above equation leads to:

$$L = 2.5 \text{ [m]}$$

Solved Examples

Example 10: Determining the eccentricity of a square footing

Find the allowable eccentricity e_x for a square footing of a side $L = 2.5$ [m] and an allowable load of $P_{all} = 620$ [kN]. The footing is placed at depth of $D_f = 2.0$ [m] in a clay layer of cohesion $c = 43$ [kN/m²] and unit weight $\gamma = 17.5$ [kN/m³], Figure 10. The factor of safety is $F_b = 2.5$ [-].

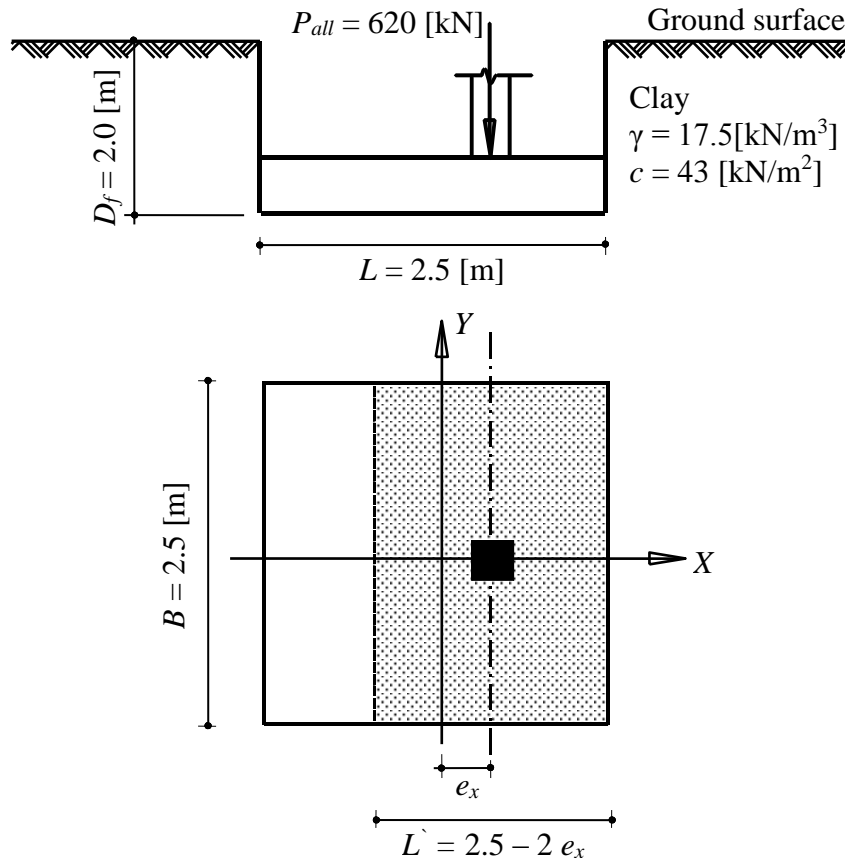


Figure 10: Footing subjected to an eccentric load

Bearing capacity factors N_c , N_q and N_γ

Bearing capacity factors for the clay can be determined from:

$$N_c = 5.0, N_q = 1.0 \text{ and } N_\gamma = 0.0$$

Shape factors λ_c and λ_q

Shape factors for the square footing subjected to an eccentric load are:

$$\lambda_c = \lambda_q = 1 + 0.3 \frac{L'}{B}$$

$$\lambda_c = \lambda_q = 1 + 0.3 \frac{L'}{2.5} = 1 + 0.12 L'$$

Solved Examples

Allowable bearing capacity of the soil q_{all}

The allowable bearing capacity of the soil is given by:

$$q_{all} = \frac{P}{A_f} = \frac{620}{2.5 \hat{L}} = \frac{248}{\hat{L}}$$

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity may be determined from:

$$q_{ult} = q_{all} F_b = \frac{248}{\hat{L}} \cdot 2.5 = \frac{620}{\hat{L}}$$

But the ultimate bearing capacity equation for the clay is given by:

$$q_{ult} = c N_c \lambda_c + \gamma_1 D_f N_q \lambda_q$$

$$q_{ult} = 43 \times 5 \times (1 + 0.12 \hat{L}) + 17.5 \times 2.0 \times 1.0 \times (1 + 0.12 \hat{L})$$

$$q_{ult} = 250 \times (1 + 0.12 \hat{L}) \quad (1)$$

Equating the above two equations 1 and 2 gives:

$$\frac{620}{\hat{L}} = 250 \times (1 + 0.12 \hat{L})$$

or

$$0.12 \hat{L}^2 + \hat{L} - 2.48 = 0.0 \quad (2)$$

Solving the above equation leads to:

$$\hat{L} = 2.0 [\text{m}]$$

but

$$\hat{L} = L - 2 e_x$$

$$2.0 = 2.5 - 2 e_x$$

$$e_x = 0.25 [\text{m}]$$

Solved Examples

Example 11: Determining the allowable horizontal force

Find the allowable horizontal force H that can be carried by a square footing of a side $L = 2.5$ [m]. The footing carries a vertical load of $V = 442$ [kN] and rests on a clay layer having cohesion of $c = 40$ [kN/m²] and unit weight of $\gamma = 18$ [kN/m³]. Foundation depth is $D_f = 2.0$ [m] from the ground surface, Figure 11. The factor of safety is $F_b = 2.5$ [-].

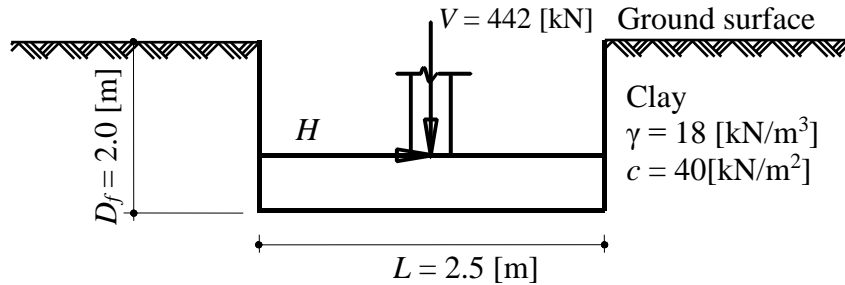


Figure 11: Square footing subjected to horizontal load

Ultimate bearing capacity V_b , H_b and q_{ult} :

The vertical ultimate bearing capacity is given by:

$$V_b = F_b V = 2.5 \times 442 = 1105 \text{ [kN]}$$

While the ultimate bearing capacity of the soil is given by:

$$q_{ult} = \frac{V_b}{A_f} = \frac{1105}{2.5^2} = 176.8 \text{ [kN/m}^2\text{]}$$

Inclination factors i_c and i_q

The inclination factors for the clay are:

$$i_q = 1.0$$

$$i_c = 0.5 + 0.5 \sqrt{1 - \frac{H_b}{c A_f}}$$

$$i_c = 0.5 + 0.5 \sqrt{1 - \frac{H_b}{40 \times 2.5^2}}$$

The inclination factor i_c can be rewritten in relation to H_b as:

$$i_c = 0.5 + 0.5 \sqrt{1 - \frac{H_b}{250}} \quad (1)$$

Bearing capacity factors N_c , N_q and N_γ

Bearing capacity factors for the clay can be determined from:

Solved Examples

$$N_c = 5.0, N_q = 1.0 \text{ and } N_\gamma = 0.0$$

Shape factors λ_c and λ_q

Shape factors for the square footing are:

$$\lambda_c = \lambda_q = 1.3$$

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity of the soil is given by:

$$q_{ult} = c N_c \lambda_c i_c + \gamma_1 D_f N_q \lambda_q i_q$$
$$q_{ult} = 40 \times 5.0 \times 1.3 i_c + 18 \times 2.0 \times 1.0 \times 1.3 \times 1.0$$

or

$$q_{ult} = 260 i_c + 46.8 \quad (2)$$

Horizontal forces H_b and H

Substituting Eq. (1) in Eq. (2), gives:

$$176.8 = 260 \left(0.5 + 0.5 \sqrt{1 - \frac{H_b}{250}} \right) + 46.8$$

Solving the above equation, gives the horizontal ultimate bearing capacity as:

$$H_b = 250 \text{ [kN]}$$

The allowable horizontal force is given by:

$$H = \frac{H_b}{F_b} = \frac{250}{2.5} = 100 \text{ [kN]}$$

Solved Examples

Example 12: Determining the side of a square footing carrying an inclined load

Find the side L of a square footing carrying a vertical load of $V = 442$ [kN] and a horizontal load of $H = 100$ [kN]. The footing rests on a clay layer having cohesion of $c = 40$ [kN/m²] and unit weight of $\gamma = 18$ [kN/m³]. Foundation depth is $D_f = 2.0$ [m] from the ground surface, Figure 12. The factor of safety is $F_b = 2.5$ [-].

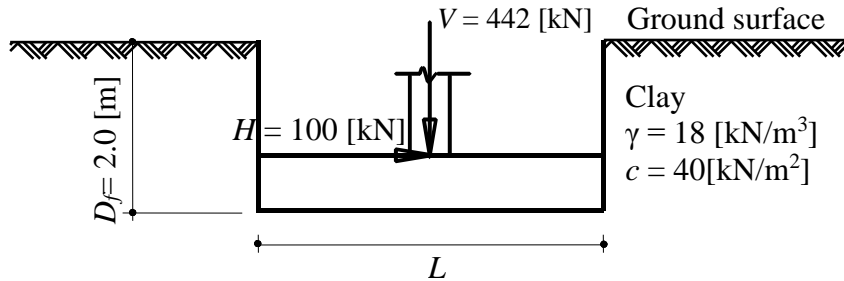


Figure 12: Square footing subjected to an inclined load

Ultimate bearing capacity H_b , V_b and q_{ult} :

The horizontal and vertical ultimate bearing capacities are given by:

$$H_b = F_b H = 2.5 \times 100 = 250 \text{ [kN]}$$

$$V_b = F_b V = 2.5 \times 442 = 1105 \text{ [kN]}$$

while the ultimate bearing capacity of the soil is given by:

$$q_{ult} = \frac{V_b}{A_f} = \frac{1105}{A_f} \quad (1)$$

Inclination factors i_q and i_c

The inclination factors for the clay are:

$$i_q = 1.0$$

$$i_c = 0.5 + 0.5 \sqrt{1 - \frac{H_b}{c A_f}}$$

$$i_c = 0.5 + 0.5 \sqrt{1 - \frac{250}{40 \times A_f}}$$

The inclination factor i_c can be rewritten in relation to A_f as:

$$i_c = 0.5 + 0.5 \sqrt{1 - \frac{6.25}{A_f}} \quad (2)$$

Solved Examples

Bearing capacity factors N_c , N_q and N_γ

Bearing capacity factors for the clay can be determined from:

$$N_c = 5.0, N_q = 1.0 \text{ and } N_\gamma = 0.0$$

Shape factors λ_c and λ_q

Shape factors for the square footing are:

$$\lambda_c = \lambda_q = 1.3$$

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity of the soil is given by:

$$q_{ult} = c N_c \lambda_c i_c + \gamma_1 D_f N_q \lambda_q i_q$$
$$q_{ult} = 40 \times 5.0 \times 1.3 i_c + 18 \times 2.0 \times 1.0 \times 1.3 \times 1.0$$

or

$$q_{ult} = 260 i_c + 46.8 \quad (3)$$

Footing area A_f and side L

Substituting Eq. (1) and (2) in Eq. (3), gives:

$$\frac{1105}{A_f} = 260 \left(0.5 + 0.5 \sqrt{1 - \frac{6.25}{A_f}} \right) + 46.8$$

Rearranging the equation, leads to:

$$A_f^2 - 19.85 A_f + 85 = 0$$

Solving the equation, gives:

$$A_f = 6.25 \text{ [m}^2\text{]} \text{ and } L = 2.5 \text{ [m]}$$

Solved Examples

Example 13: Determining the allowable bearing capacity for layered subsoil

A rectangular footing of $L = 2.5$ [m] and $B = 2.0$ [m] is placed at depth of $D_f = 2.0$ [m] in layered subsoil as shown in Figure 13. Find the allowable bearing capacity for a factor of safety $F_b = 2.5$ [-].

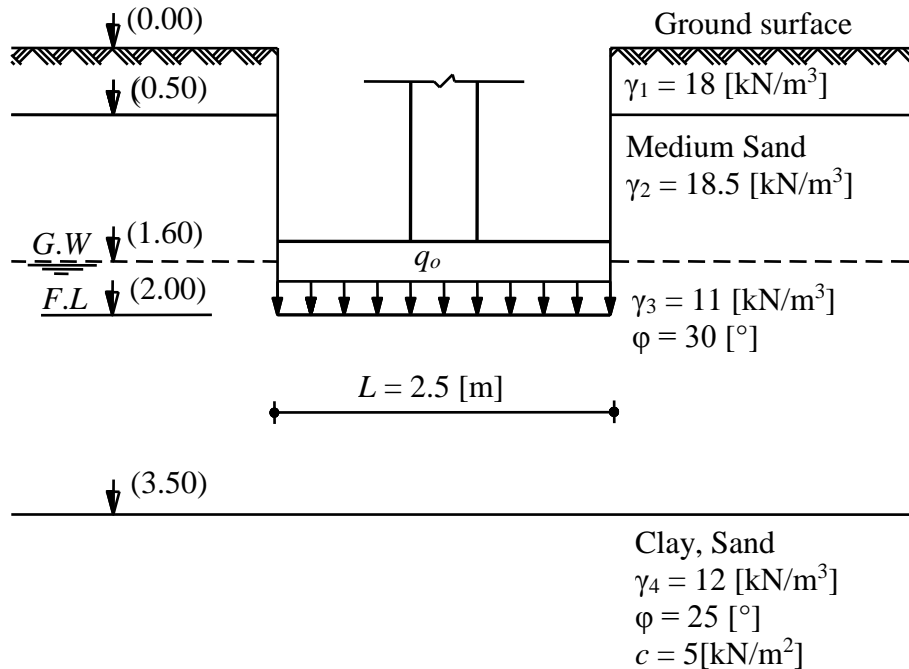


Figure 13: capacity for medium sand layer

1. Bearing capacity for medium sand layer

Assume all layers under the footing are replaced by equivalent medium sand layer.

Soil weight until the foundation level is given by:

$$\gamma_a D_f = \sum_{i=1}^n \gamma_i h_i = \gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3$$

$$\gamma_a D_f = 18 \times 0.5 + 18.5 \times 1.1 + 11 \times 0.4$$

$$\gamma_a D_f = 33.75 \text{ [kN/m}^2\text{]}$$

Shape factors λ_q and λ_γ

Shape factors for the rectangular footing are:

$$\lambda_q = 1 + 0.3 \frac{B}{L} = 1 + 0.3 \frac{2.0}{2.5} = 1.24$$

$$\lambda_\gamma = 1 - 0.3 \frac{B}{L} = 1 - 0.3 \frac{2.0}{2.5} = 0.76$$

Solved Examples

Bearing capacity factors N_q , N_c and N_γ for medium sand layer

Bearing capacity factors can be determined from:

$$N_q = e^{\pi \tan \phi} \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$N_q = e^{\pi \tan 30} \tan^2 \left(45 + \frac{30}{2} \right) = 18.4$$

$$N_\gamma = (N_q - 1) \tan \phi$$

$$N_\gamma = (18.4 - 1) \tan 30 = 10.05$$

Ultimate bearing capacity for medium sand q_{ult1}

The ultimate bearing capacity of the soil is given by:

$$q_{ult1} = \gamma_a D_f N_q \lambda_q + \gamma_4 B N_\gamma \lambda_\gamma$$

$$q_{ult1} = 33.75 \times 18.4 \times 1.24 + 11 \times 2.0 \times 10.05 \times 0.76$$

$$q_{ult1} = 938.07 \text{ [kN/m}^2\text{]}$$

$$q_o = \frac{q_{ult1}}{F_b} = \frac{938.07}{2.5} = 375.23 \text{ [kN/m}^2\text{]} \quad (1)$$

2. Bearing capacity for medium sand layer

New footing dimensions

$$B = 3.5 \text{ [m]}, L = 4 \text{ [m]}$$

Shape factors λ_c , λ_q and λ_γ

Shape factors for the rectangular footing are:

$$\lambda_c = \lambda_q = 1 + 0.3 \frac{B}{L} = 1 + 0.3 \frac{3.5}{4.0} = 1.2625$$

$$\lambda_\gamma = 1 - 0.3 \frac{B}{L} = 1 - 0.3 \frac{3.5}{4.0} = 0.7375$$

Solved Examples

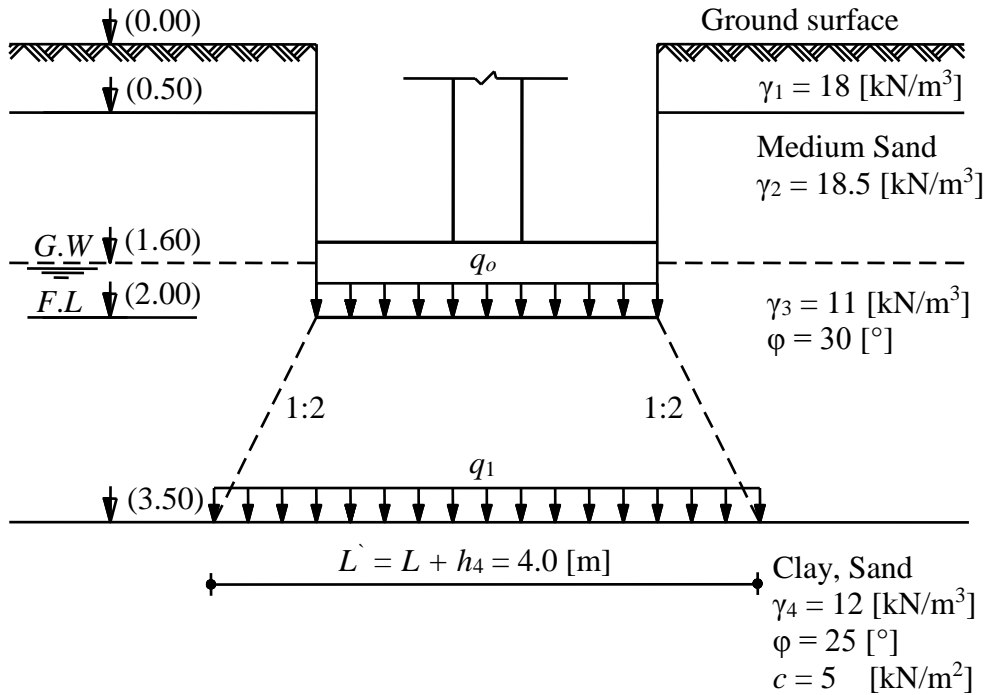


Figure 14: Bearing capacity for clay, sand layer

Assume all layers under the clay, sand layer are replaced by equivalent clay, sand layer as shown in Figure 14.

Soil weight until the foundation level is given by:

$$\gamma_a D_f = \sum_{i=1}^n \gamma_i h_i = \gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3 + \gamma_4 h_4$$

$$\gamma_a D_f = 18 \times 0.5 + 18.5 \times 1.1 + 11 \times 0.4 + 11 \times 1.5$$

$$\gamma_a D_f = 50.25 \text{ [kN/m}^2\text{]}$$

Bearing capacity factors N_q , N_c and N_γ for clay, sand layer

Bearing capacity factors can be determined from:

$$N_q = e^{\pi \tan \phi} \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$N_q = e^{\pi \tan 25} \tan^2 \left(45 + \frac{25}{2} \right) = 10.66$$

Solved Examples

$$N_c = (N_q - 1) \cot \phi$$

$$N_c = (10.66 - 1) \cot 25 = 20.72$$

$$N_\gamma = (N_q - 1) \tan \phi$$

$$N_\gamma = (10.66 - 1) \tan 25 = 4.5$$

Ultimate bearing capacity for clay, sand q_{ult2}

The ultimate bearing capacity of the soil is given by:

$$q_{ult2} = c N_c \lambda_c + \gamma_a D_f N_q \lambda_q + \gamma_4 B N_\gamma \lambda_\gamma$$

$$q_{ult2} = 5 \times 20.72 \times 1.2625 + 50.25 \times 10.66 \times 1.2625 + 12 \times 3.5 \times 4.5 \times 0.7375$$

$$q_{ult2} = 946.46 \text{ [kN/m}^2\text{]}$$

$$q_1 = \frac{q_{ult2}}{F_b} = \frac{946.46}{2.5} = 378.58 \text{ [kN/m}^2\text{]}$$

But

$$q_o = q_1 \frac{A_2}{A_1}$$

then

$$q_o = q_1 \frac{(B + h_4)(L + h_4)}{B L}$$

$$q_o = 378.58 \frac{(2 + 1.5)(2.5 + 1.5)}{2 \times 2.5}$$

$$q_o = 1060.04 \text{ [kN/m}^2\text{]} \quad (2)$$

By comparing equations (1) and (2) the allowable bearing capacity for the two layers (Medium sand and clay, sand) is the smallest value of q_o , then the allowable bearing capacity value is $q_o = 375.227$ [kN/ m2] and the ultimate bearing capacity value is $q_{ult} = 938.068$ [kN/ m2].

Solved Examples

Example 14: Determining the allowable vertical load

A rectangular footing of $L = 3.0$ [m] and $B = 2.5$ [m] and carrying a horizontal load of $H = 100$ [kN] and vertical load V . The footing is placed at depth of $D_f = 2.0$ [m] in layered subsoil as shown in Figure 15. Determine the allowable bearing capacity for a factor of safety $F_b = 2.5$ [-].

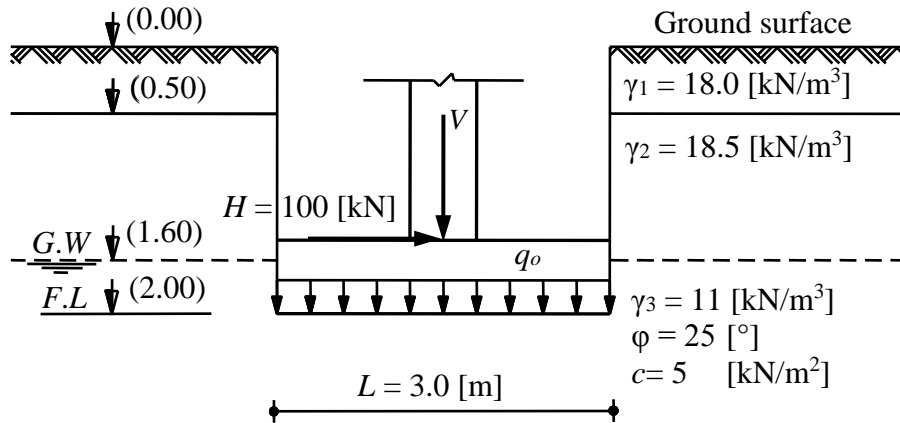


Figure 15: Rectangular footing subjected to an inclined load

Horizontal and vertical ultimate bearing capacities H_b , V_b :

The horizontal ultimate bearing capacity is given by:

$$H_b = F_b H = 2.5 \times 100 = 250 \text{ [kN]}$$

Average soil weight until the foundation level

Average soil weight until the foundation level is given by:

$$\gamma_a D_f = \sum_{i=1}^n \gamma_i h_i = \gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3$$

$$\gamma_a D_f = 18 \times 0.5 + 18.5 \times 1.1 + 11 \times 0.4$$

$$\gamma_a D_f = 33.75 \text{ [kN/m}^2\text{]}$$

Shape factors λ_q and λ_γ

Shape factors for the rectangular footing are:

$$\lambda_c = \lambda_q = 1 + 0.3 \frac{B}{L} = 1 + 0.3 \frac{2.5}{3.0} = 1.25$$

$$\lambda_\gamma = 1 - 0.3 \frac{B}{L} = 1 - 0.3 \frac{2.5}{3.0} = 0.75$$

Solved Examples

Bearing capacity factors N_q , N_c and N_γ

Bearing capacity factors can be determined from:

$$N_q = e^{\pi \tan \phi} \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$N_q = e^{\pi \tan 30} \tan^2 \left(45 + \frac{25}{2} \right) = 10.66$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_c = (10.66 - 1) \cot 25 = 20.72$$

$$N_\gamma = (N_q - 1) \tan \phi$$

$$N_\gamma = (10.66 - 1) \tan 25 = 4.5$$

Inclination factors i_q and i_c

The inclination factors for the soil can be determined from:

$$i_q = \left[1 - 0.7 \frac{H_b}{V_b + A_f c \cot \phi} \right]^3$$

$$i_q = \left[1 - 0.7 \frac{250}{1250 + 3.0 \times 2.5 \times 5 \times \cot 25} \right]^3 = 0.66$$

$$i_\gamma = \left[1 - \frac{H_b}{V_b + A_f c \cot \phi} \right]^3$$

$$i_\gamma = \left[1 - \frac{250}{1250 + 3.0 \times 2.5 \times 5 \times \cot 25} \right]^3 = 0.54$$

$$i_c = i_q - \frac{1 - i_q}{N_q - 1}$$

$$i_c = 0.66 - \frac{1 - 0.66}{10.66 - 1} = 0.63$$

Solved Examples

Ultimate bearing capacity q_{ult}

The ultimate bearing capacity of the soil is given by:

$$q_{ult} = c N_c \lambda_c i_c + \gamma_a D_f N_q \lambda_q i_q + \gamma_2 B N_\gamma \lambda_\gamma i_\gamma$$

$$q_{ult} = 5 \times 20.72 \times 1.25 \times 0.63 + 33.75 \times 10.66 \times 1.25 \times 0.66 + 11 \times 2.5 \times 4.5 \times 0.75 \times 0.54$$

$$q_{ult} = 428.52 \text{ [kN/m}^2\text{]}$$

Allowable bearing capacity q_{all}

The allowable bearing capacity of the soil is given by:

$$q_{all} = \frac{q_{ult}}{F_b} = \frac{428.52}{2.5} = 171.41 \text{ [kN/m}^2\text{]}$$

2- Design of Strip Footings

Solved Examples

Example 15: Design of a strip footing to support a brick wall

Design a strip footing to support a brick wall of thickness $b = 0.4$ [m], carrying a load of $p = 450$ [kN/m]. The allowable soil pressure is $q_{all} = 175$ [kN/m²] at a depth of $D_f = 1.25$ [m], Figure 17. Average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

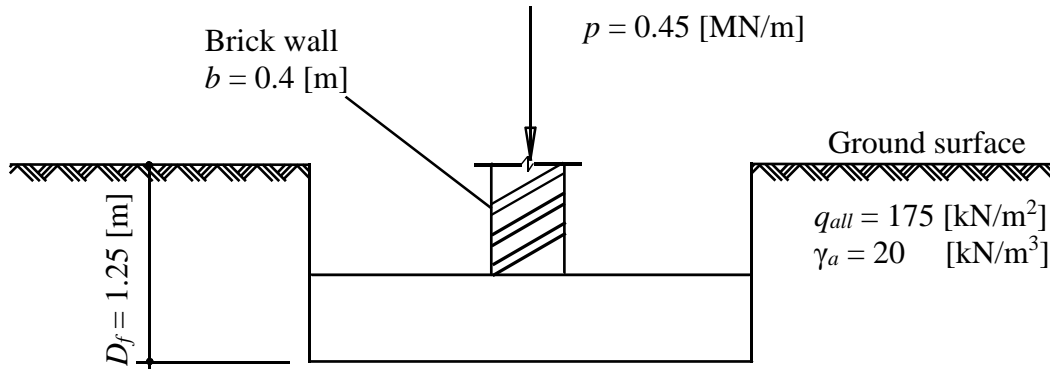


Figure 17: Strip footing to support a brick wall

1. Determining footing dimensions

Take 1.0 [m] strip, then

$$P = p \times 1.0 = 450 \times 1 = 450 \text{ [kN]}$$

Total load at the foundation level is given by:

$$P_T = \frac{P}{\left(1 - \frac{\gamma_a D_f}{q_{all}}\right)}$$

$$P_T = \frac{450}{\left(1 - \frac{20 \times 1.25}{175}\right)} = 525 \text{ [kN]}$$

$$A_f = \frac{P_T}{q_{all}} = \frac{525}{175} = 3.0 \text{ [m}^2\text{]}$$

take $B = 3.0$ [m] strip footing

2. Computing the contact pressure under the footing q_o

$$q_o = \frac{P}{B} = \frac{0.45}{3.0} = 0.15 \text{ [MN/m]}$$

3. Determining the depth required to resist the moment d_m

The maximum bending moment for the strip footing to support a brick wall lies at a distance $b/4 = 0.1$ [m] from the face of the brick wall as shown in Figure 18.

Solved Examples

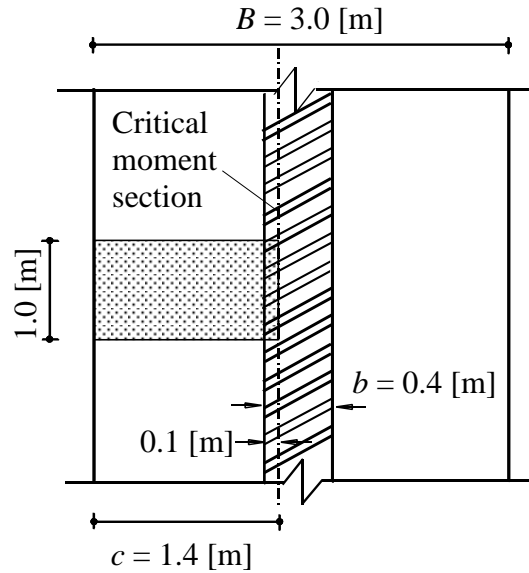


Figure 18: Critical section of moment for a strip footing to support a brick wall

$$M = q_o \frac{c^2}{2}$$

$$M = 0.15 \frac{1.4^2}{2} = 0.147 \text{ [MN.m]}$$

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.147}{1.0}} = 0.294 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.294 \text{ [m]}$, then the section is designed as singly reinforced section.

The corresponding k_1 for $d = 0.45 \text{ [m]}$ is given by:

$$k_1 = 0.45 \sqrt{\frac{1.0}{0.147}} = 1.174$$

From Table 2 at $k_1 = 1.175$, $f_c = 5.5 \text{ [MN/m}^2\text{]}$ and $k_2 = 181$

Solved Examples

4. Computing the area of steel reinforcement

The required area of steel reinforcement per meter A_s is:

$$A_s = \frac{M}{k_2 d}$$

$$A_s = \frac{0.147}{181 \times 0.45} = 0.001805 \text{ [m}^2\text{/m]}$$

$$A_s = 18.05 \text{ [cm}^2\text{/m]}$$

Chosen steel $9\Phi 16/\text{m} = 18.1 \text{ [cm}^2\text{/m]}$

5. Check of shear

The critical shear section lies a distance $d = 0.45 \text{ [m]}$ from the face of the wall as shown in Figure 19.

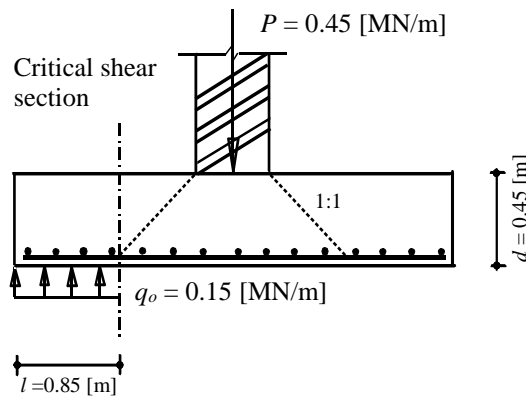


Figure 19: Critical section for shear

Stresses

Soil pressure under the footing

$$q_o = 0.15 \text{ [MN/m}^2\text{]}$$

Allowable shear strength for concrete C 250

$$q_{sall} = 0.9 \text{ [MN/m}^2\text{]}$$

Check for section capacity

The shear force Q_s is

$$Q_s = q_o l$$

$$Q_s = 0.15 \times 0.85 = 0.1275 \text{ [MN]}$$

The shear stress q_s is given by:

$$q_s = \frac{Q_s}{B d}$$

Solved Examples

$$q_s = \frac{0.1275}{1.0 \times 0.45} = 0.283 \text{ [MN/m}^2\text{]}$$

$q_{sall} = 0.9 \text{ [MN/m}^2\text{]} > q_s = 0.283 \text{ [MN/m}^2\text{]}$, the section is safe for shear.

Figure 20 shows the footing dimensions and reinforcement.

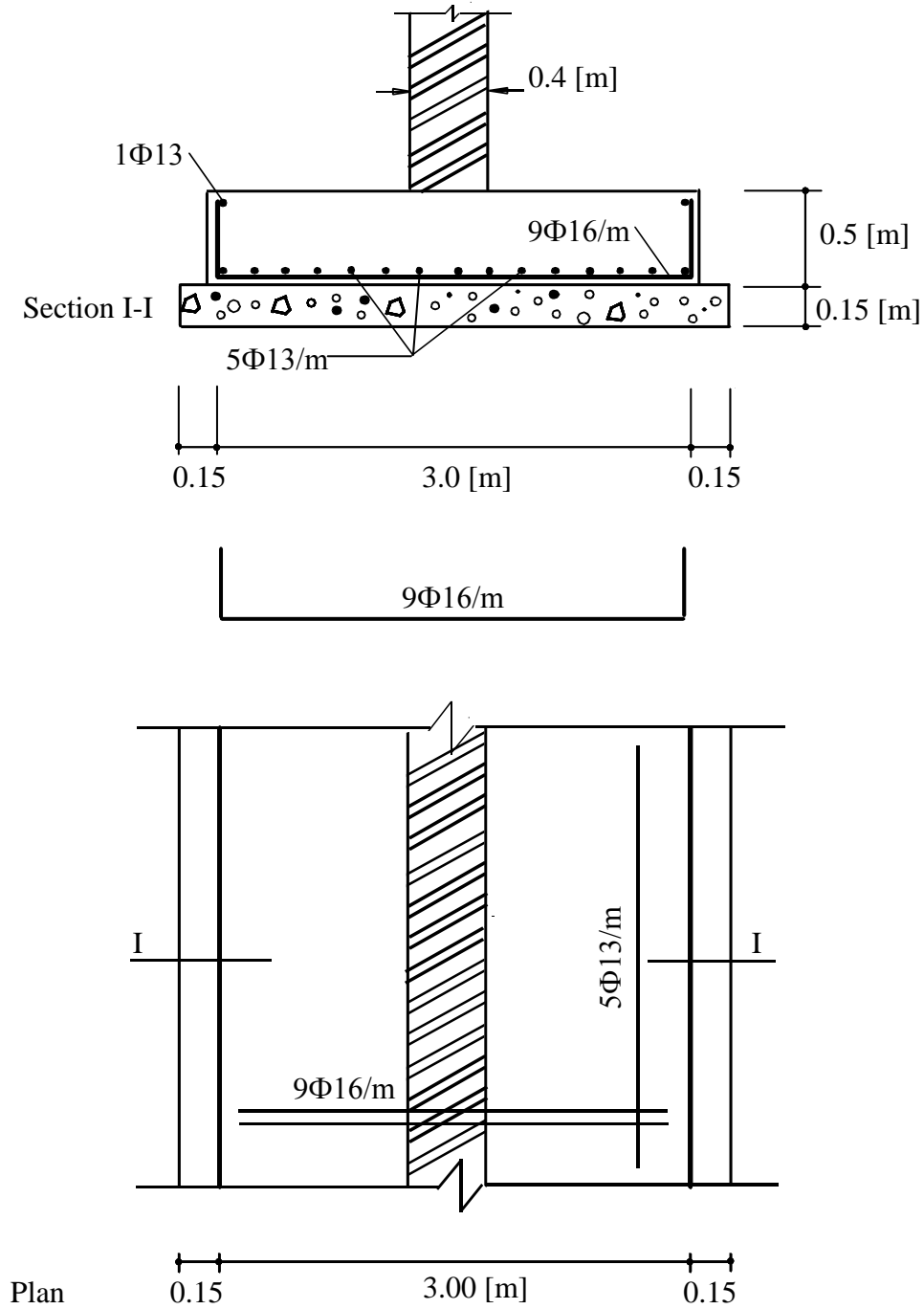


Figure 20: Footing dimensions and reinforcement

Solved Examples

Example 16: Design of a strip footing to support a concrete wall

Design a strip footing to support a concrete wall of thickness $b = 0.25$ [m] and reinforced by $5\Phi 13/m$ in both directions, carrying a load of $P = 400$ [kN/m]. The allowable soil pressure is $q_{all} = 185$ [kN/m²] at a depth of $D_f = 1.5$ [m], Figure 21. Average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

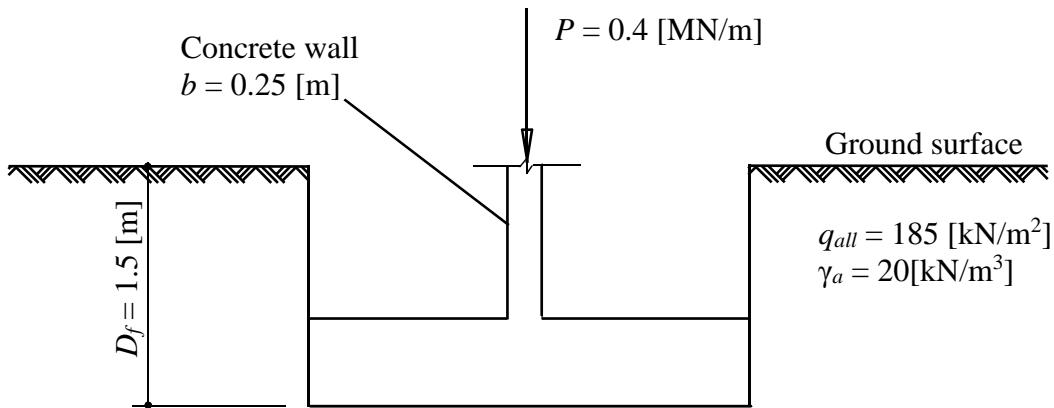


Figure 21: Strip footing to support a concrete wall

The design is conducted in steps as follows:

1. Determining footing dimensions

Take 1.0 [m] strip, then

$$P = p \times 1.0 = 400 \times 1 = 400 \text{ [kN]}$$

Total load at the foundation level is given by:

$$P_T = \frac{P}{\left(1 - \frac{\gamma_a D_f}{q_{all}}\right)}$$

$$P_T = \frac{400}{\left(1 - \frac{20 \times 1.5}{185}\right)} = 477.42 \text{ [kN]}$$

$$A_f = \frac{P_T}{q_{all}} = \frac{477.42}{185} = 2.58 \text{ [m}^2\text{]}$$

take $B = 2.6$ [m] strip footing

2. Computing the contact pressure under the footing q_o

$$q_o = \frac{P}{B} = \frac{0.4}{2.6} = 0.154 \text{ [MN/m]}$$

3. Determining the depth required to resist the moment d_m

The maximum bending moment lies at the wall face as shown in Figure 22.

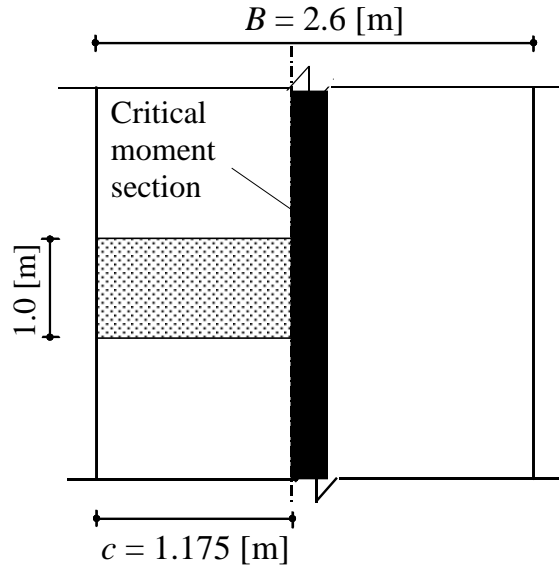


Figure 22: Critical section of moment for a strip footing to support a concrete wall

$$M = q_o \frac{c^2}{2}$$

$$M = 0.154 \frac{1.175^2}{2} = 0.106 \text{ [MN.m]}$$

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.106}{1.0}} = 0.249 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.249 \text{ [m]}$, then the section is designed as singly reinforced section.

The corresponding k_1 for $d = 0.45 \text{ [m]}$ is given by:

$$k_1 = 0.45 \sqrt{\frac{1.0}{0.106}} = 1.38$$

Solved Examples

From Table 2 at $k_1 = 1.387$, $f_c = 4.5$ [MN/m²] and $k_2 = 183$

4. Computing the area of steel reinforcement

The required area of steel reinforcement per meter A_s is:

$$A_s = \frac{M}{k_2 d}$$

$$A_s = \frac{0.106}{183 \times 0.45} = 0.001287 \text{ [m}^2\text{/m]}$$

$$A_s = 12.87 \text{ [cm}^2\text{/m]}$$

Chosen steel $7\Phi 16/\text{m} = 14.1$ [cm²/m]

5. Check of shear

The critical shear section lies a distance $d = 0.45$ [m] from the face of the wall as shown in Figure 23.

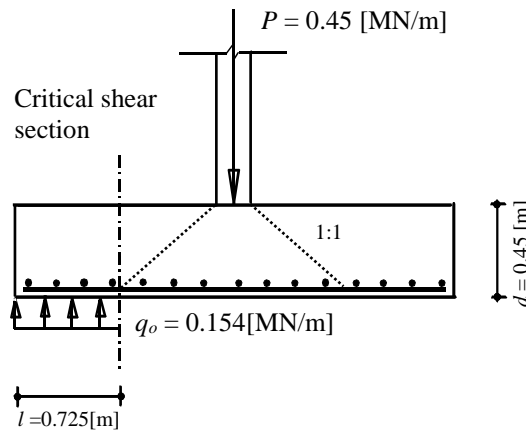


Figure 23: Critical section for shear

Stresses

Soil pressure under the footing

$$q_o = 0.154 \text{ [MN/m}^2\text{]}$$

Allowable shear strength for concrete C 250

$$q_{sall} = 0.9 \text{ [MN/m}^2\text{]}$$

Check for section capacity

The shear force Q_s is

$$Q_s = q_o l$$

$$Q_s = 0.154 \times 0.725 = 0.112 \text{ [MN]}$$

The shear stress q_s is given by:

Solved Examples

$$q_s = \frac{Q_s}{B d}$$

$$q_s = \frac{0.112}{1.0 \times 0.45} = 0.249 \text{ [MN/m}^2\text{]}$$

$q_{sall} = 0.9 \text{ [MN/m}^2\text{]} > q_s = 0.249 \text{ [MN/m}^2\text{]}$, the section is safe for shear.

Figure 24 shows the footing dimensions and reinforcement.

Solved Examples

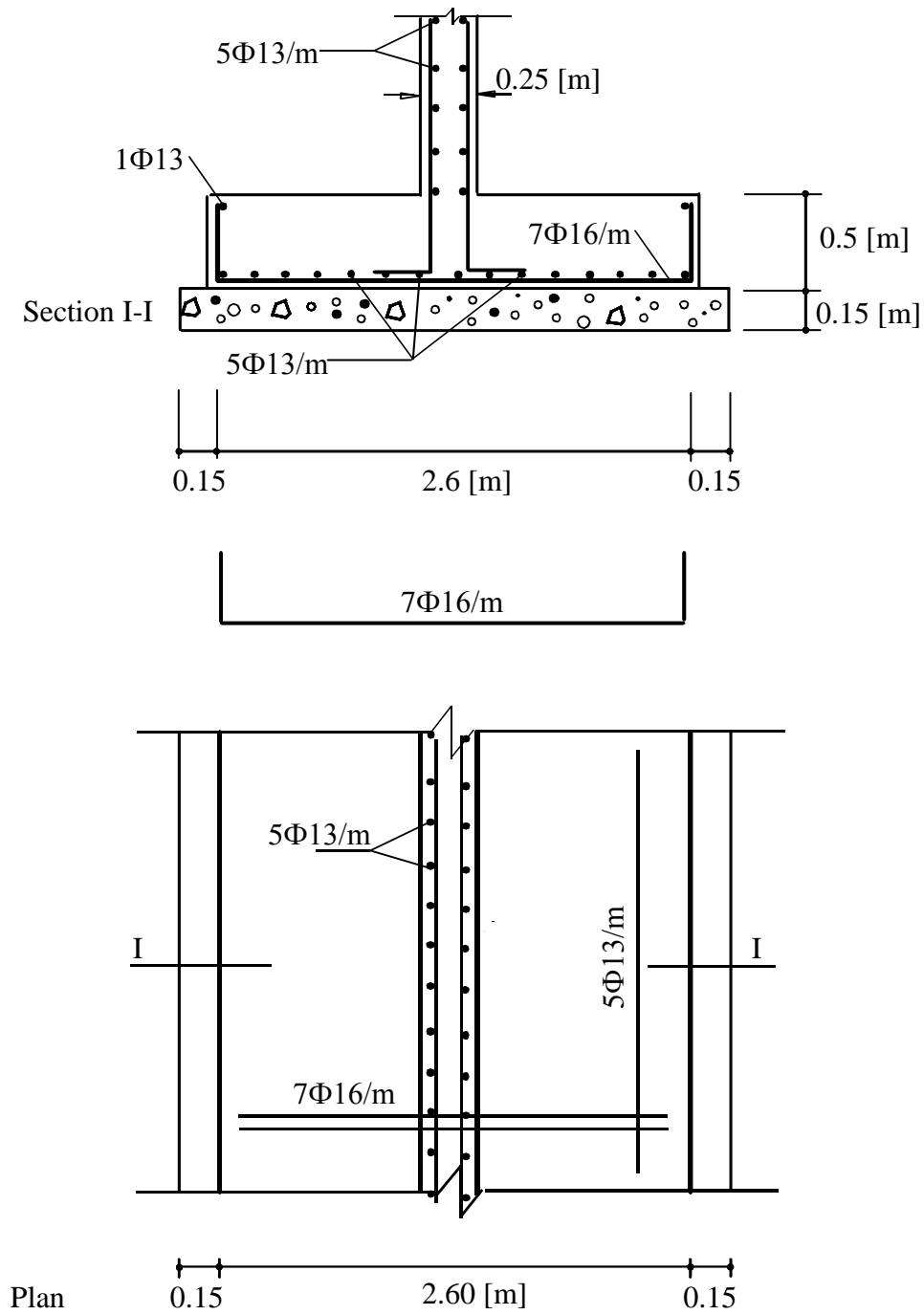


Figure 24: Footing dimensions and reinforcement

3- Design of Isolated Footings

Solved Examples

Example 17: Design of a square isolated footing of column $0.50 \times 0.50 \text{ [m}^2\text{]}$

Design a square isolated footing of column $0.50 \times 0.50 \text{ [m}^2\text{]}$ reinforced by $4\Phi 19$, carrying a load of $P = 1125 \text{ [kN]}$. The allowable soil pressure is $q_{all} = 198 \text{ [kN/m}^2\text{]}$ at a depth of $D_f = 1.5 \text{ [m]}$ and average unit weight of the soil and concrete is $\gamma_a = 20 \text{ [kN/m}^3\text{]}$, Figure 25.

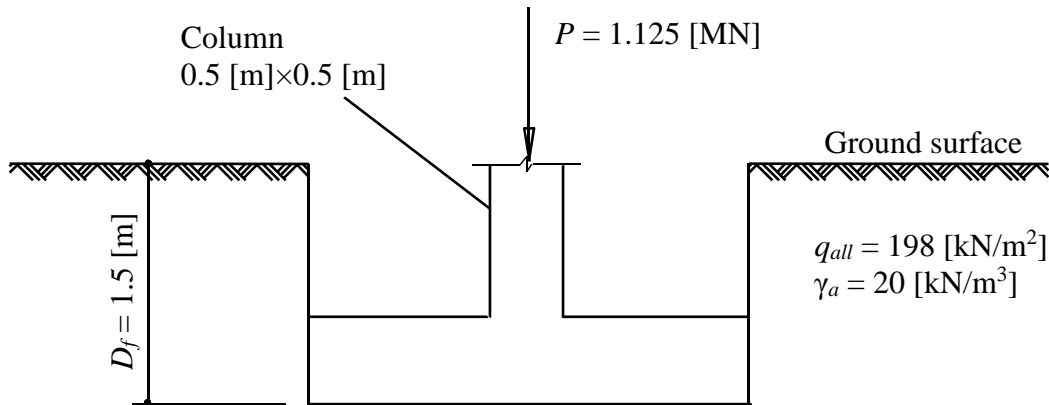


Figure 25: Square isolated footing

The design is conducted in steps as follows:

1. Determining footing dimensions

$$P_T = \frac{P}{\left(1 - \frac{\gamma_a D_f}{q_{all}}\right)}$$

$$P_T = \frac{1125}{\left(1 - \frac{20 \times 1.5}{198}\right)} = 1326 \text{ [kN]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{P_T}{q_{all}} = \frac{1326}{198} = 6.70 \text{ [m}^2\text{]}$$

take $A_f = 2.6 \times 2.6 = 6.76 \text{ [m}^2\text{]}$ square footing

2. Computing the contact pressure under the footing q_o

$$q_o = \frac{P}{A_f} = \frac{1.125}{6.76} = 0.166 \text{ [MN/m}^2\text{]}$$

3. Determining the depth required to resist the moment d_m

The maximum bending moment lies at the column face as shown in Figure 26.

Solved Examples

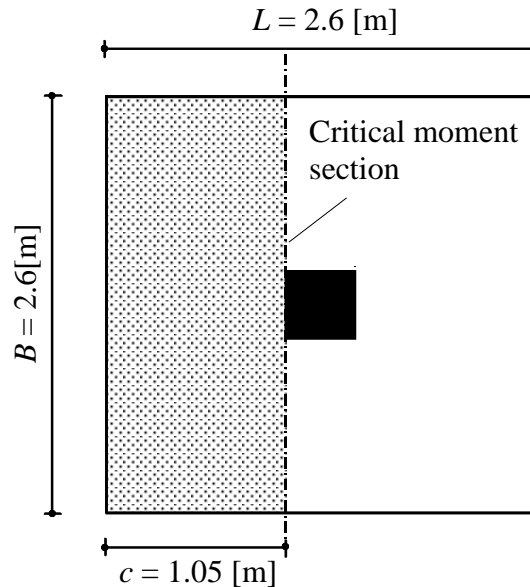


Figure 26: Critical section of moment

For two way footing slab, moment may be reduced by 85%, then the critical moment is:

$$M = 0.85 q_o B \frac{c^2}{2}$$

$$M = 0.85 \times 0.166 \times 2.6 \frac{1.05^2}{2} = 0.202 \text{ [MN.m]}$$

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.202}{2.6}} = 0.214 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.214 \text{ [m]}$, then the section is designed as singly reinforced section. The corresponding k_1 for $d = 0.45 \text{ [m]}$ is given by:

$$k_1 = 0.45 \sqrt{\frac{2.6}{0.202}} = 1.61$$

From Table 2 at $k_1 = 1.532$, $f_c = 4.0 \text{ [MN/m}^2\text{]}$ and $k_2 = 185$

Solved Examples

4. Computing the area of steel reinforcement

The required area of steel reinforcement per meter A_s is:

$$A_s = \frac{M}{k_2 d}$$

$$A_s = \frac{0.202}{185 \times 0.45} = 0.002426 \text{ [m}^2\text{/2.6 m]}$$

$$A_s = 24.26 \text{ [cm}^2\text{/2.6 m]} = 9.33 \text{ [cm}^2\text{/m]}$$

Chosen steel $5\Phi 16/\text{m} = 10.1 \text{ [cm}^2\text{/m]}$ in both directions

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.225 \text{ [m]}$ from the face of the column as shown in Figure 27.

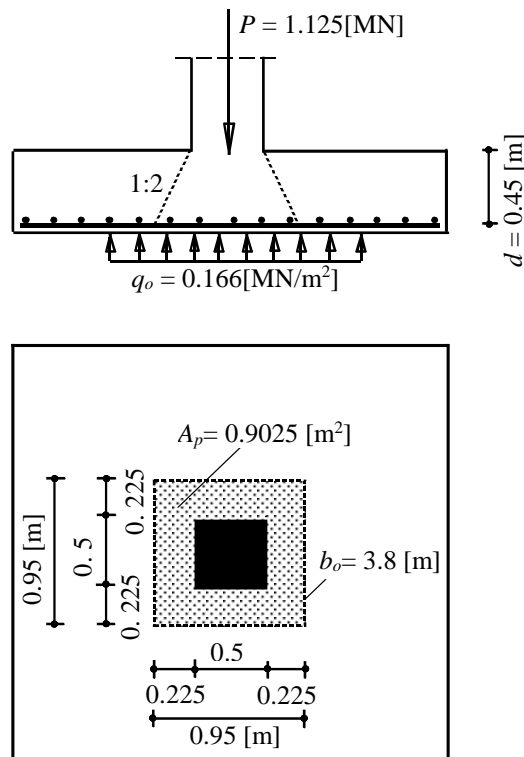


Figure 27: Critical section for punching shear

Geometry (Figure 27)

Effective depth of the section

$$d = 0.45 \text{ [m]}$$

Column side

$$a = b = 0.5 \text{ [m]}$$

Area of critical punching shear section

$$A_p = (a + d)^2 = 0.9025 \text{ [m}^2\text{]}$$

Solved Examples

Perimeter of critical punching shear section $b_o = 4(a + d) = 3.8$ [m]

Loads and stresses

Column load $P = 1.125$ [MN]
Soil pressure under the column $q_o = 0.166$ [MN/m²]
Main value of shear strength for concrete C 250 $q_{cp} = 0.9$ [MN/m²]

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.125 - 0.166 \times 0.9025 = 0.98 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{0.98}{3.8 \times 0.45} = 0.573 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9$ [MN/m²] $> q_p = 0.573$ [MN/m²], the section is safe for punching shear.

Figure 28 shows the footing dimensions and reinforcement.

Solved Examples

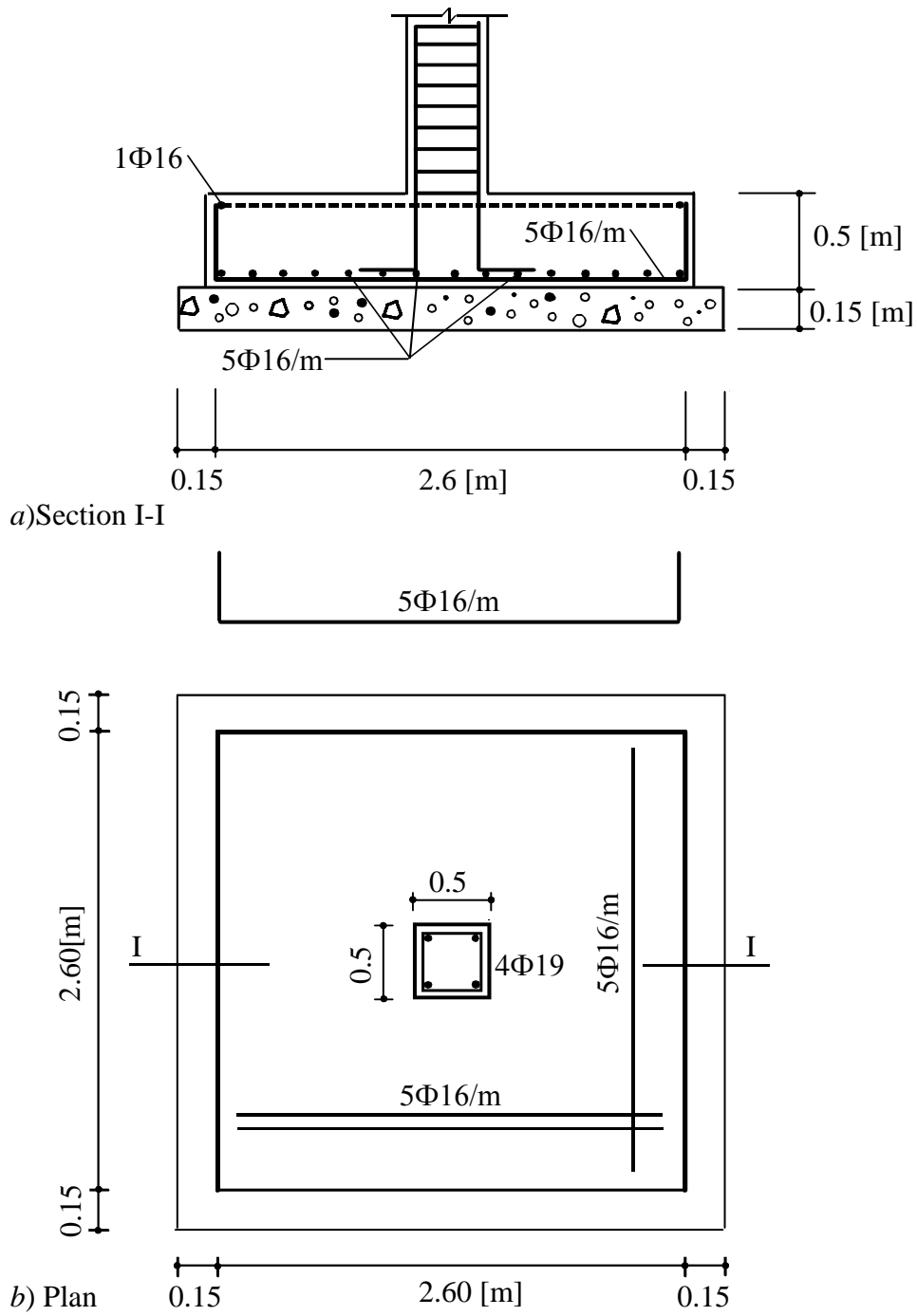


Figure 28: Footing dimensions and reinforcement

Example 18: Design of a square isolated footing of column 0.30×0.60 [m²]

Design a square isolated footing of column 0.30×0.60 [m²] reinforced by 6Φ19, carrying a load of $p = 1260$ [kN]. The allowable soil pressure is $q_{all} = 165$ [kN/m²] at a depth of $D_f = 1.25$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³], Figure 29.

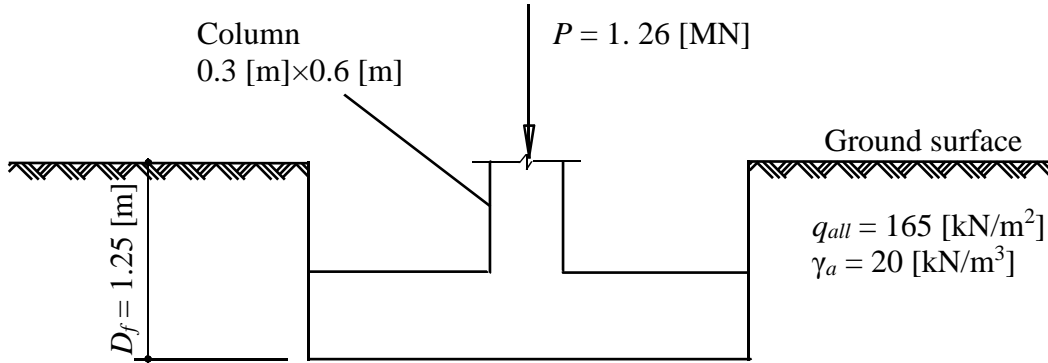


Figure 29: Square isolated footing

The design is conducted in steps as follows:

1. Determining footing dimensions

$$P_T = \frac{P}{\left(1 - \frac{\gamma_a D_f}{q_{all}}\right)}$$

$$P_T = \frac{1260}{\left(1 - \frac{20 \times 1.25}{165}\right)} = 1485 \text{ [kN]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{P_T}{q_{all}} = \frac{1485}{165} = 9.0 \text{ [m}^2\text{]}$$

take $A_f = 3.0 \times 3.0 = 9.0$ [m²] square footing

2. Computing the contact pressure under the footing q_o

$$q_o = \frac{P}{A_f} = \frac{1.26}{9.0} = 0.14 \text{ [MN/m}^2\text{]}$$

3. Determining the depth required to resist the moment d_m

The maximum bending moment lies at the column face in y-direction as shown in Figure 30.

Solved Examples

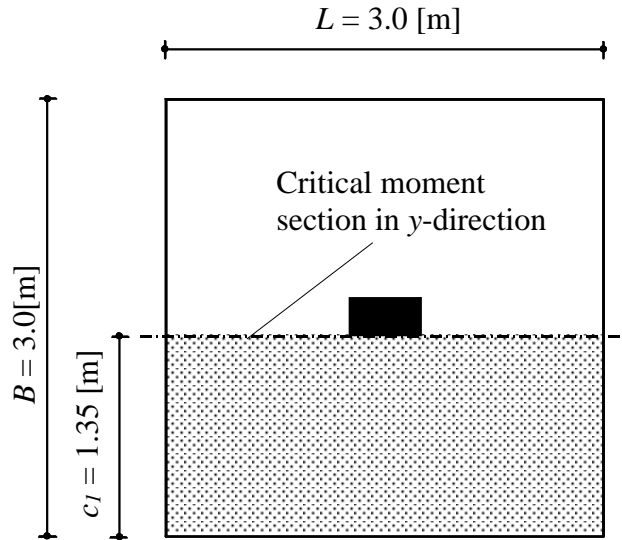


Figure 30: Critical section of moment in y-direction

For two way footing slab, moment may be reduced by 85%, then the critical moment is:

$$M_y = 0.85 q_o L \frac{c_1^2}{2}$$

$$M_y = 0.85 \times 0.14 \times 3.0 \frac{1.35^2}{2} = 0.325 \text{ [MN.m]}$$

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_y}{L}}$$

$$d_m = 0.766 \sqrt{\frac{0.325}{3.0}} = 0.252 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.252 \text{ [m]}$, then the section is designed as singly reinforced section. The corresponding k_1 for $d = 0.45 \text{ [m]}$ is given by:

$$k_1 = 0.45 \sqrt{\frac{3.0}{0.325}} = 1.37$$

From Table 2 at $k_1 = 1.387$, $f_c = 4.5 \text{ [MN/m}^2\text{]}$ and $k_2 = 183$

Solved Examples

4. Computing the area of steel reinforcement

The required area of steel reinforcement per meter A_{sy} in y - direction is:

$$A_{sy} = \frac{M_y}{k_2 d}$$

$$A_{sy} = \frac{0.325}{183 \times 0.45} = 0.003947 \text{ [m}^2\text{/3.0 m]}$$

$$A_{sy} = 39.47 \text{ [cm}^2\text{/3.0 m]} = 13.16 \text{ [cm}^2\text{/m]}$$

Chosen steel $5\Phi 19/\text{m} = 14.2 \text{ [cm}^2\text{/m]}$

The critical section of moment in x -direction is shown in Figure 31. The maximum moment in x -direction is given by:

$$M_x = 0.85 q_o B \frac{c_2^2}{2}$$

$$M_x = 0.85 \times 0.14 \times 3.0 \frac{1.2^2}{2} = 0.257 \text{ [MN.m]}$$

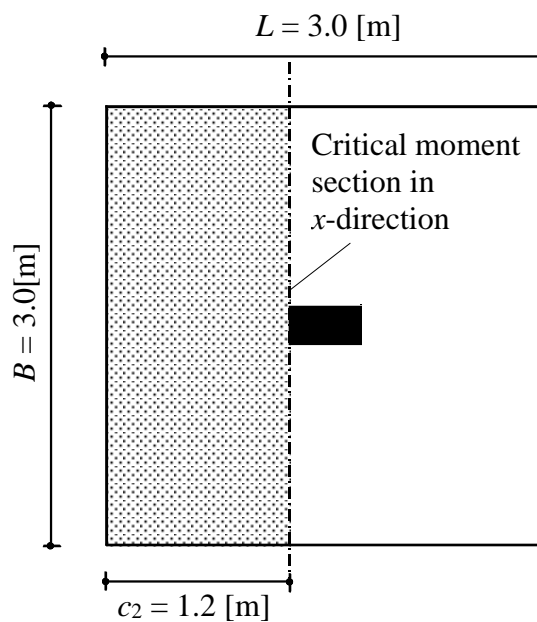


Figure 31: Critical section of moment in x -direction

The required area of steel reinforcement per meter A_{sx} in x - direction is:

Solved Examples

$$A_{sx} = \frac{M_x}{k_2 d}$$

$$A_{sx} = \frac{0.257}{183 \times 0.45} = 0.003121 \text{ [m}^2\text{/3.0 m]}$$

$$A_{sx} = 31.21 \text{ [cm}^2\text{/3.0 m]} = 10.40 \text{ [cm}^2\text{/m]}$$

Chosen steel $5\Phi 19/\text{m} = 14.2 \text{ [cm}^2\text{/m]}$

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.225 \text{ [m]}$ from the face of the column as shown in Figure 32.

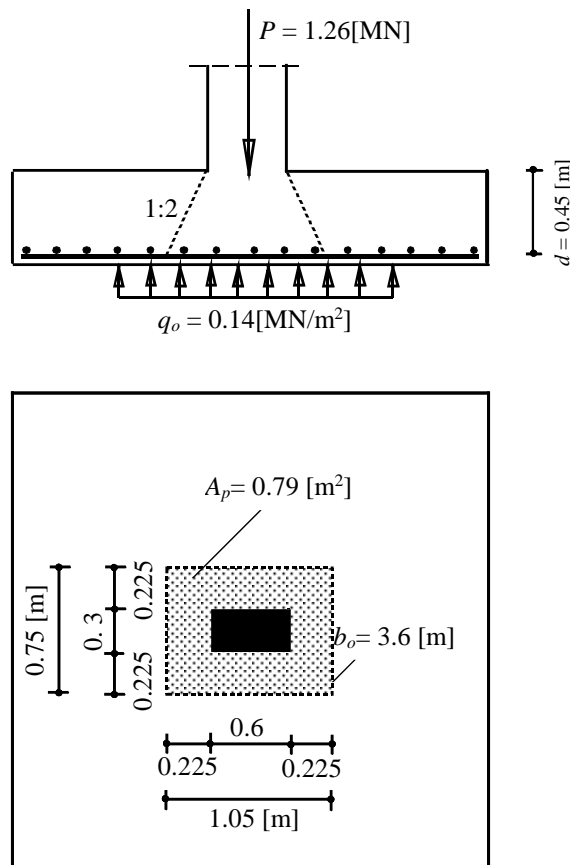


Figure 32: Critical section for punching shear

Geometry (Figure 32)

Effective depth of the section	$d = 0.45 \text{ [m]}$
Column width	$a = 0.3 \text{ [m]}$
Column length	$b = 0.6 \text{ [m]}$
Area of critical punching shear section	$A_p = (a + d)(b + d) = 0.79 \text{ [m}^2\text]}$

Solved Examples

Perimeter of critical punching shear section $b_o = 2(a + d + 2d) = 3.6$ [m]

Loads and stresses

Column load $P = 1.26$ [MN]
Soil pressure under the column $q_o = 0.14$ [MN/m²]
Main value of shear strength for concrete C 250 $q_{cp} = 0.9$ [MN/m²]

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.26 - 0.14 \times 0.79 = 1.15 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{1.15}{3.6 \times 0.45} = 0.71 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.3}{0.6}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9$ [MN/m²] $> q_p = 0.71$ [MN/m²], the section is safe for punching shear.

Figure 33 shows the footing dimensions and reinforcement.

Solved Examples

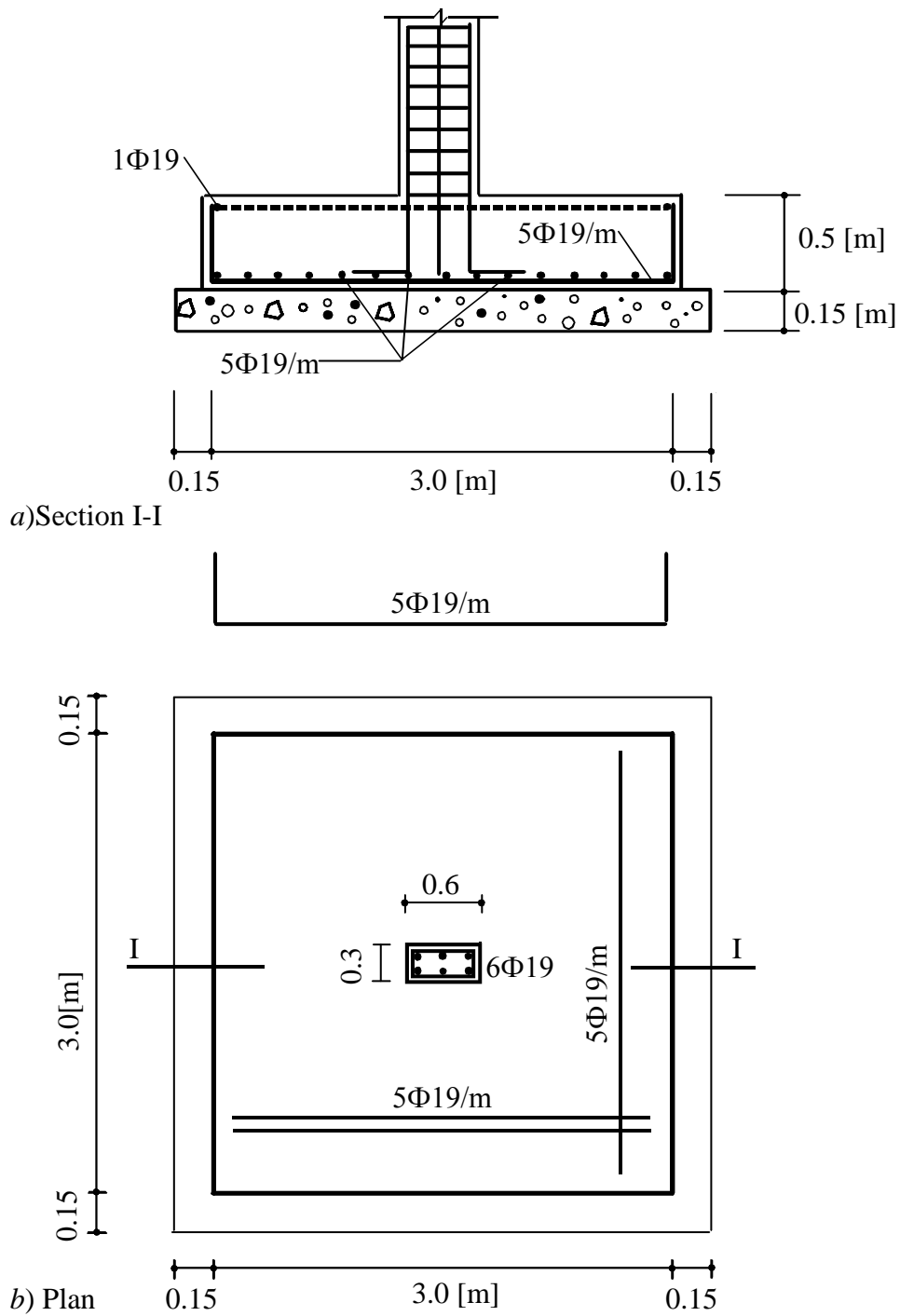


Figure 33: Footing dimensions and reinforcement

Solved Examples

Example 19: Design of a rectangular isolated footing

Design a rectangular isolated footing of column 0.30×0.90 [m²] reinforced by $8\Phi 19$, carrying a load of $P = 1500$ [kN]. The allowable soil pressure is $q_{all} = 190$ [kN/m²] at a depth of $D_f = 1.6$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³], Figure 34.

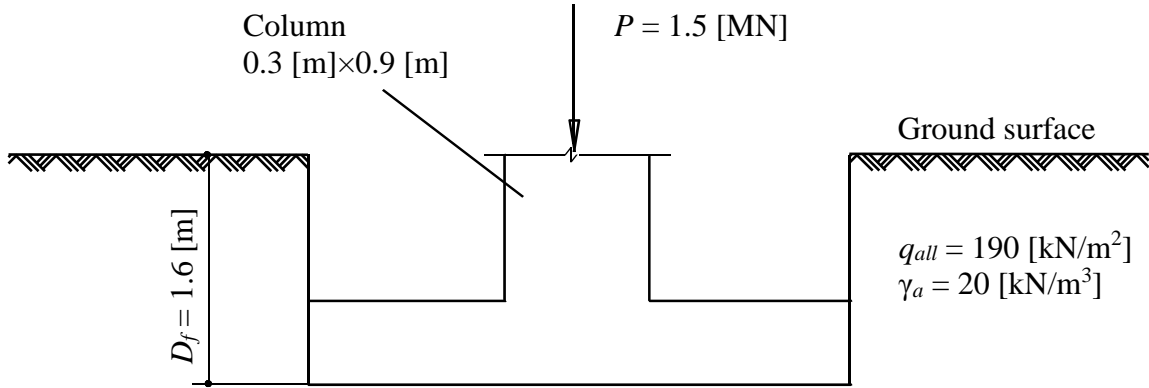


Figure 34: Rectangular isolated footing

The design is conducted in steps as follows:

1. Determining footing dimensions

$$P_T = \frac{P}{\left(1 - \frac{\gamma_a D_f}{q_{all}}\right)}$$

$$P_T = \frac{1500}{\left(1 - \frac{20 \times 1.6}{190}\right)} = 1804 \text{ [kN]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{P_T}{q_{all}} = \frac{1804}{190} = 9.49 \text{ [m}^2\text{]}$$

take $A_f = 3.4 \times 2.8 = 9.52$ [m²] rectangular footing

The area is chosen to be a rectangular with an offset of $c = 1.25$ [m] in both sides.

2. Computing the contact pressure under the footing q_o

$$q_o = \frac{P}{A_f} = \frac{1.50}{9.52} = 0.158 \text{ [MN/m}^2\text{]}$$

3. Determining the depth required to resist the moment d_m

The maximum bending moment lies at the column face in y-direction as shown in Figure 35.

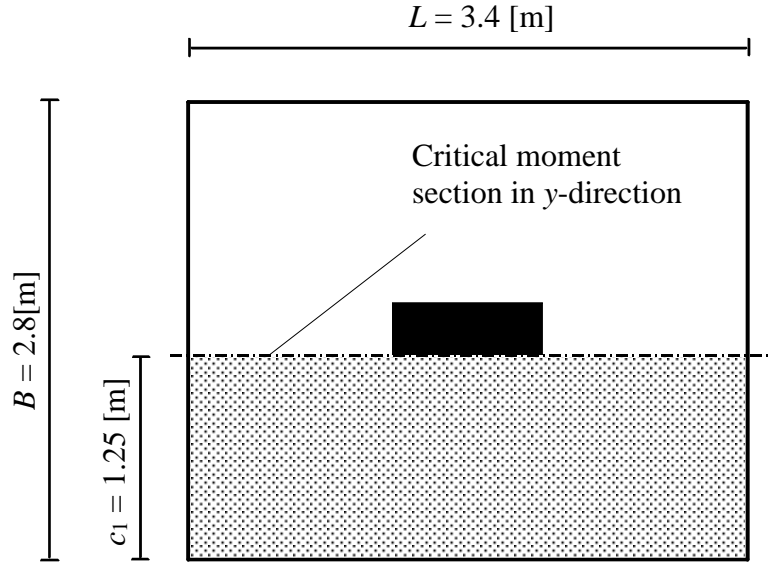


Figure 35: Critical section of moment in y-direction

For two way footing slab, moment may be reduced by 85%, then the critical moment is:

$$M_y = 0.85 q_o L \frac{c_1^2}{2}$$

$$M_y = 0.85 \times 0.158 \times 3.4 \frac{1.25^2}{2} = 0.357 \text{ [MN.m]}$$

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_y}{L}}$$

$$d_m = 0.766 \sqrt{\frac{0.357}{3.4}} = 0.25 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.25 \text{ [m]}$, then the section is designed as singly reinforced section. The corresponding k_1 for $d = 0.45 \text{ [m]}$ is given by:

$$k_1 = 0.45 \sqrt{\frac{3.4}{0.357}} = 1.39$$

Solved Examples

From Table 2 at $k_1 = 1.387$, $f_c = 4.5$ [MN/m²] and $k_2 = 183$

4. Computing the area of steel reinforcement

The required area of steel reinforcement per meter A_{sy} in y - direction is:

$$A_{sy} = \frac{M_y}{k_2 d}$$

$$A_{sy} = \frac{0.357}{183 \times 0.45} = 0.004335 \text{ [m}^2\text{/3.4 m]}$$

$$A_{sy} = 43.35 \text{ [cm}^2\text{/3.4 m]} = 12.75 \text{ [cm}^2\text{/m]}$$

Chosen steel $5\Phi 19/\text{m} = 14.2$ [cm²/m]

The critical section of moment in x -direction is shown in Figure 36. The maximum moment in x -direction is given by:

$$M_x = 0.85 q_o B \frac{c_2^2}{2}$$

$$M_x = 0.85 \times 0.158 \times 2.8 \frac{1.25^2}{2} = 0.294 \text{ [MN.m]}$$

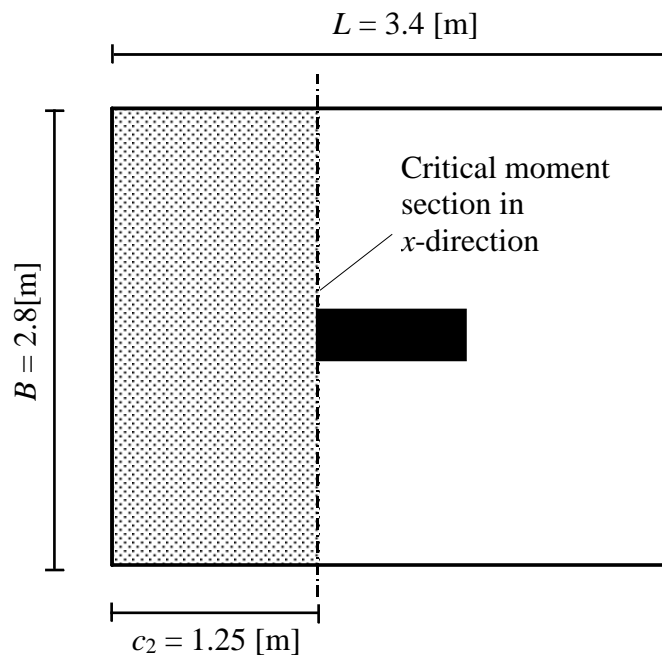


Figure 36: Critical section of moment in x -direction

The required area of steel reinforcement per meter A_{sx} in x - direction is:

Solved Examples

$$A_{sx} = \frac{M_x}{k_2 d}$$

$$A_{sx} = \frac{0.294}{183 \times 0.45} = 0.003570 \text{ [m}^2\text{/2.8 m]}$$

$$A_{sx} = 35.70 \text{ [cm}^2\text{/2.8 m]} = 12.75 \text{ [cm}^2\text{/m]}$$

Chosen steel $5\Phi 19/\text{m} = 14.2 \text{ [cm}^2\text{/m]}$

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.225 \text{ [m]}$ from the face of the column as shown in Figure 37.

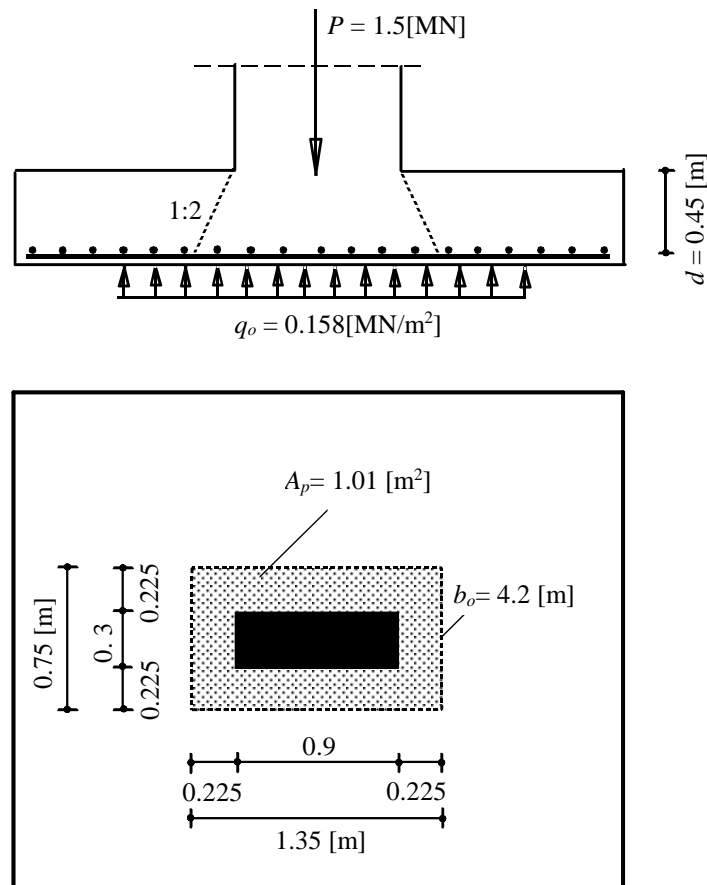


Figure 37: Critical section for punching shear

Geometry (Figure 37)

Effective depth of the section	$d = 0.45 \text{ [m]}$
Column width	$a = 0.3 \text{ [m]}$
Column length	$b = 0.9 \text{ [m]}$
Area of critical punching shear section	$A_p = (a + d)(b + d) = 1.01 \text{ [m}^2\text]}$

Solved Examples

Perimeter of critical punching shear section $b_o = 2(a + d + 2d) = 4.2$ [m]

Loads and stresses

Column load $P = 1.5$ [MN]
Soil pressure under the column $q_o = 0.158$ [MN/m²]
Main value of shear strength for concrete C 250 $q_{cp} = 0.9$ [MN/m²]

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.5 - 0.158 \times 1.01 = 1.34$$
 [MN]

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{1.34}{4.2 \times 0.45} = 0.71$$
 [MN/m²]

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.3}{0.9}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.75$$
 [MN/m²]

$q_{pall} = 0.75$ [MN/m²] $> q_p = 0.71$ [MN/m²], the section is safe for punching shear.

Figure 38 shows the footing dimensions and reinforcement.

Solved Examples

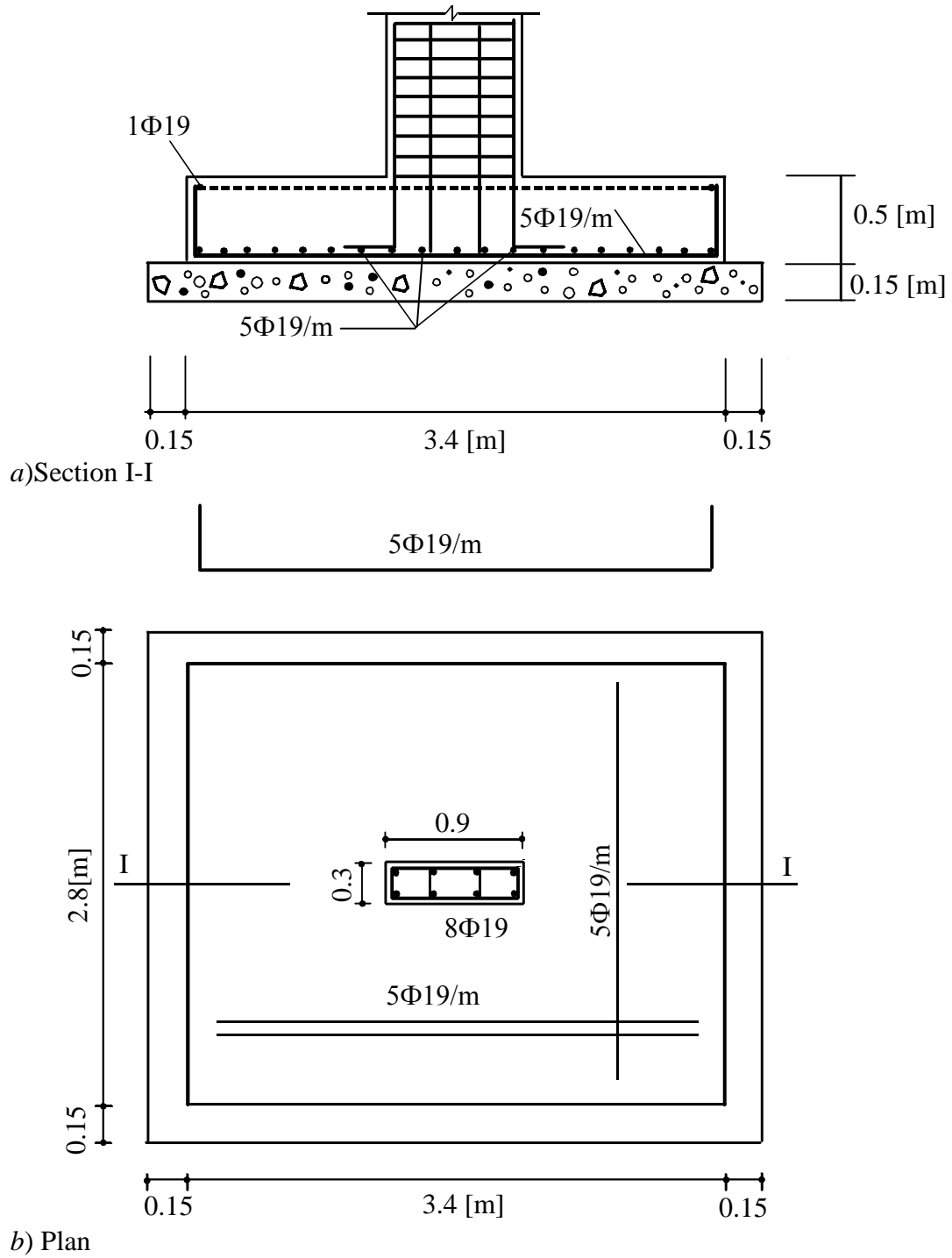


Figure 38: Footing dimensions and reinforcement

Solved Examples

Example 20: Design of a footing subjected to an axial load and moment

Design a square isolated footing of column $0.50 \times 0.50 \text{ [m}^2\text{]}$ reinforced by $4\Phi 19$, carrying a load of $P = 1500 \text{ [kN]}$ and moment of $M = 750 \text{ [kN.m]}$, Figure 39.

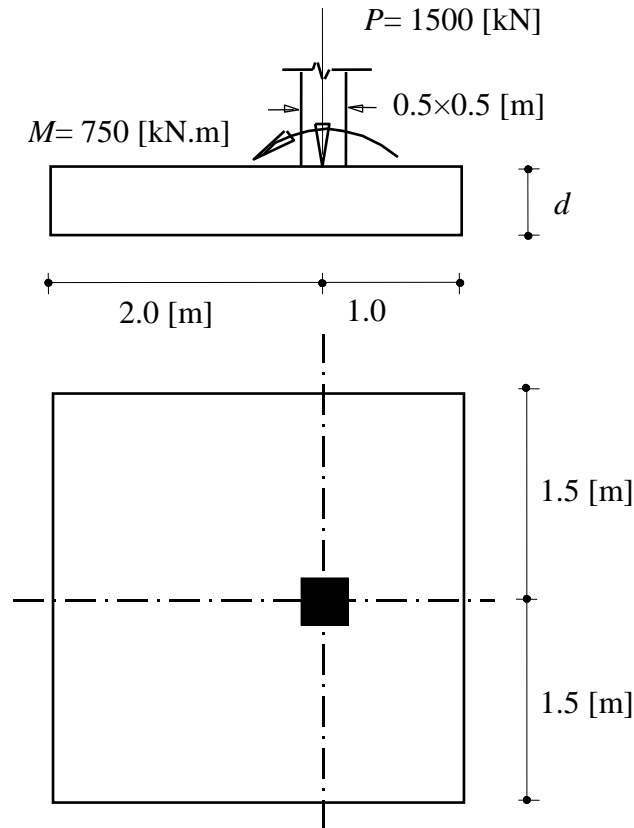


Figure 39: Footing subjected to an axial load and moment

The design is conducted in steps as follows:

1. Computing the contact pressure under the footing q_o

Total moment about y-axis is given by (Figure 40):

$$M_t = P \times e - M = 1500 \times 0.5 - 750 = 0.0 \text{ [kN.m]}$$

A total moment $M_t = 0.0 \text{ [kN.m]}$ leads to a uniform contact pressure at the base of the footing.

The contact pressure at the base of the footing is

$$q_o = \frac{P}{A_f} = \frac{1.50}{9.0} = 0.167 \text{ [MN/m}^2\text{]}$$

Solved Examples

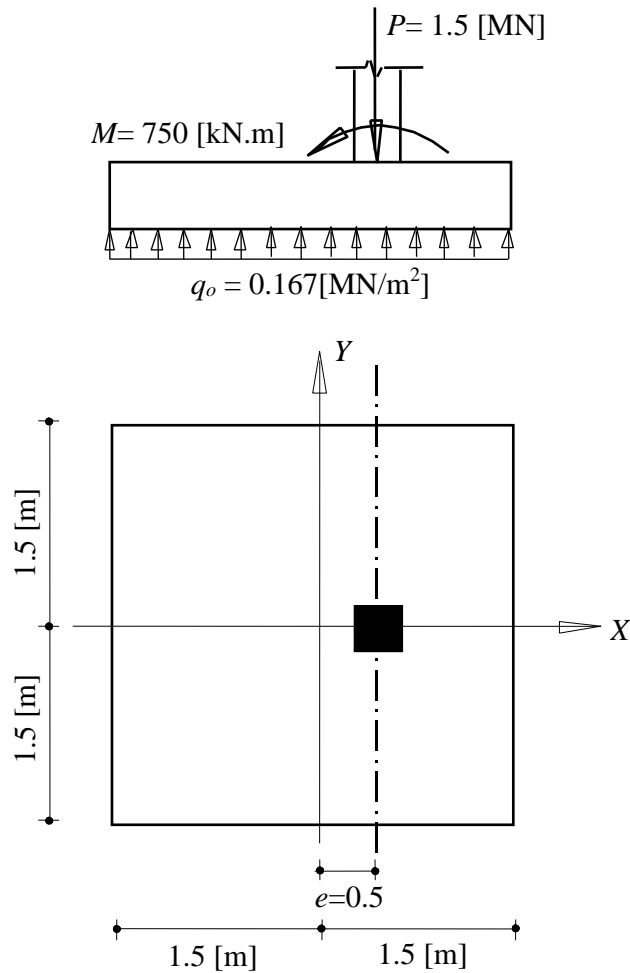


Figure 40: Contact pressure under the base of the footing

2. Determining the depth required to resist the moment d_m

The critical section of moment in x -direction is shown in Figure 41. For two way footing slab, moment may be reduced by 85%. The maximum moment in x -direction is given by:

$$M_x = 0.85 q_o B \frac{c_1^2}{2}$$

$$M_x = 0.85 \times 0.167 \times 3.0 \frac{1.75^2}{2} = 0.652 \text{ [MN.m]}$$

Solved Examples

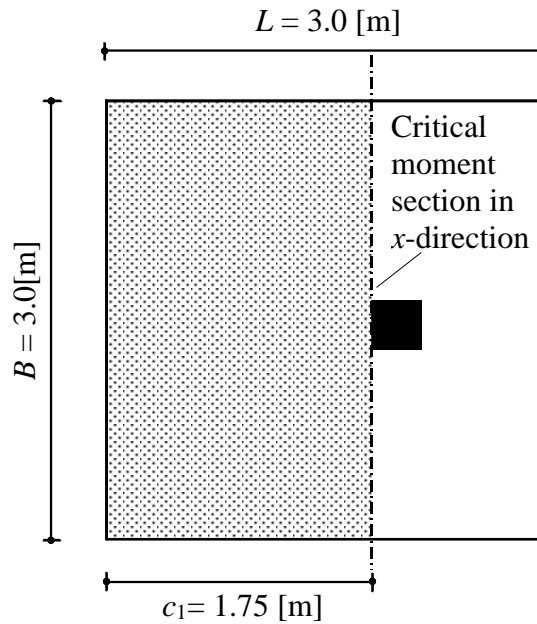


Figure 41: Critical section of moment in x -direction

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_x}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.652}{3.0}} = 0.36 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.36 \text{ [m]}$, then the section is designed as singly reinforced section. The corresponding k_1 for $d = 0.45 \text{ [m]}$ is given by:

$$k_1 = 0.45 \sqrt{\frac{3.0}{0.652}} = 0.965$$

From Table 2 at $k_1 = 0.965$, $f_c = 7.0 \text{ [MN/m}^2\text{]}$ and $k_2 = 177$

3. Computing the area of steel reinforcement

The required area of steel reinforcement per meter A_{sx} in x - direction is:

$$A_{sx} = \frac{M_x}{k_2 d}$$

Solved Examples

$$A_{sx} = \frac{0.652}{177 \times 0.45} = 0.008186 \text{ [m}^2\text{/3.0 m]}$$

$$A_{sx} = 81.86 \text{ [cm}^2\text{/3.0 m]} = 27.29 \text{ [cm}^2\text{/m]}$$

Chosen steel $10\Phi 19/\text{m} = 28.4 \text{ [cm}^2\text{/m]}$

The maximum bending moment lies at the column face in y-direction as shown in Figure 42.

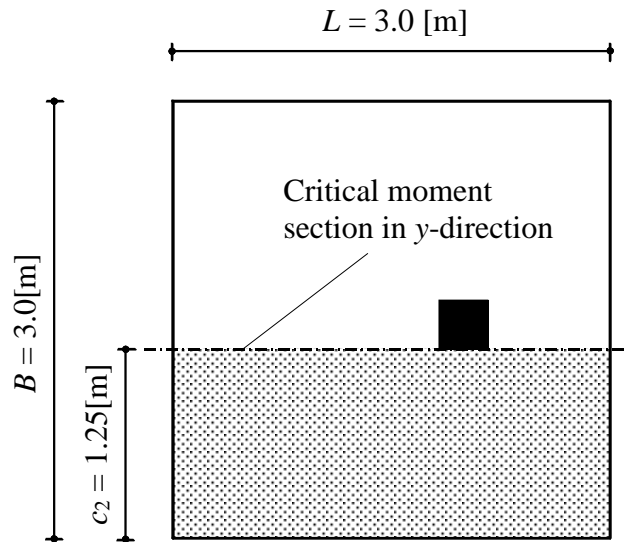


Figure 42: Critical section of moment in y-direction

$$M_y = 0.85 q_o L \frac{c_2^2}{2}$$

$$M_y = 0.85 \times 0.167 \times 3.0 \frac{1.25^2}{2} = 0.333 \text{ [MN.m]}$$

The required area of steel reinforcement per meter A_{sy} in y- direction is:

$$A_{sy} = \frac{M_y}{k_2 d}$$

$$A_{sy} = \frac{0.333}{177 \times 0.45} = 0.004181 \text{ [m}^2\text{/3.0 m]}$$

$$A_{sy} = 41.81 \text{ [cm}^2\text{/3.0 m]} = 13.94 \text{ [cm}^2\text{/m]}$$

Chosen steel $5\Phi 19/\text{m} = 14.2 \text{ [cm}^2\text{/m]}$

Solved Examples

4. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.225$ [m] from the face of the column as shown in Figure 43.

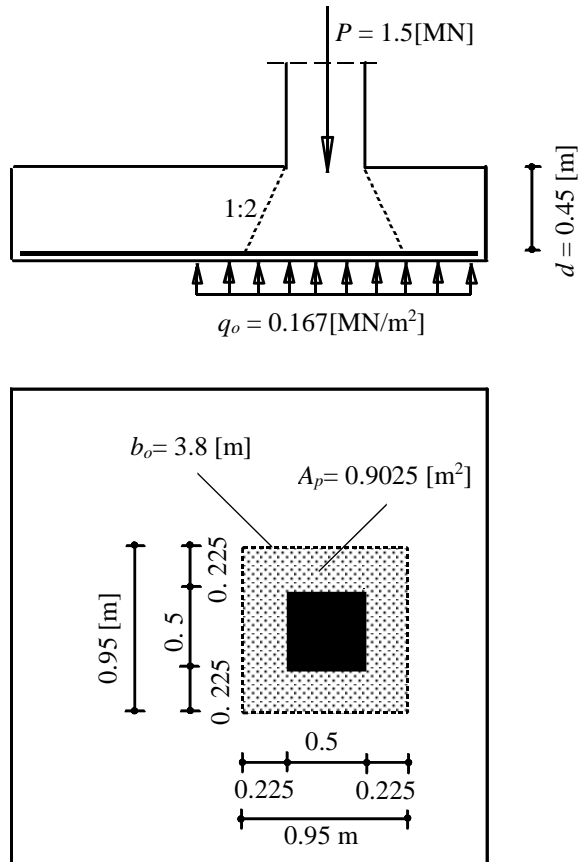


Figure 43: Critical section for punching shear

Geometry (Figure 43)

Effective depth of the section	$d = 0.45$ [m]
Column side	$a = b = 0.5$ [m]
Area of critical punching shear section	$A_p = (a + d)^2 = 0.9025$ [m ²]
Perimeter of critical punching shear section	$b_o = 4(a + d) = 3.8$ [m]

Loads and stresses

Column load	$P = 1.5$ [MN]
Soil pressure under the column	$q_o = 0.167$ [MN/m ²]
Main value of shear strength for concrete C 250	$q_{cp} = 0.9$ [MN/m ²]

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

Solved Examples

$$Q_p = 1.5 - 0.167 \times 0.9025 = 1.35 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{1.35}{3.8 \times 0.45} = 0.79 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

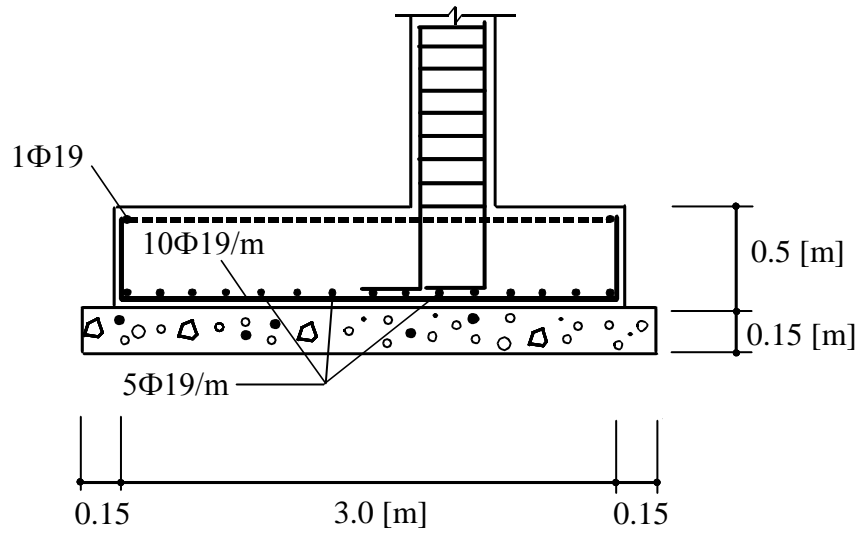
$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

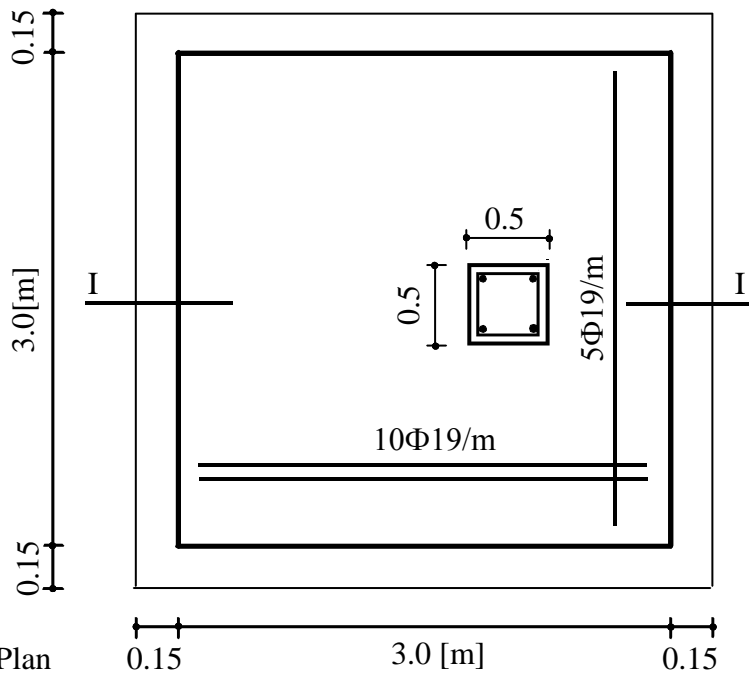
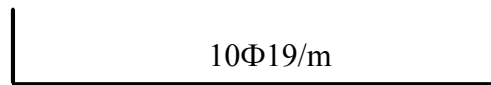
$q_{pall} = 0.9 \text{ [MN/m}^2\text{]} > q_p = 0.79 \text{ [MN/m}^2\text{]}$, the section is safe for punching shear.

Figure 44 shows the footing dimensions and reinforcement.

Solved Examples



a) Section I-I



b) Plan

Figure 44: Footing dimensions and

Solved Examples

Example 21: Design of a circular footing for a circular column

Design a circular footing for a column 0.6 [m] in diameter, reinforced by 16Φ16 and carrying a load of 1500 [kN] as shown in Figure 45. The allowable soil pressure $q_{all} = 185$ [kN/m²] at a depth of $D_f = 1.75$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

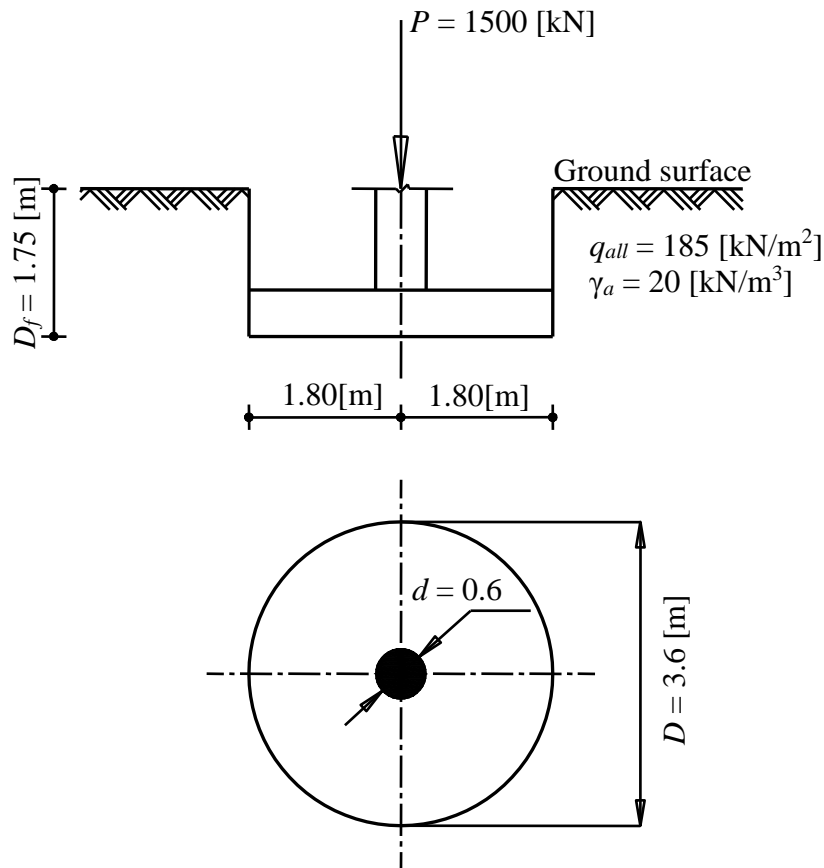


Figure 45: Circular footing for a circular column

The design is conducted in steps as follows:

1. Determining the footing diameter D

$$P_T = \frac{P}{\left(1 - \frac{\gamma_a \times D_f}{q_{all}}\right)}$$

$$P_T = \frac{1500}{\left(1 - \frac{20 \times 1.75}{185}\right)} = 1850 \text{ [kN]}$$

Area of footing A_f is obtained from:

Solved Examples

$$A_f = \frac{P_T}{q_{all}} = \frac{1850}{185} = 10 \text{ [m}^2\text{]}$$

$$\text{take } D = 3.60 \text{ [m], } A_f = (\pi/4) \times (3.60)^2 = 10.179 \text{ [m}^2\text{]}$$

2. Computing the contact pressure under the footing q_o

The contact pressure per meter square under the base of the footing will be uniform, it's given by:

$$q_o = \frac{P}{A_f} = \frac{1.5}{10.179} = 0.147 \text{ [MN/m}^2\text{]}$$

3. Determining the maximum bending moment

The maximum bending moment acting on the footing occurs at the side face of the column in y-y direction due to the shaded area shown in Figure 46.

Area of the shaded part:

$$A_{sh} = 0.25 \left(\frac{\pi D^2}{4} - \frac{\pi d^2}{4} \right)$$

$$A_{sh} = 0.25 \left(\frac{\pi 3.6^2}{4} - \frac{\pi 0.6^2}{4} \right) = 2.474 \text{ [m}^2\text{]}$$

Solved Examples

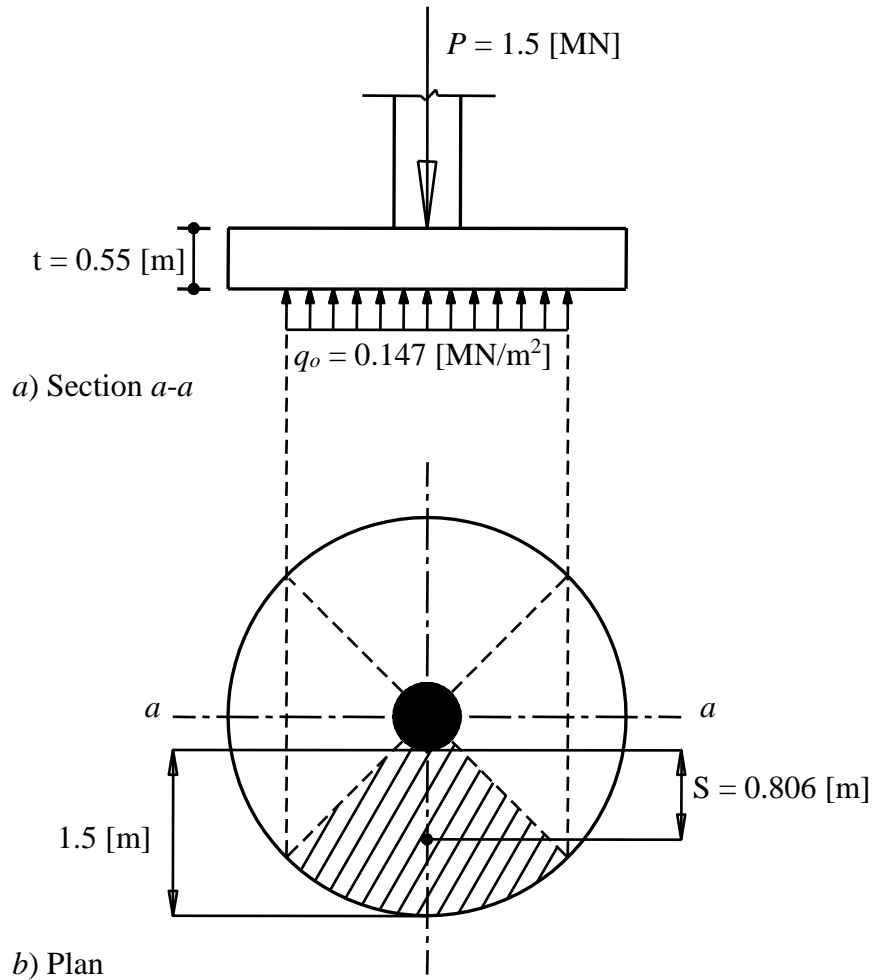


Figure 46: Load and bending moment acting on the footing

Distance of the centroid of the shaded area from the face of the column is given by:

$$S = 0.3 \left(\frac{D^2 + d^2 + Dd}{D + d} \right) - 0.5d$$

$$S = 0.3 \left(\frac{3.6^2 + 0.6^2 + 3.6 \times 0.6}{3.6 + 0.6} \right) - 0.5 \times 0.6 = 0.806 \text{ [m]}$$

Load acting on the shaded area is given by:

$$Q = q_o A_{sh}$$

$$Q = 0.147 \times 2.47 = 0.364 \text{ [MN]}$$

The maximum bending moment M_{max} is given by:

$$M_{max} = QS$$

Solved Examples

$$M_{max} = 0.364 \times 0.806 = 0.293 \text{ [MN.m]}$$

Equivalent breadth of the shaded area may be taken as the average shaded circularly perimeter from the column and the footing.

$$B = 0.5 \left(\frac{\pi D}{4} + \frac{\pi d}{4} \right)$$

$$B = 0.5 \left(\frac{\pi \cdot 3.6}{4} + \frac{\pi \cdot 0.6}{4} \right) = 1.649 \text{ [m]}$$

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.293}{1.649}} = 0.323 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.323 \text{ [m]}$ and $t = 0.5 \text{ [m]}$.

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d_f/2 = 0.225 \text{ [m]}$ from the face of the column as shown in Figure 47.

Geometry (Figure 47)

Effective depth of the section	$d_f = 0.45 \text{ [m]}$
Column diameter	$d = 0.6 \text{ [m]}$
Area of critical punching shear section	$A_p = \pi (d + d_f)^2 / 4 = 0.866 \text{ [m}^2\text{]}$
Perimeter of critical punching shear section	$b_o = \pi (d + d_f) = 3.3 \text{ [m]}$

Loads and stresses

Column load	$P = 1.5 \text{ [MN]}$
Soil pressure under the column	$q_o = 0.147 \text{ [MN/m}^2\text{]}$
Main value of shear strength for concrete C 250	$q_{cp} = 0.9 \text{ [MN/m}^2\text{]}$

Solved Examples

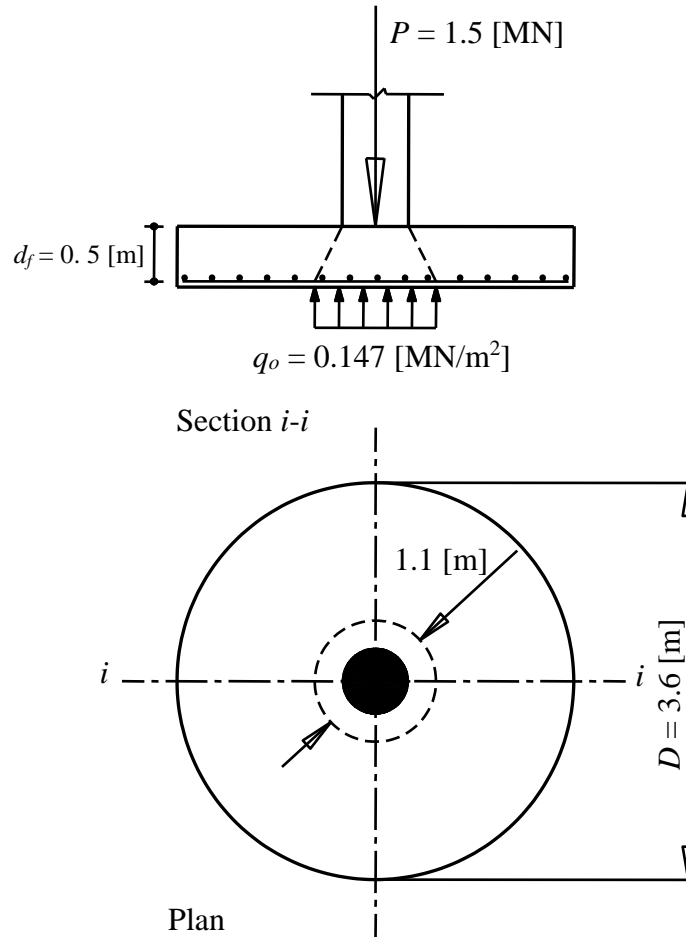


Figure 47: Critical section for punching shear

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.5 - 0.147 \times 0.866 = 1.373 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d_f}$$

$$q_p = \frac{1.373}{3.3 \times 0.45} = 0.924 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

Solved Examples

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = (0.5 + 1) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9 \text{ [MN/m}^2\text{]} \approx q_p = 0.924 \text{ [MN/m}^2\text{]}$, the section is safe for punching shear.

Determining the depth required to resist punching shear d_p

$$d_p = \frac{1.373}{3.3 \times 0.9} = 0.462 \text{ [m]}$$

Take $d = 0.5 \text{ [m]} > d_m = 0.462 \text{ [m]}$ and $t = 0.55 \text{ [m]}$, then the section is designed as singly reinforced section.

The corresponding k_1 for $d = 0.5 \text{ [m]}$ is given by:

$$k_1 = 0.5 \sqrt{\frac{1.649}{0.293}} = 1.19$$

From Table 2 at $k_1 = 1.19$, $f_c = 6.0 \text{ [MN/m}^2\text{]}$ and $k_2 = 179$

6. Computing the area of steel reinforcement

Minimum area of steel reinforcement $A_{s \min} = 0.15\% A_c = 0.0015 \times 55 \times 100 = 8.25 \text{ [cm}^2/\text{m]}$

Take $A_{s \min} = 5\Phi 16 = 10.053 \text{ [cm}^2/\text{m]}$

The required area of steel reinforcement A_s is:

$$A_s = \frac{M_{\max}}{k_2 d_f}$$

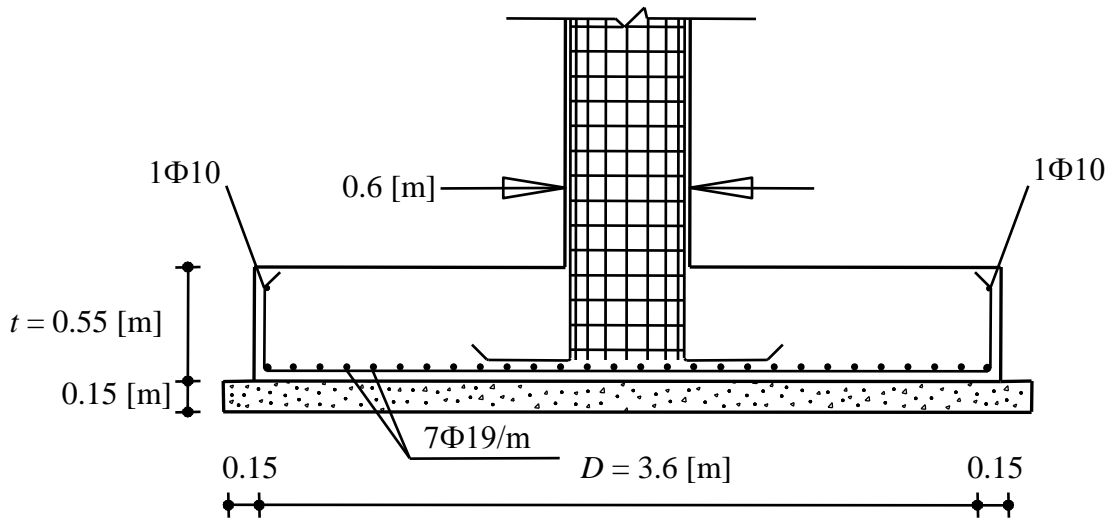
$$A_s = \frac{0.293}{179 \times 0.5} = 0.003274 \text{ [m}^2/1.649 \text{ m]}$$

$$A_s = 32.74 \text{ [cm}^2/1.649 \text{ m]} = 19.85 \text{ [cm}^2/\text{m]}$$

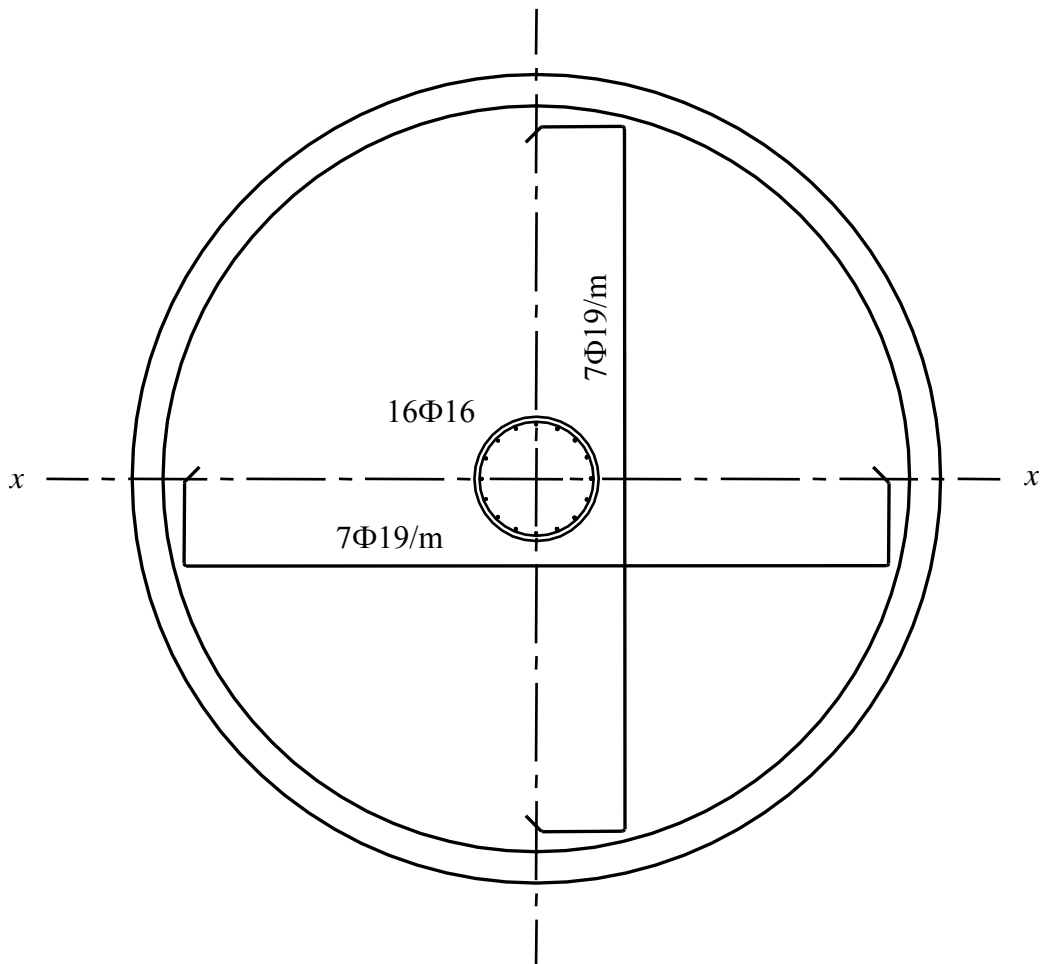
Chosen steel $7\Phi 19/\text{m} = 19.90 \text{ [cm}^2/\text{m]}$

Figure 48 shows the footing dimensions and reinforcement.

Solved Examples



a) Section $x-x$



b) Plan

Figure 48: Footing dimensions and reinforcement

Solved Examples

Example 22: Design of a rectangular combined footing for two interior columns (1)

Design a combined footing for two equal columns, each of 0.50×0.50 [m], reinforced by $4\Phi 19$, carrying a load of $P = 1500$ [kN] as shown in Figure 49. The distance center to center of columns is 3.0 [m]. The allowable soil pressure is $q_{all} = 210$ [kN/m²] at a depth of $D_f = 2.0$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

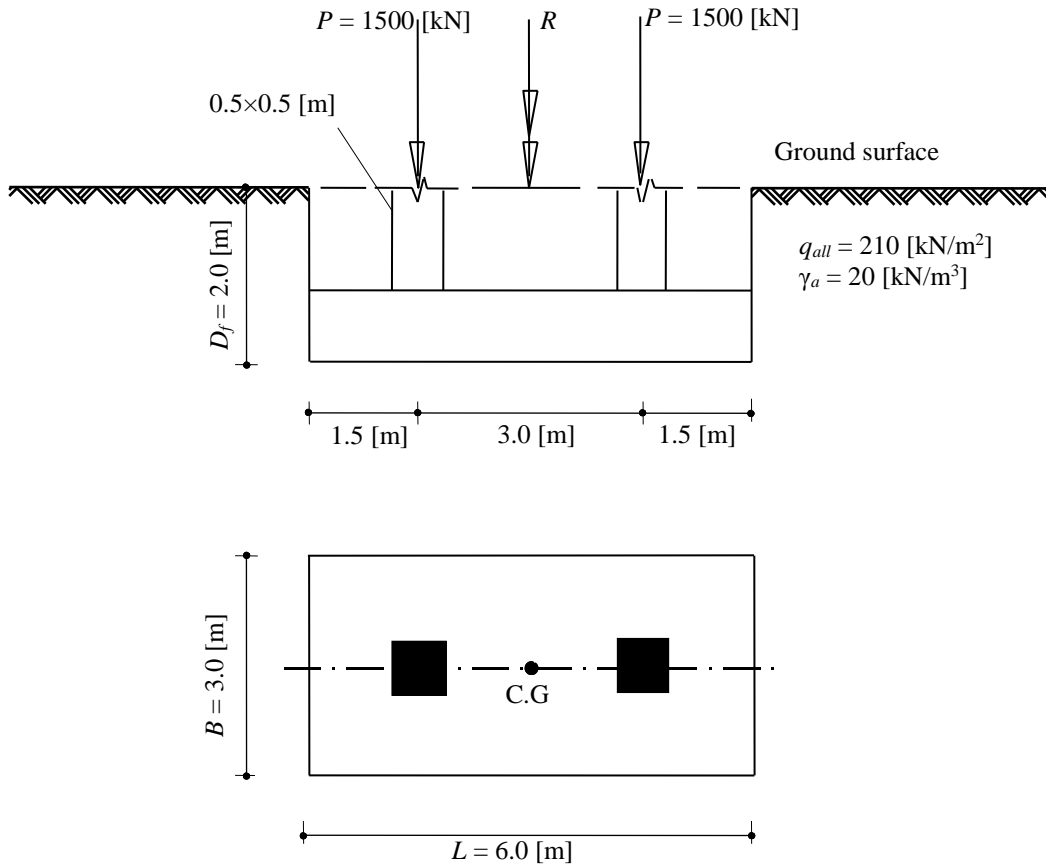


Figure 49: Rectangular combined footing

1. Determining footing sides L and B

Resultant of loads R at the ground surface level is given by:

$$R = 2P = 2 \times 1500 = 3000 \text{ [kN]}$$

Resultant of total loads R_T at the foundation level is given by:

$$R_T = \frac{R}{\left(1 - \frac{\gamma_a D_f}{q_{all}}\right)}$$

Solved Examples

$$R_T = \frac{3000}{\left(1 - \frac{20 \times 2.0}{210}\right)} = 3705.88 \text{ [kN]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R_T}{q_{all}} = \frac{3705.88}{210} = 17.65 \text{ [m}^2\text{]}$$

take $A_f = 3.0 \times 6.0 = 18.0 \text{ [m}^2\text{]}$ rectangular combined footing

2. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{3000}{3 \times 6} = 166.67 \text{ [kN/m}^2\text{]} = 0.167 \text{ [MN/m}^2\text{]}$$

Now, the rectangular footing is treated as a beam footing. The contact pressure per meter is given by:

$$q = q_o B = 0.167 \times 3.0 = 0.5 \text{ [MN/m]}$$

Figure 50 shows the load diagram and moment diagram for the beam footing.

3. Determining the maximum moment M_{max}

From Figure 50, the maximum bending moment occurs at the face of the column which is:

$$M_{max} = q \frac{c_1^2}{2}$$

$$M_{max} = 0.5 \times \frac{1.25^2}{2} = 0.391 \text{ [MN.m]}$$

Solved Examples

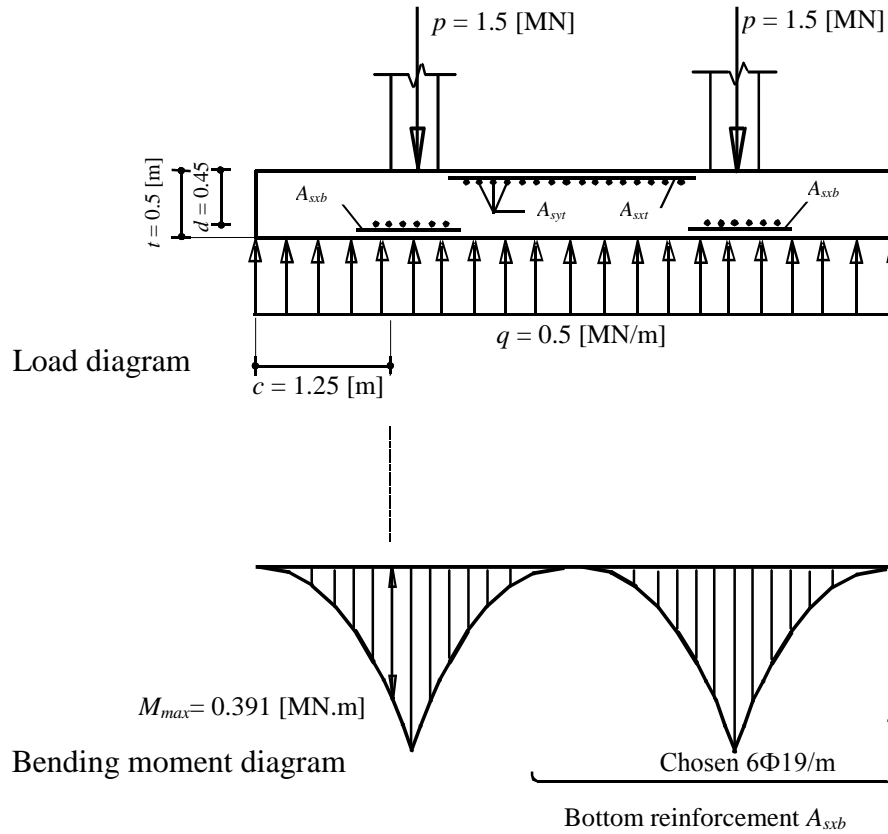


Figure 50: Contact pressure, bending moment and critical section

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.391}{3.0}} = 0.28 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.28 \text{ [m]}$, then the section is designed as singly reinforced section.

The corresponding k_1 for $d = 0.45 \text{ [m]}$ is given by:

$$k_1 = 0.45 \sqrt{\frac{3.0}{0.391}} = 1.25$$

Solved Examples

From Table 2 at $k_1 = 1.27$, $f_c = 5.0$ [MN/m²] and $k_2 = 182$

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.225$ [m] from the face of the column as shown in Figure 51.

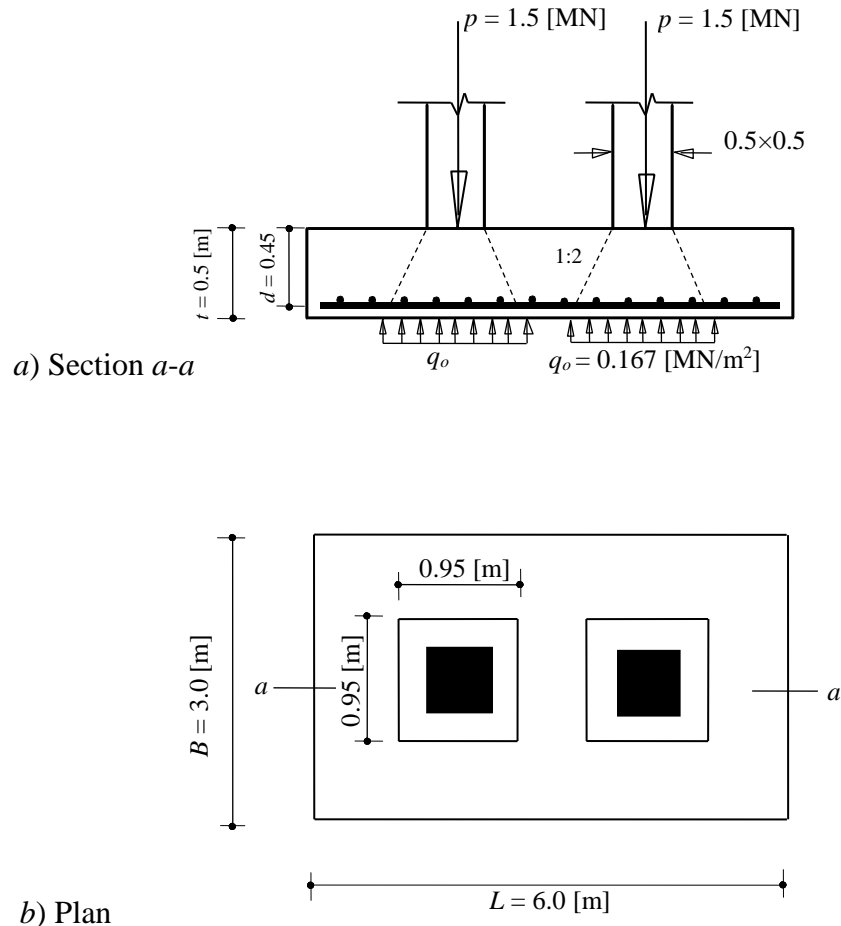


Figure 51: Critical section for punching shear

Geometry (Figure 51)

Effective depth of the section	$d = 0.45$ [m]
Column side	$a = b = 0.5$ [m]
Area of critical punching shear section	$A_p = (a + d)^2 = 0.9025$ [m ²]
Perimeter of critical punching shear section	$b_o = 4(a + d) = 3.8$ [m]

Loads and stresses

Column load	$P = 1.5$ [MN]
Soil pressure under the column	$q_o = 0.167$ [MN/m ²]
Main value of shear strength for concrete C 250	$q_{cp} = 0.9$ [MN/m ²]

Check for section capacity

Solved Examples

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.5 - 0.167 \times 0.9025 = 1.35 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{1.35}{3.8 \times 0.45} = 0.79 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9 \text{ [MN/m}^2\text{]} > q_p = 0.79 \text{ [MN/m}^2\text{]}$, the section is safe for punching shear.

6. Computing the area of steel reinforcement

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 50 \times 100 = 7.5 \text{ [cm}^2/\text{m]}$

Take $A_{smin} = 5\Phi 16 = 10.1 \text{ [cm}^2/\text{m]}$.

The required area of steel reinforcement A_s is:

$$A_{sxb} = \frac{M_{\max}}{k_2 d}$$

$$A_{sxb} = \frac{0.391}{182 \times 0.45} = 0.004774 \text{ [m}^2/3.0 \text{ m]}$$

$$A_{sxb} = 47.74 \text{ [cm}^2/3.0 \text{ m]} = 15.91 \text{ [cm}^2/\text{m]}$$

Chosen steel $6\Phi 19/\text{m} = 17.0 \text{ [cm}^2/\text{m]}$

Solved Examples

7. Computing the steel reinforcement in transverse direction

The transverse bending moment may be approximately determined at each column by assuming that the column load is distributed outward at 45 [°] from the face of the column over an appropriate area as shown in Figure 52. Both sides of this area should not be greater than the breadth of the footing.

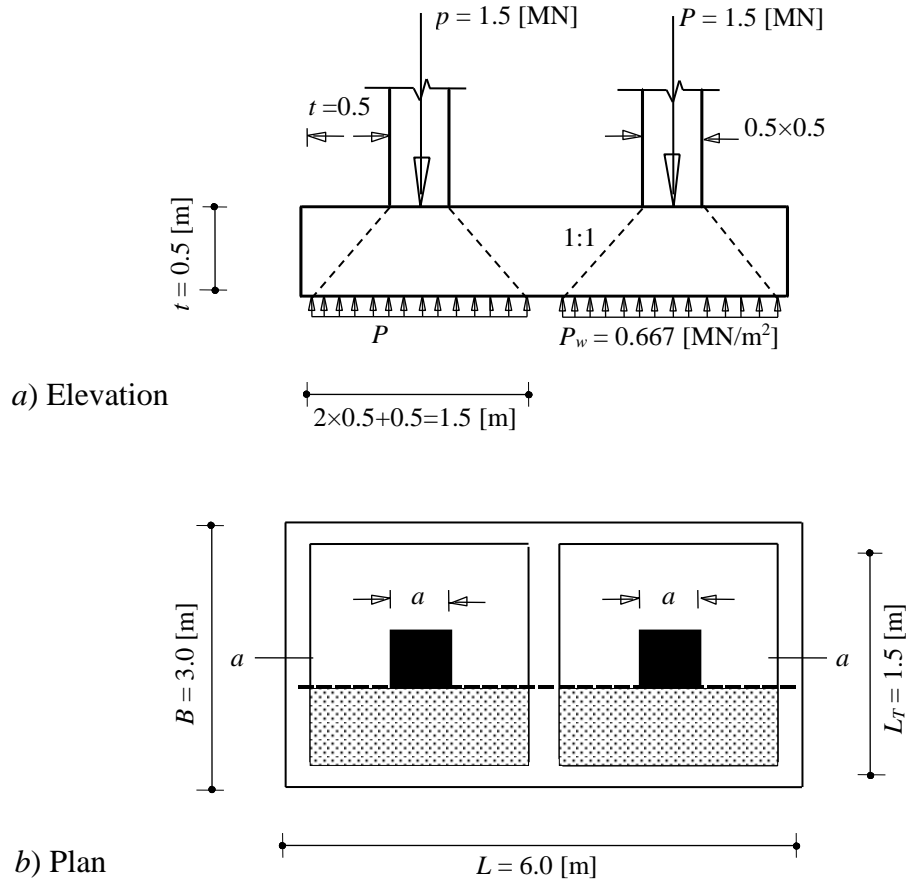


Figure 52: Sections for determining transverse moments

Side of the loaded area L_T is given by:

$$L_T = a + 2t = 0.5 + 2 \times 0.5 = 1.5 \text{ [m]}$$

The transverse bending moment M_T under the column is given by:

$$M_T = \frac{P}{8} \frac{(L_T - a)^2}{L_T} = \frac{1.5}{8} \frac{(1.5 - 0.5)^2}{1.5} = 0.125 \text{ [MN.m]}$$

The required area of steel reinforcement in transverse direction under the column A_{sT} is:

$$A_{sT} = \frac{M_T}{k_2 d}$$

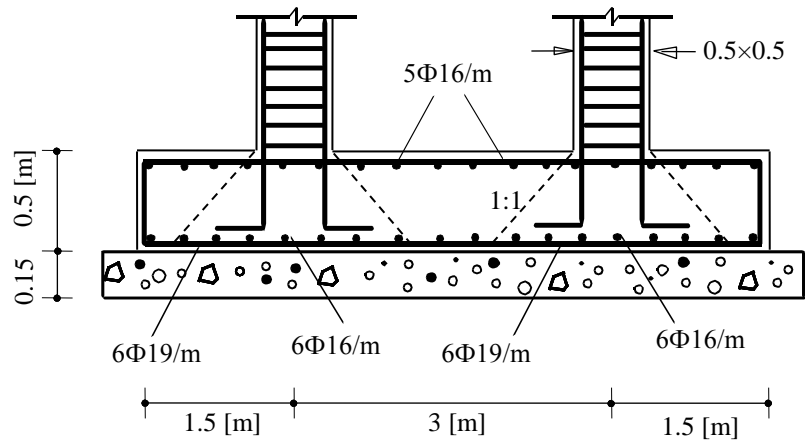
Solved Examples

$$A_{sT} = \frac{0.125}{182 \times 0.45} = 0.001526 \text{ [m}^2\text{/1.5m]}$$

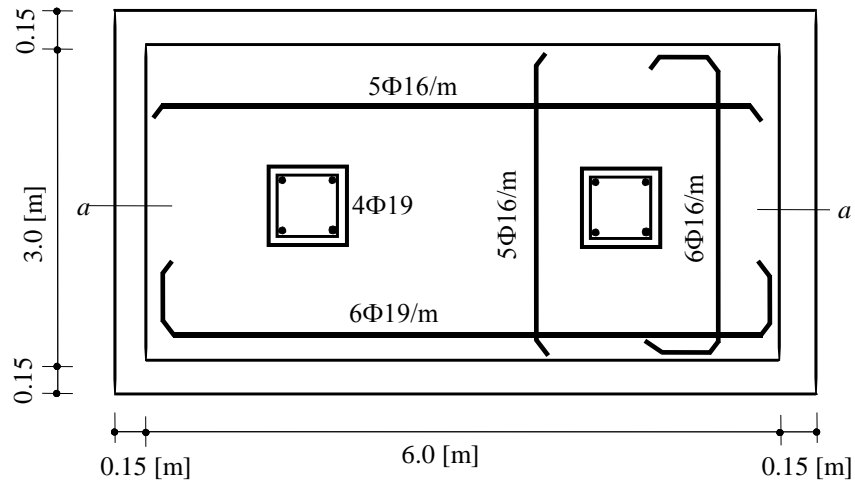
$$A_{sT} = 15.26 \text{ [cm}^2\text{/1.5m]} = 10.17 \text{ [cm}^2\text{/m]}$$

Chosen steel $6\Phi 16/\text{m} = 12.10 \text{ [cm}^2\text{/m]}$.

The details of reinforcement in plan and section *a-a* through the footing are shown in Figure 53.



a) Section *a-a*



b) Plan

Figure 53: Details of reinforcement in plan and section *a-a* through the footing

Solved Examples

Example 23: Design of a rectangular combined footing for two interior columns (2)

Design a combined footing for two equal columns, each of 0.50×0.30 [m], reinforced by $4\Phi 19$, carrying a load of $P = 1000$ [kN] as shown in Figure 54. The distance center to center of columns is 4.5 [m]. The allowable soil pressure is $q_{all} = 150$ [kN/m²] at a depth of $D_f = 1.5$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

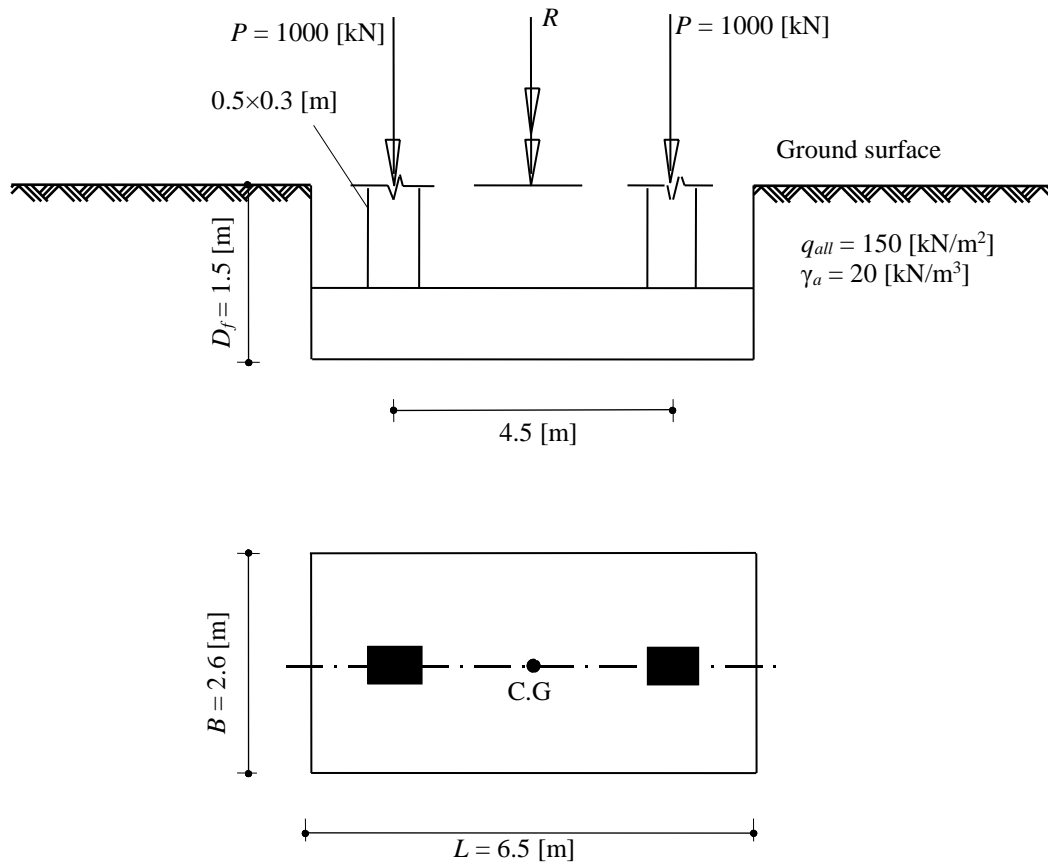


Figure 54: Rectangular combined footing

1. Determining footing sides L and B

Resultant of loads R at the ground surface level is given by:

$$R = 2P = 2 \times 1000 = 2000 \text{ [kN]}$$

Resultant of total loads R_T at the foundation level is given by:

$$R_T = \frac{R}{\left(1 - \frac{\gamma_a D_f}{q_{all}}\right)}$$

Solved Examples

$$R_T = \frac{2000}{\left(1 - \frac{20 \times 1.5}{150}\right)} = 2500 \text{ [kN]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R_T}{q_{all}} = \frac{2500}{150} = 16.67 \text{ [m}^2\text{]}$$

take $A_f = 2.6 \times 6.5 = 16.9 \text{ [m}^2\text{]}$ rectangular combined footing

2. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{2000}{2.6 \times 6.5} = 118.34 \text{ [kN/m}^2\text{]} = 0.118 \text{ [MN/m}^2\text{]}$$

Now, the rectangular footing is treated as a beam footing. The contact pressure per meter is given by:

$$q = q_o B = 0.118 \times 2.6 = 0.308 \text{ [MN/m]}$$

Figure 55 shows the load diagram and moment diagram for the beam footing.

3. Determining the maximum moment M_{max}

From Figure 55, the maximum bending moment occurs at the face of the column which is:

$$M_{c1} = q \frac{c^2}{2}$$

$$M_{c1} = 0.308 \times \frac{1.0^2}{2} = 0.154 \text{ [MN.m]}$$

$$M_{max} = P \frac{S}{2} - q \frac{(L/2)^2}{2}$$

$$M_{max} = 1 \times \frac{4.5}{2} - 0.308 \times \frac{(6.5/2)^2}{2} = 0.623 \text{ [MN.m]}$$

Solved Examples

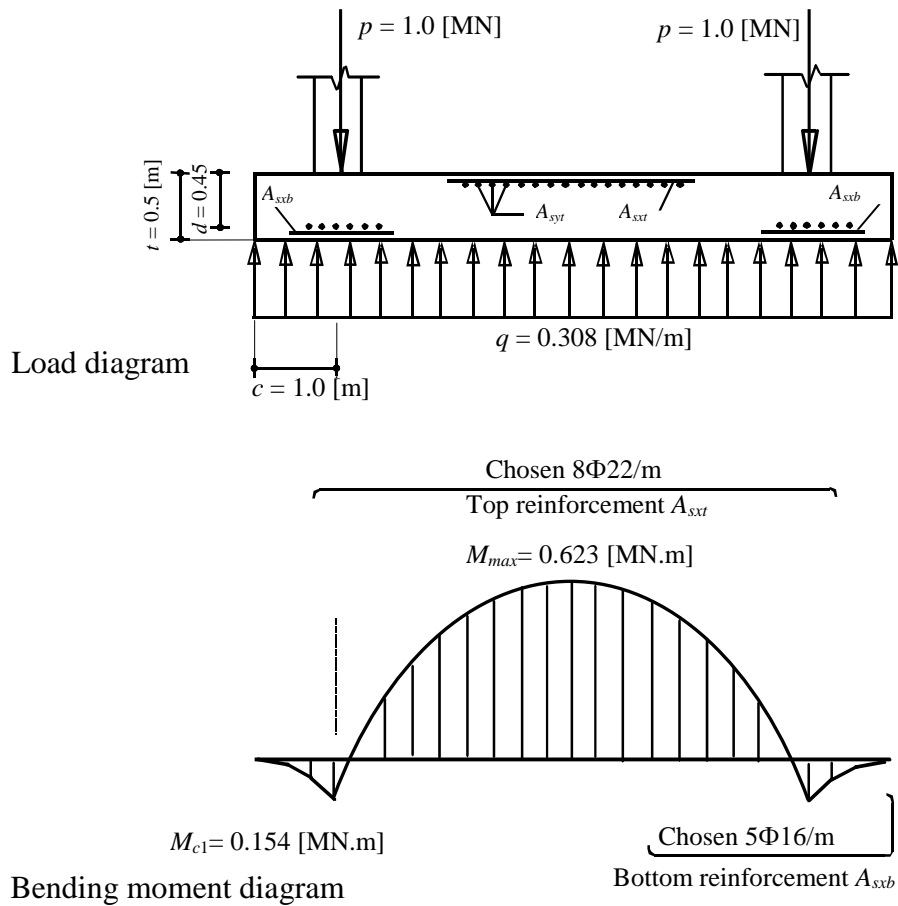


Figure 55: Contact pressure, bending moment and critical section

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5$ [MN/m²] and $f_s = 200$ [MN/m²], the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient k_2 [MN/m²] to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172$ [MN/m²].

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.623}{2.6}} = 0.37 \text{ [m]}$$

Take $d = 0.45$ [m] $> d_m = 0.37$ [m], then the section is designed as singly reinforced section.

The corresponding k_1 for $d = 0.45$ [m] is given by:

Solved Examples

$$k_1 = 0.45 \sqrt{\frac{2.6}{0.623}} = 0.919$$

From Table 2 at $k_1 = 0.968$, $f_c = 7.0$ [MN/m²] and $k_2 = 177$

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.225$ [m] from the face of the column as shown in Figure 56.

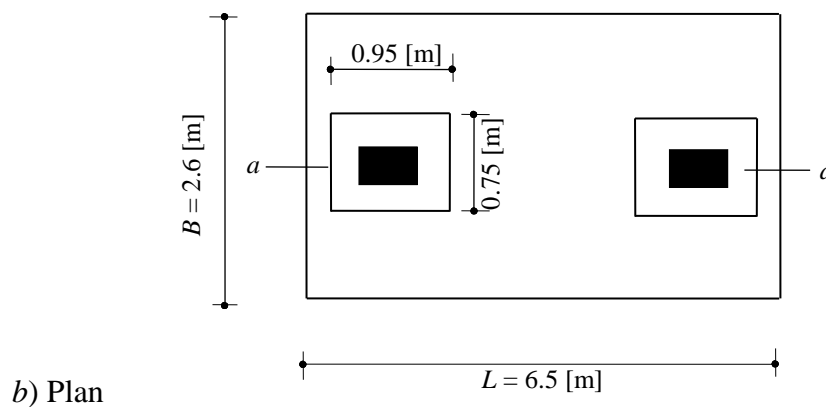
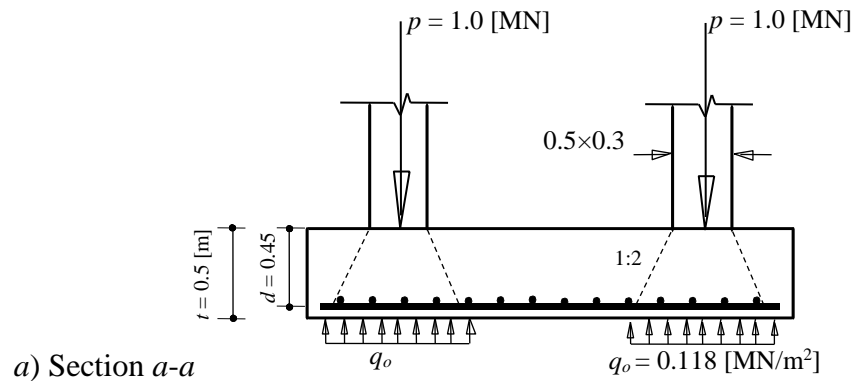


Figure 56: Critical section for punching shear

Geometry (Figure 56)

Effective depth of the section	$d = 0.45$ [m]
Column side	$a = 0.5$ [m] and $b = 0.3$ [m]
Area of critical punching shear section	$A_p = (a + d)(b + d) = 0.7125$ [m ²]
Perimeter of critical punching shear section	$b_o = 2(a + b + 2d) = 3.4$ [m]

Loads and stresses

Column load	$P = 1.0$ [MN]
-------------	----------------

Solved Examples

Soil pressure under the column $q_o = 0.118$ [MN/m²]

Main value of shear strength for concrete C 250 $q_{cp} = 0.9$ [MN/m²]

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.0 - 0.118 \times 0.7125 = 0.916 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{0.916}{3.4 \times 0.45} = 0.6 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9$ [MN/m²] $> q_p = 0.6$ [MN/m²], the section is safe for punching shear.

6. Computing the area of steel reinforcement

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 50 \times 100 = 7.5$ [cm²/m]

Take $A_{smin} = 5\Phi 16 = 10.1$ [cm²/m].

The required area of steel reinforcement A_{sxb} is:

$$A_{sxb} = \frac{M_{c1}}{k_2 d}$$

$$A_{sxb} = \frac{0.154}{177 \times 0.45} = 0.00193 \text{ [m}^2\text{/2.6 m]}$$

$$A_{sxb} = 19.33 \text{ [cm}^2\text{/2.6 m]} = 7.44 \text{ [cm}^2\text{/m]}$$

Chosen steel $5\Phi 16/\text{m} = 10.1$ [cm²/m]

Solved Examples

The required area of steel reinforcement A_{sxt} is:

$$A_{sxt} = \frac{M_{\max}}{k_2 d}$$

$$A_{sxt} = \frac{0.623}{177 \times 0.45} = 0.00782 \text{ [m}^2\text{/2.6 m]}$$

$$A_{sxt} = 78.22 \text{ [cm}^2\text{/2.6 m]} = 30.08 \text{ [cm}^2\text{/m]}$$

Chosen steel $8\Phi 22/\text{m} = 30.4 \text{ [cm}^2\text{/m]}$

7. Computing the steel reinforcement in transverse direction

The transverse bending moment may be approximately determined at each column by assuming that the column load is distributed outward at 45° from the face of the column over an appropriate area as shown in Figure 57. Both sides of this area should not be greater than the breadth of the footing.

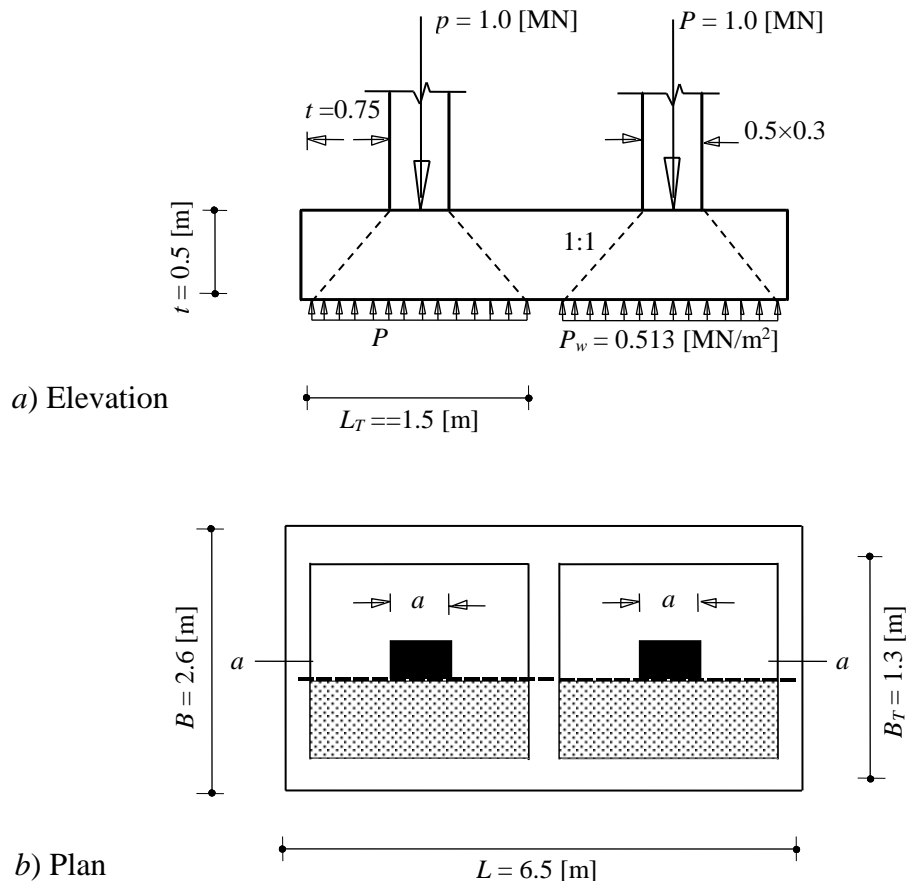


Figure 57: Sections for determining transverse moments

Side of the loaded area L_T is given by:

$$L_T = a + 2t = 0.5 + 2 \times 0.5 = 1.5 \text{ [m]}$$

Solved Examples

Side of the loaded area B_T is given by:

$$B_T = b + 2 t = 0.3 + 2 \times 0.5 = 1.3 \text{ [m]}$$

The transverse bending moment M_T under the column is given by:

$$M_T = \frac{P}{8} \frac{(B_T - b)^2}{B_T} = \frac{1.0}{8} \frac{(1.3 - 0.3)^2}{1.3} = 0.096 \text{ [MN.m]}$$

The required area of steel reinforcement in transverse direction under the column A_{sT} is:

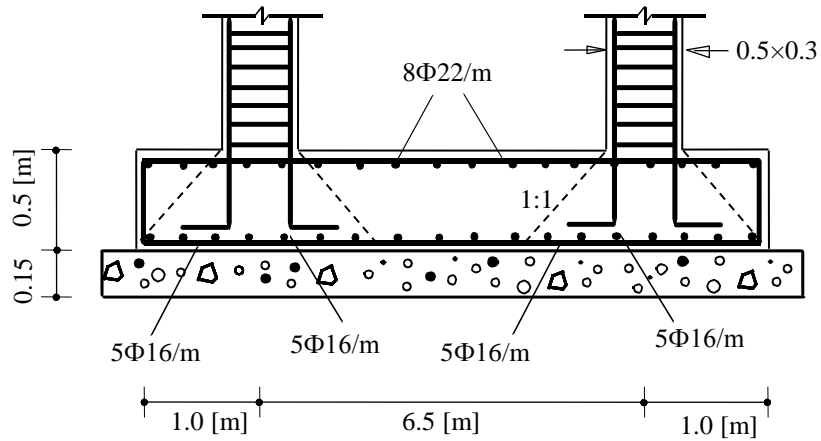
$$A_{sT} = \frac{M_T}{k_2 d}$$

$$A_{sT} = 12.08 \text{ [cm}^2\text{/1.5m]} = 8.05 \text{ [cm}^2\text{/m]}$$

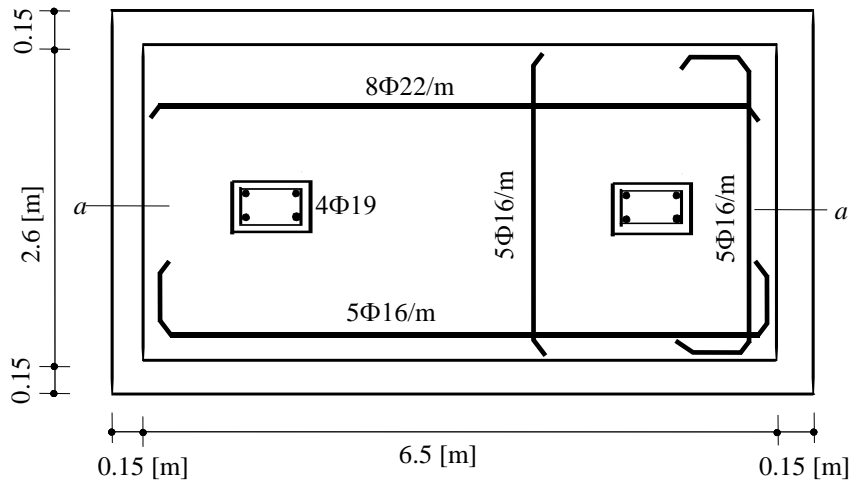
Chosen steel $5\Phi 16/\text{m} = 10.10 \text{ [cm}^2\text{/m]}$.

The details of reinforcement in plan and section $a-a$ through the footing are shown in Figure 58.

Solved Examples



a) Section *a-a*



b) Plan

Figure 58: Details of reinforcement in plan and section *a-a* through the footing

Solved Examples

Example 24: Design of a rectangular combined footing for two edge columns

Design a rectangular combined footing for two equal edge columns, each carrying a load of $P = 1125$ [kN] as shown in Figure 59. Column sides are 0.50×0.50 [m²] while column reinforcement is $4\Phi 19$. The allowable soil pressure is $q_{all} = 210$ [kN/m²] at a depth of $D_f = 1.5$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

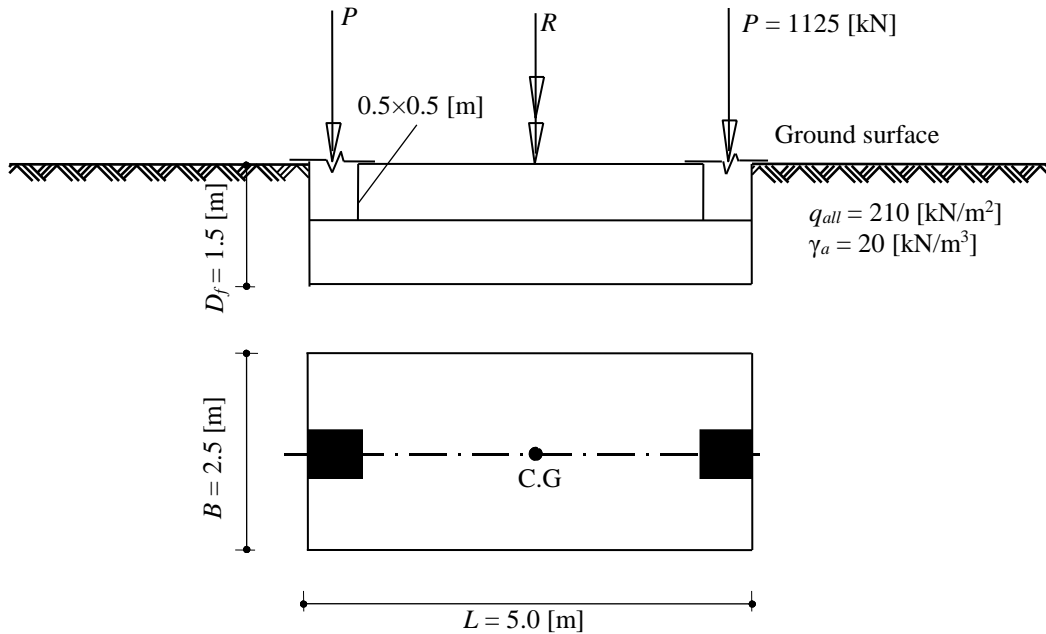


Figure 59: Combined rectangular footing

1. Determining footing sides L and B

Resultant of loads R at the ground surface level is given by:

$$R = 2P = 2 \times 1125 = 2250 \text{ [kN]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 210 - 20 \times 1.5 = 180 \text{ [kN/m}^2\text{]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R}{q_{net}} = \frac{2250}{180} = 12.5 \text{ [m}^2\text{]}$$

take $A_f = 2.5 \times 5.0 = 12.5$ [m²] rectangular combined footing

2. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

Solved Examples

$$q_o = \frac{R}{A_f} = \frac{2250}{12.5} = 180 \text{ [kN/m}^2\text{]} = 0.18 \text{ [MN/m}^2\text{]}$$

Now, the rectangular footing is treated as a beam footing. The contact pressure per meter is given by:

$$q = q_o B = 0.18 \times 2.5 = 0.45 \text{ [MN/m]}$$

Figure 60 shows the load diagram and moment diagram for the beam footing.

3. Determining the maximum moment M_{max}

From Figure 60, the maximum bending moment in the longitudinal direction occurs at the middle of the footing which is:

$$M_{max} = P \left(\frac{l}{2} - c \right) - q \frac{l^2}{8} = 1.125 \left(\frac{5.0}{2} - 0.25 \right) - 0.45 \frac{5.0^2}{8} = 1.125 \text{ [MN.m]}$$

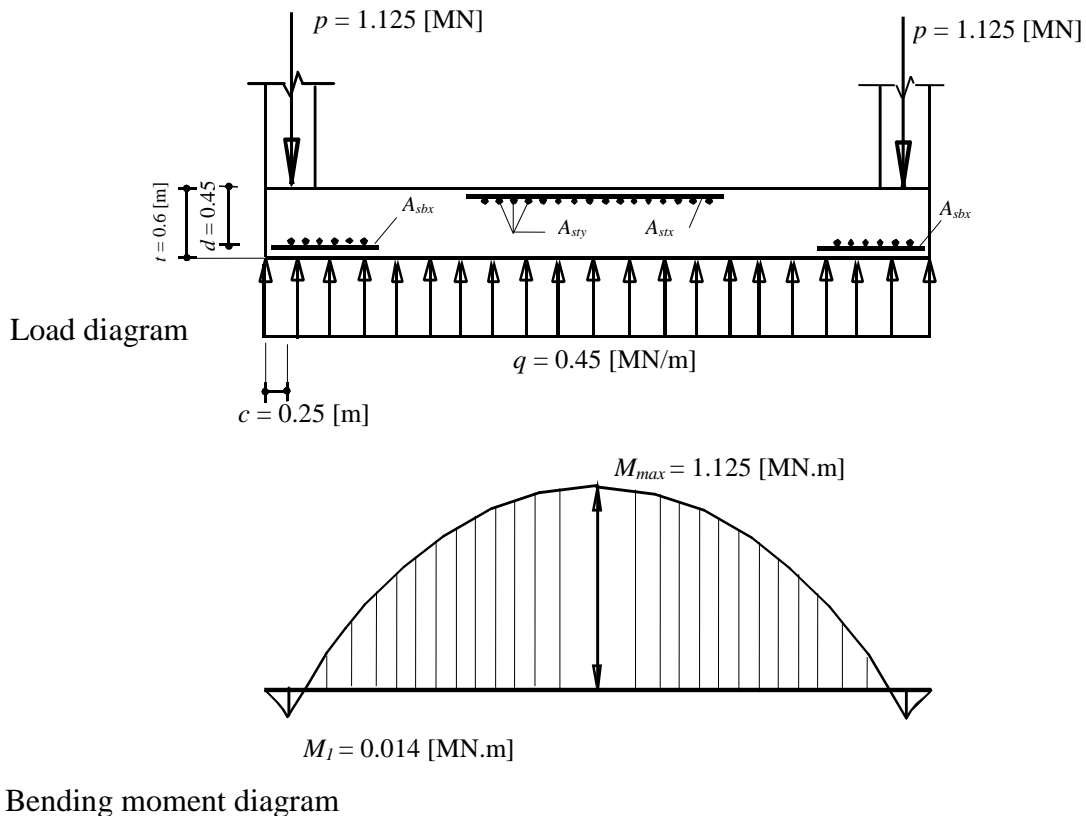


Figure 60: Contact pressure, bending moment and critical sections

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

Solved Examples

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{1.125}{2.5}} = 0.51 \text{ [m]}$$

Take $d = 0.55 \text{ [m]} > d_m = 0.51 \text{ [m]}$, then the section is designed as singly reinforced section.

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.275 \text{ [m]}$ from the face of the column as shown in Figure 61.

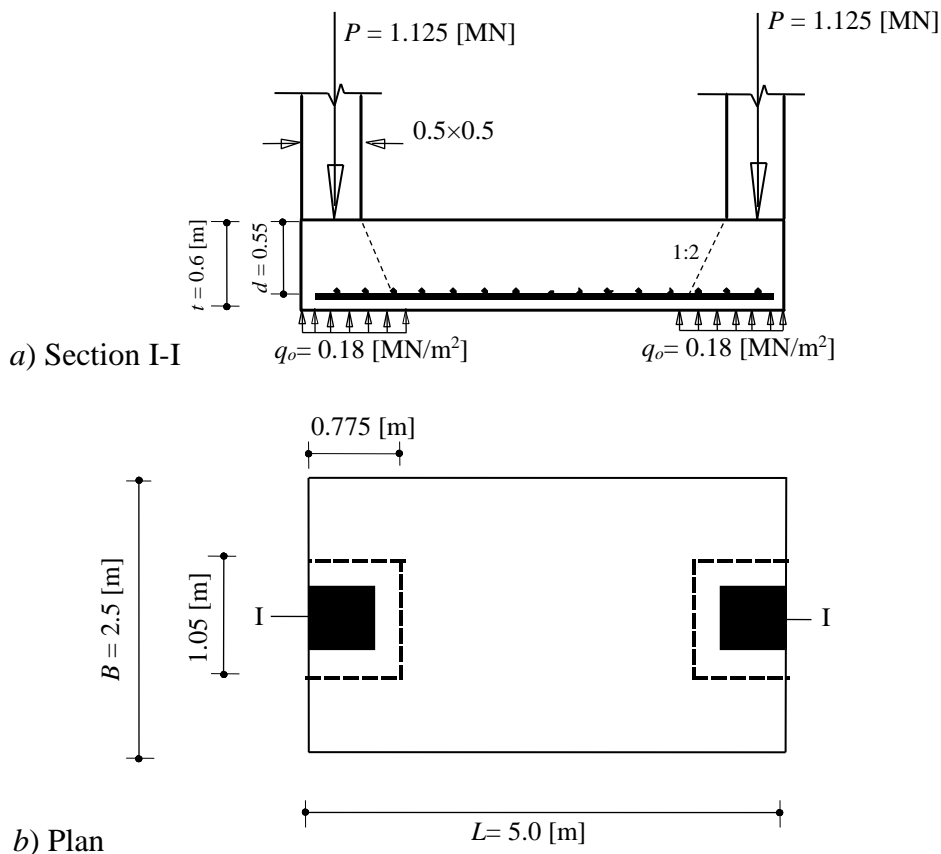


Figure 61: Critical section for punching shear

Geometry (Figure 61)

Effective depth of the section	$d = 0.55 \text{ [m]}$
Column side	$a = b = 0.5 \text{ [m]}$
Area of critical punching shear section	$A_p = (a + d/2) (b + d) = 0.81 \text{ [m}^2\text{]}$
Perimeter of critical punching shear section	$b_o = 2 (a + d/2) + (b + d) = 2.6 \text{ [m]}$

Solved Examples

Loads and stresses

Column load	$P = 1.125$ [MN]
Soil pressure under the column	$q_o = 0.18$ [MN/m ²]
Main value of shear strength for concrete C 250	$q_{cp} = 0.9$ [MN/m ²]

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.125 - 0.18 \times 0.81 = 0.98 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{0.98}{2.6 \times 0.55} = 0.69 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9$ [MN/m²] $> q_p = 0.69$ [MN/m²], the section is safe for punching shear.

6. Computing the area of steel reinforcement

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 60 \times 100 = 9.0$ [cm²/ m]
Take $A_{smin} = 5\Phi 16 = 10.1$ [cm²/ m].

The required area of steel reinforcement A_s is:

$$A_{sxt} = \frac{M_{\max}}{k_2 d}$$

$$A_{sxt} = \frac{1.125}{172 \times 0.55} = 0.011892 \text{ [m}^2\text{/2.5 m]}$$

Solved Examples

$$A_{sxt} = 118.92 \text{ [cm}^2/2.5 \text{ m]} = 47.57 \text{ [cm}^2/\text{m]}$$

Chosen steel $13\Phi 22/\text{m} = 49.4 \text{ [cm}^2/\text{m]}$

7. Computing the steel reinforcement in transverse direction

The transverse bending moment may be approximately determined at each column by assuming that the column load is distributed outward at 45° from the face of the column over an appropriate area as shown in Figure 62. Both sides of this area should not be greater than the breadth of the footing.

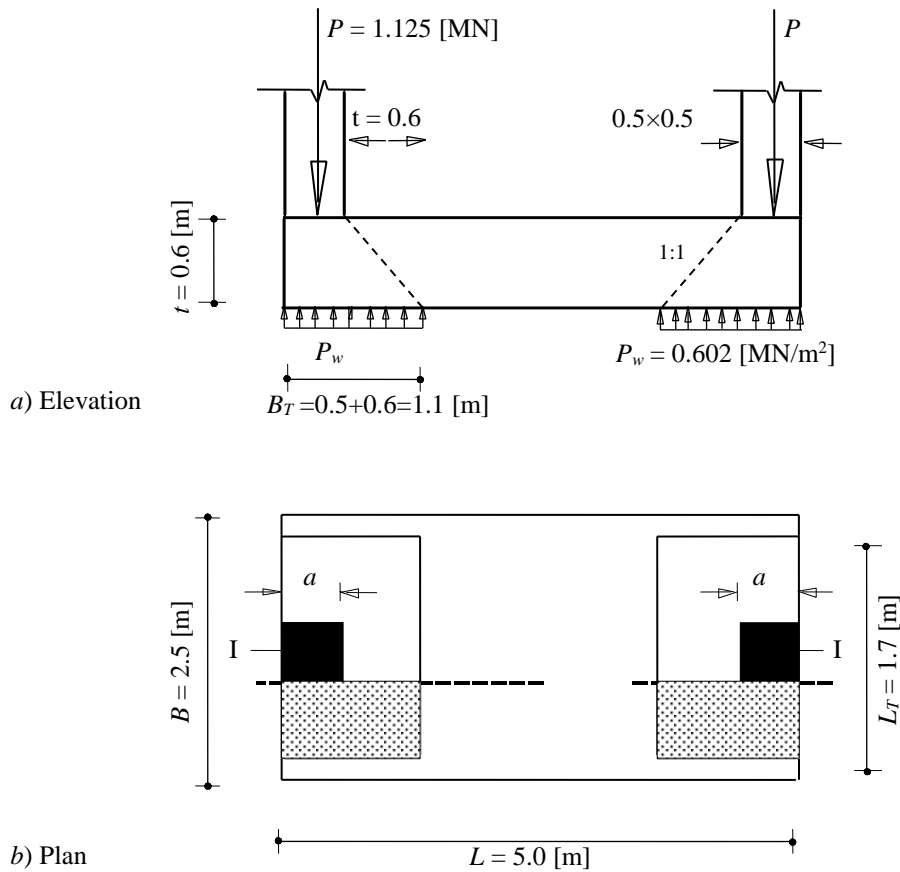


Figure 62: sections for determining transverse moments

Length of the loaded area L_T is given by:

$$L_T = a + 2t = 0.5 + 2 \times 0.6 = 1.7 \text{ [m]}$$

Breadth of the loaded area B_T is given by:

$$B_T = a + t = 0.5 + 0.6 = 1.1 \text{ [m]}$$

Distributed load P_w under the column is:

$$P_w = \frac{P}{L_T B_T} = \frac{1.125}{1.7 \times 1.1} = 0.602 \text{ [MN/m}^2\text{]}$$

Solved Examples

The transverse bending moment M_T under the column is given by:

$$M_T = P_w B_T \frac{t^2}{2} = 0.602 \times 1.1 \frac{0.6^2}{2} = 0.119 \text{ [MN.m]}$$

The required area of steel reinforcement in transverse direction under the column A_{sT} is:

$$A_{sT} = \frac{M_T}{k_2 d}$$

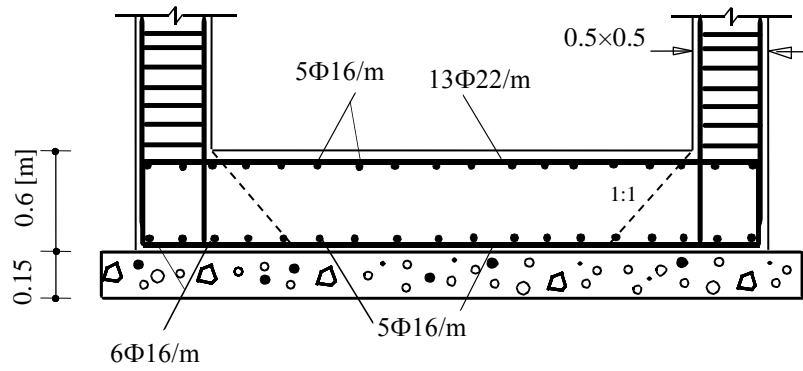
$$A_{sT} = \frac{0.119}{172 \times 0.55} = 0.001258 \text{ [m}^2\text{/1.1m]}$$

$$A_{sT} = 12.58 \text{ [cm}^2\text{/1.1m]} = 11.44 \text{ [cm}^2\text{/m]}$$

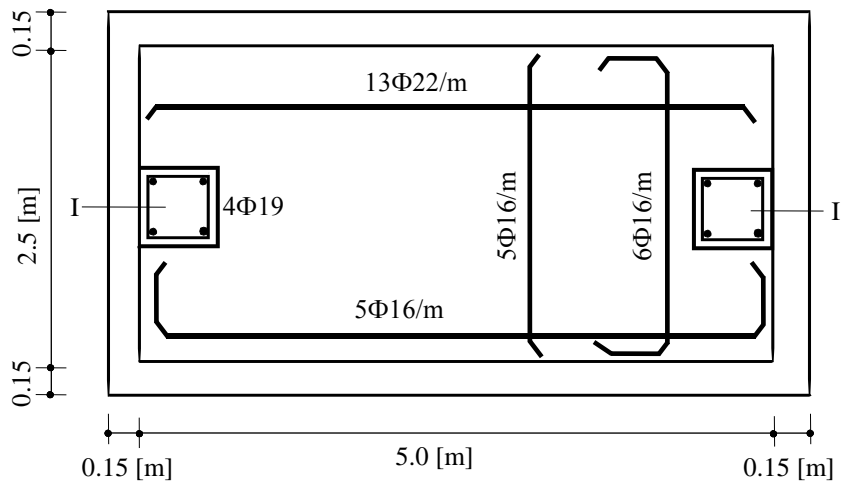
Chosen steel $6\Phi 16/\text{m} = 12.1 \text{ [cm}^2\text{/m]}$.

The details of reinforcement in plan and section *a-a* through the footing are shown in Figure 63.

Solved Examples



a) Section I-I



b) Plan

Figure 63: Details of reinforcement in plan and section *a-a* through the footing

Solved Examples

Example 25: Design of a rectangular combined footing for two unequal columns

Design a rectangular combined footing for an exterior column C_1 of a load of 1050 [kN] and an interior column C_2 of a load of 1800 [kN] as shown in Figure 64. C_1 is 0.50×0.50 [m²] and reinforced by $6\Phi 22$, while column C_2 is 0.60×0.50 [m²] and reinforced by $10\Phi 22$. The allowable soil pressure $q_{all} = 185$ [kN/m²] at a depth of $D_f = 1.90$ [m], and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

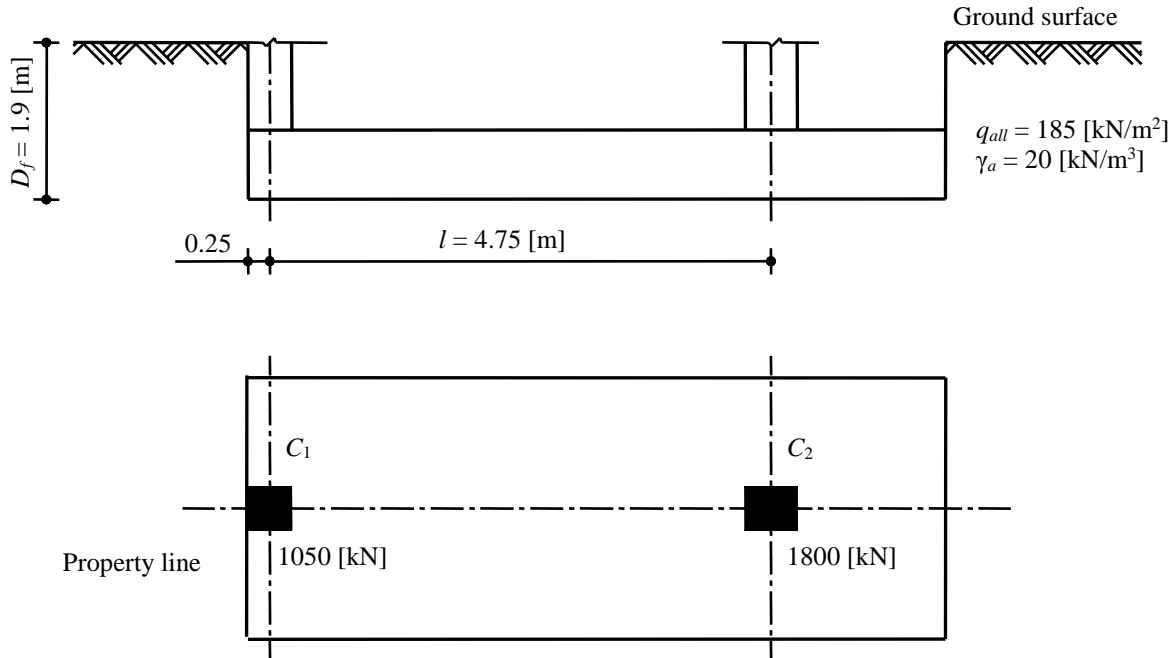


Figure 64: Combined rectangular footing

1. Determining point of application of the resultant force

Resultant force R equals:

$$R = P_1 + P_2 = 1050 + 1800 = 2850 \text{ [kN]}$$

Take the moment about C_1 to get the distance S :

$$P_2 l = R S$$

$$1800 \times 4.75 = 2850 \times S$$

$$S = 3.0 \text{ [m]}$$

2. Determining footing sides L and B

$$L = 2 X_p = 2 (0.25 + S) = 2 (0.25 + 3.0) = 6.5 \text{ [m]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 185 - 20 \times 1.9 = 147 \text{ [kN/m}^2\text{]}$$

Solved Examples

Area of footing A_f is obtained from:

$$A_f = \frac{R}{q_{net}} = \frac{2850}{147} = 19.39 \text{ [m}^2\text{]}$$

take $A_f = 6.5 \times 3.0 = 19.50 \text{ [m}^2\text{]}$ rectangular combined footing, as shown in Figure 65.

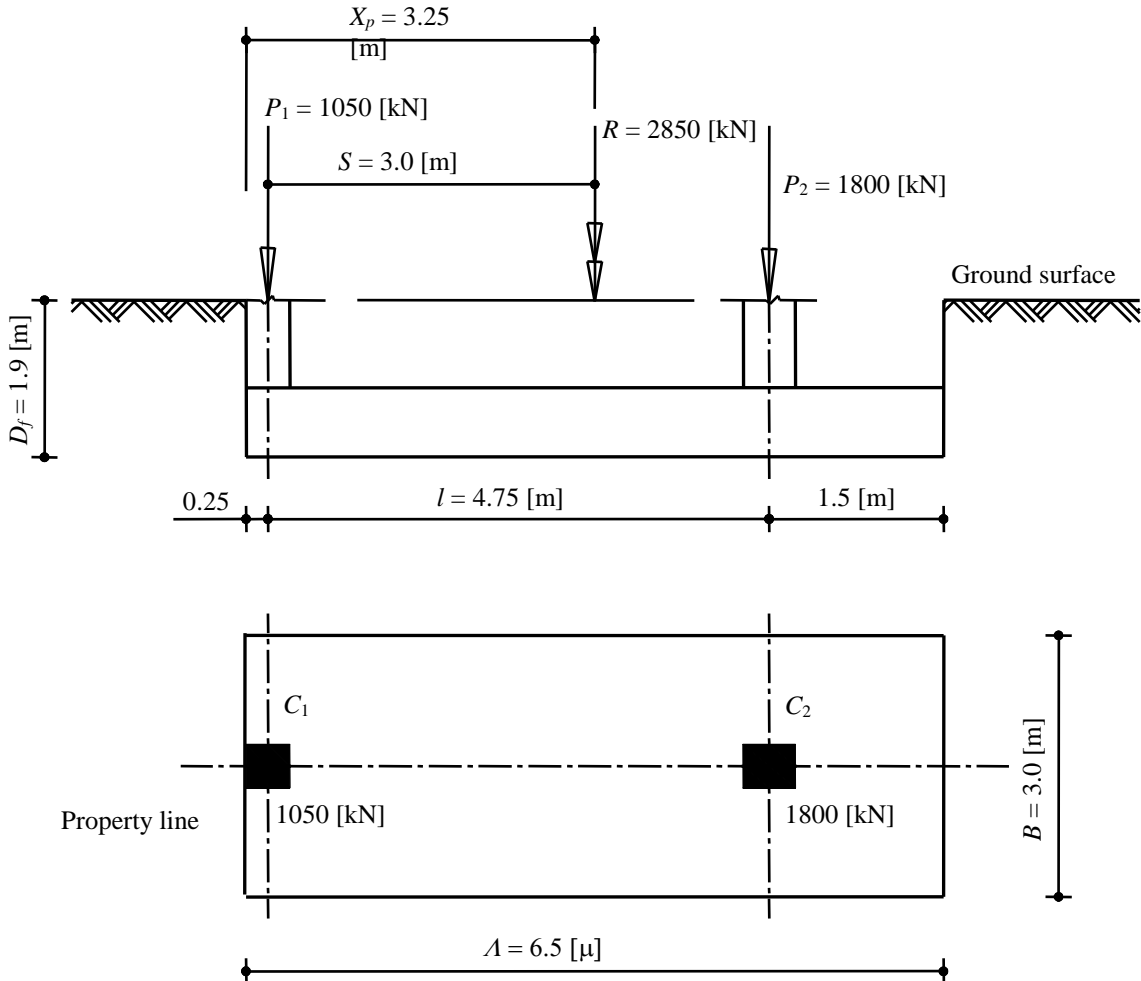


Figure 65: The combined rectangular ribbed footing with property line and column loads

3. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{2850}{19.5} = 146.15 \text{ [kN/m}^2\text{]} = 0.146 \text{ [MN/m}^2\text{]}$$

Now, the rectangular footing is treated as a beam footing. The contact pressure per meter under the footing is given by:

$$q = q_o B = 0.146 \times 3.0 = 0.438 \text{ [MN/m]}$$

Solved Examples

4. Determining the maximum moment M_{max}

Figure 66 shows the load diagram, bending moment and shear force diagram for the beam footing.

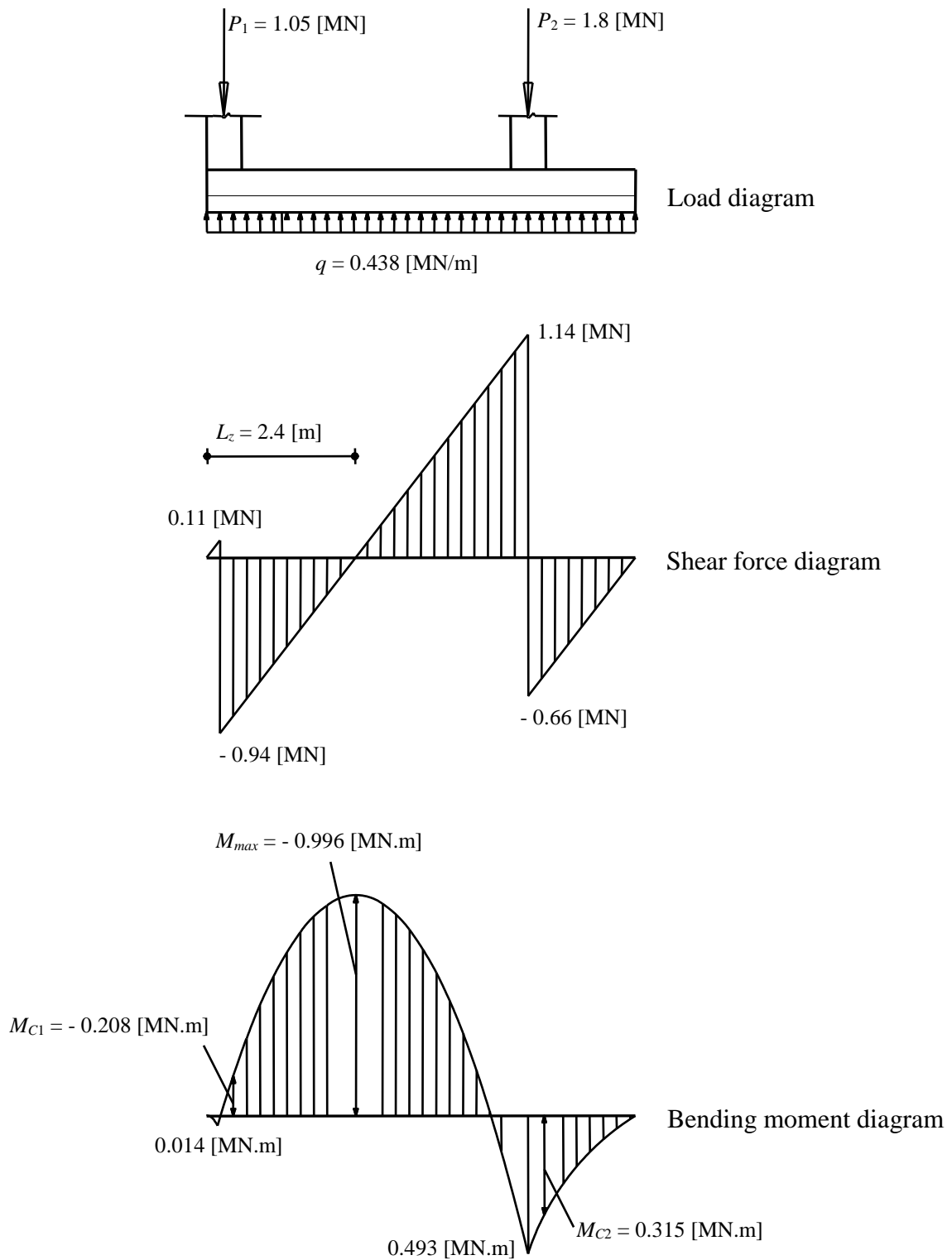


Figure 66: Load, shear force and bending moment diagrams with critical sections

Solved Examples

The critical bending moments acting on column sides are:

$$M_{C1} = 0.438 \times (0.5)^2 / 2 - 1.05 \times 0.25 = -0.208 \text{ [MN.m]}$$

$$M_{C2} = 0.438 \times (1.5)^2 / 2 = 0.493 \text{ [MN.m]}$$

The maximum bending moment occurs at the position of zero shear. Point of zero shear locates at l_z from the outer face of the footing.

$$P_1 = ql_z$$

$$1.05 = 0.438 \times l_z$$

$$l_z = 2.4 \text{ [m]}$$

$$M_{max} = q (l_z)^2 / 2 - P_1 (l_z - C_1) = 0.438 \times (2.4)^2 / 2 - 1.05 (2.4 - 0.25) = -0.996 \text{ [MN.m]}$$

5. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.996}{3}} = 0.44 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.44 \text{ [m]}$ and $t = 0.5 \text{ [m]}$.

6. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.225 \text{ [m]}$ from the face of the column as shown in Figure 67.

Column C1

Geometry (Figure 67)

Effective depth of the section	$d = 0.45 \text{ [m]}$
Column side	$a = b = 0.5 \text{ [m]}$
Area of critical punching shear section	$A_p = (a + d/2) (b + d) = 0.689 \text{ [m}^2\text{]}$
Perimeter of critical punching shear section	$b_o = 2 (a + d/2) + (b + d) = 2.4 \text{ [m]}$

Solved Examples

Loads and stresses

Column load	$P = 1.05$ [MN]
Soil pressure under the column	$q_o = 0.146$ [MN/m ²]
Main value of shear strength for concrete C 250	$q_{cp} = 0.9$ [MN/m ²]

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.05 - 0.146 \times 0.689 = 0.949 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{0.949}{2.4 \times 0.45} = 0.879 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9$ [MN/m²] $> q_p = 0.879$ [MN/m²], the section is safe for punching shear.

Column C2

Geometry (Figure 67)

Effective depth of the section	$d = 0.45$ [m]
Column side	$a = 0.6$ [m] and $b = 0.5$ [m]
Area of critical punching shear section	$A_p = (a + d)(b + d) = 0.998$ [m ²]
Perimeter of critical punching shear section	$b_o = 2(a + d) + 2(b + d) = 4.0$ [m]

Loads and stresses

Column load	$P = 1.8$ [MN]
Soil pressure under the column	$q_o = 0.146$ [MN/m ²]
Main value of shear strength for concrete C 250	$q_{cp} = 0.9$ [MN/m ²]

Solved Examples

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.8 - 0.146 \times 0.998 = 1.654 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{1.654}{4.0 \times 0.45} = 0.92 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

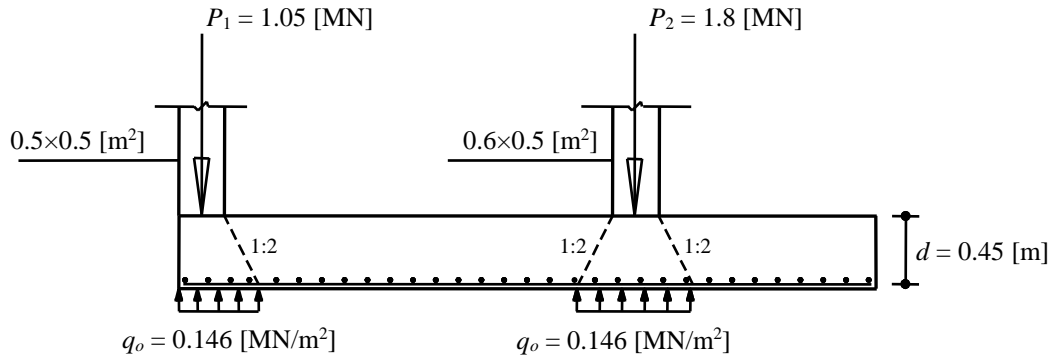
$q_{pall} = 0.9 \text{ [MN/m}^2\text{]} < q_p = 0.92 \text{ [MN/m}^2\text{]}$, the section is unsafe for punching shear.

Determining the depth required to resist punching shear d_p

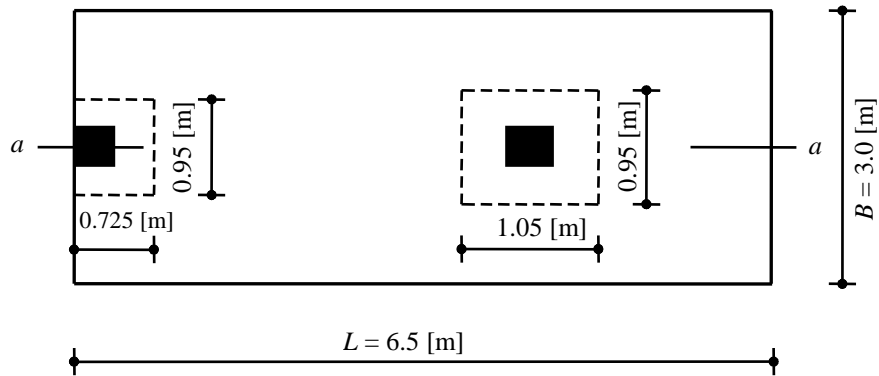
$$d_p = \frac{1.654}{4.0 \times 0.9} = 0.46 \text{ [m]}$$

Take $d = 0.50 \text{ [m]} > d_m = 0.46 \text{ [m]}$ and $t = 0.55 \text{ [m]}$, then the section is designed as singly reinforced section.

Solved Examples



a) Section a-a



b) Plan

Figure 67: Critical sections for punching shear

7. Computing the area of steel reinforcement in longitudinal direction A_s

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 55 \times 100 = 8.25 \text{ [cm}^2/\text{m]}$
 Take $A_{smin} = 5\Phi 16 = 10.1 \text{ [cm}^2/\text{m}]$.

The required area of steel reinforcement in longitudinal direction A_s is show in Table 4:

Table 4: Determination of tension reinforcement in longitudinal direction

Position	Breadth $B \text{ [m]}$	Moment $M \text{ [MN.m]}$	Calculated A_s $A_s = M / (k_2.d) / B$ $\text{[cm}^2/\text{m]}$	Chosen reinforcement A_s
$A_{sC1} = A_{sC3}$	3.0	- 0.208	8.06	$5\Phi 16/\text{m} = 10.1 \text{ cm}^2$
A_{smax}	3.0	- 0.996	38.61	$11\Phi 22/\text{m} = 41.8 \text{ cm}^2$
A_{sC2}	3.0	0.315	12.21	$7\Phi 16/\text{m} = 14.1 \text{ cm}^2$

8. Computing the area of steel reinforcement in transverse direction A_{sT}

The transverse bending moment may be approximately determined at each column by assuming that the column load is distributed outward at $45 [^\circ]$ from the face of the column over an appropriate area as shown in Figure 68. Both sides of this area should not be greater than the breadth of the footing.

Column C_1

Length of the loaded area L_{T1} is given by:

$$L_{T1} = a_1 + t = 0.5 + 0.55 = 1.05 \text{ [m]}$$

Breadth of the loaded area B_{T1} is given by:

$$B_{T1} = b_1 + 2 t = 0.5 + 2 \times 0.55 = 1.6 \text{ [m]}$$

The loaded area A_{T1} is given by:

$$A_{T1} = L_{T1} \times B_{T1} = 1.05 \times 1.6 = 1.68 \text{ [m}^2\text{]}$$

The contact pressure under the loaded area P_{w1} is given by:

$$P_{w1} = \frac{P_1}{A_{T1}} = \frac{1.05}{1.68} = 0.625 \text{ [MN/m}^2\text{]}$$

The transverse bending moment M_{T1} under the column is given by:

$$M_{T1} = P_{w1} \left(\frac{B_{T1} - b_1}{2} \right)^2 L_{T1} \times \frac{1}{2} = 0.625 \times \left(\frac{1.6 - 0.5}{2} \right)^2 \times 1.05 \times \frac{1}{2} = 0.099 \text{ [MN.m]}$$

The required area of steel reinforcement in transverse direction under the column A_{sT1} is:

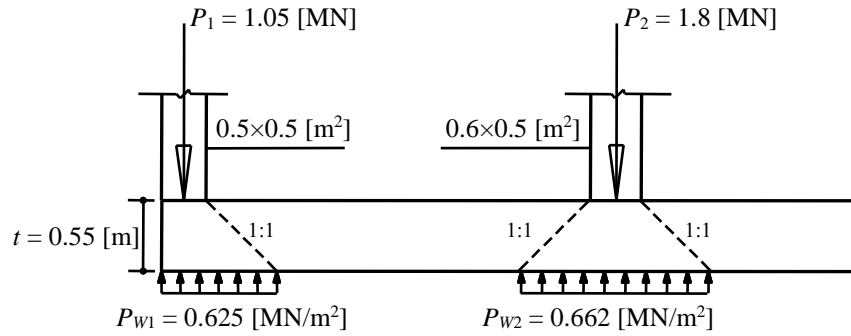
$$A_{sT1} = \frac{M_{T1}}{k_2 d}$$

$$A_{sT1} = \frac{0.099}{172 \times 0.5} = 0.001154 \text{ [m}^2\text{/1.05m]}$$

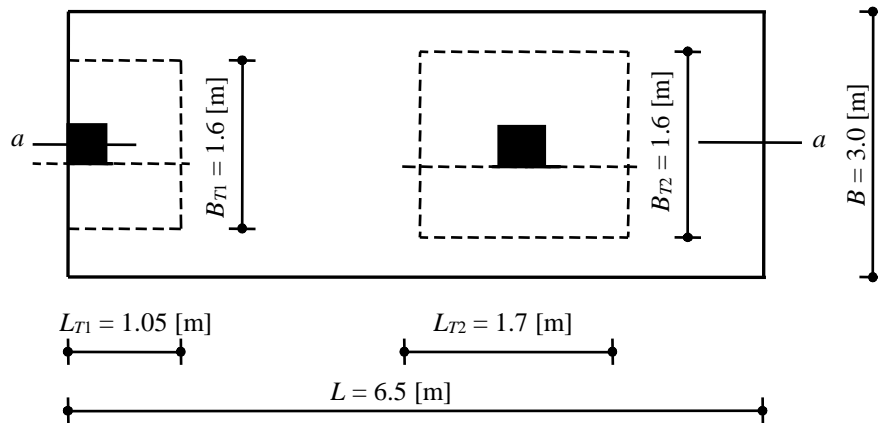
$$A_{sT1} = 11.54 \text{ [cm}^2\text{/1.05m]} = 10.99 \text{ [cm}^2\text{/m]}$$

Chosen steel $6\Phi 16/\text{m} = 12.1 \text{ [cm}^2\text{/m]}$

Solved Examples



a) Section *a-a*



b) Plan

Figure 68: sections of transverse moment

Column C_2

Length of the loaded area L_{T2} is given by:

$$L_{T2} = a_2 + 2 t = 0.6 + (2 \times 0.55) = 1.7 \text{ [m]}$$

Breadth of the loaded area B_{T2} is given by:

$$B_{T2} = b_2 + 2 t = 0.5 + (2 \times 0.55) = 1.6 \text{ [m]}$$

The loaded area A_{T2} is given by:

$$A_{T2} = L_{T2} \times B_{T2} = 1.7 \times 1.6 = 2.72 \text{ [m}^2\text{]}$$

The contact pressure under the loaded area P_{w2} is given by:

$$P_{w2} = \frac{P_2}{A_{T2}} = \frac{1.8}{2.72} = 0.662 \text{ [MN/m}^2\text{]}$$

Solved Examples

The transverse bending moment M_{T2} under the column is given by:

$$M_{T2} = P_{w2} \left(\frac{B_{T2} - b_2}{2} \right)^2 L_{T2} \times \frac{1}{2} = 0.662 \times \left(\frac{1.6 - 0.5}{2} \right)^2 \times 1.7 \times \frac{1}{2} = 0.17 \text{ [MN.m]}$$

The required area of steel reinforcement in transverse direction under the column A_{sT} is:

$$A_{sT2} = \frac{M_{T2}}{k_2 d}$$

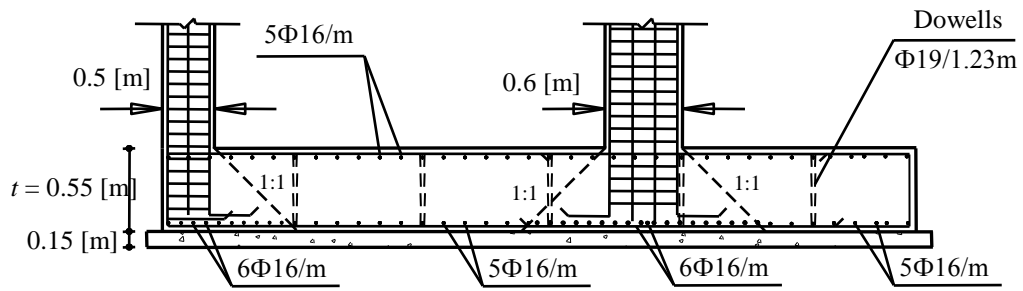
$$A_{sT2} = \frac{0.17}{172 \times 0.5} = 0.001977 \text{ [m}^2\text{/1.7m]}$$

$$A_{sT} = 19.77 \text{ [cm}^2\text{/1.7m]} = 11.63 \text{ [cm}^2\text{/m]}$$

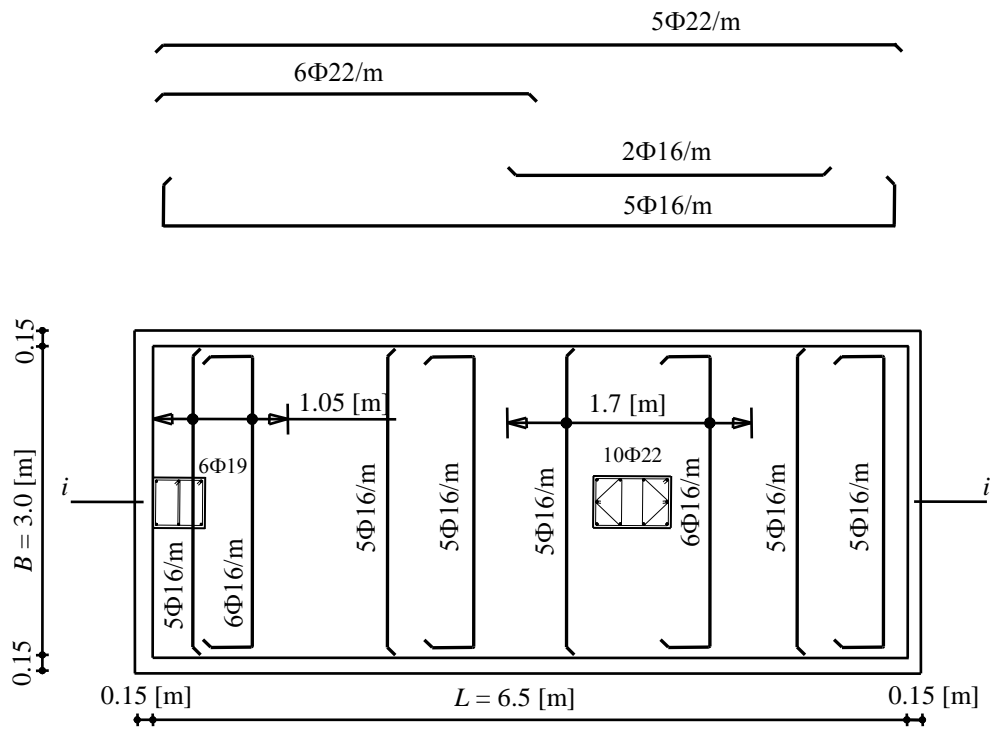
Chosen steel $6\Phi 16/\text{m} = 12.1 \text{ [cm}^2\text{/m]}$

The details of reinforcement in plan and section $i-i$ through the footing are shown in Figure 69.

Solved Examples



a) Section I-I



b) Plan with transverse reinforcement

Figure 69: Footing dimensions and reinforcement

Example 26: Design of a combined rectangular ribbed footing

Design a rectangular footing for an exterior column C_1 of a load of 1200 [kN] and an interior column C_2 of a load of 2250 [kN] as shown in Figure 70. Column C_1 is 0.50×0.65 [m²] and reinforced by $12\Phi 19$, while column C_2 is 0.60×0.65 [m²] and reinforced by $14\Phi 19$. The allowable soil pressure $q_{all} = 180$ [kN/m²] at a depth of $D_f = 1.8$ [m] and the average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³]. The footing has an inverted T- section.

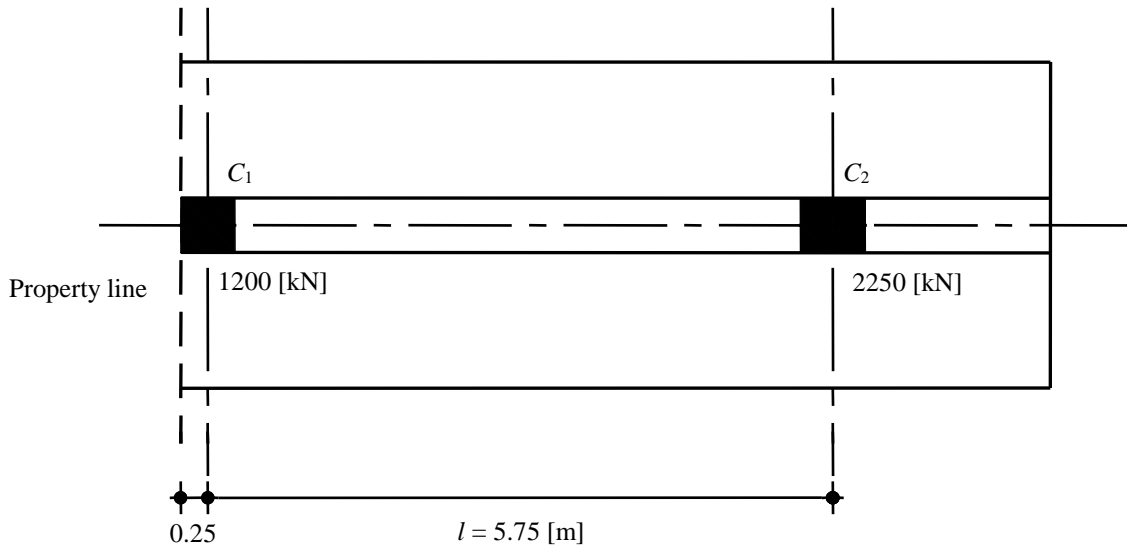


Figure 70: Plan of columns with property line and column loads

9. Determining point of application of the resultant force

Resultant force R equals:

$$R = P_1 + P_2 = 1200 + 2250 = 3450 \text{ [kN]}$$

Take the moment about C_1 to get the distance S :

$$P_2 l = R S$$

$$2250 \times 5.75 = 3450 \times S$$

$$S = 3.75 \text{ [m]}$$

10. Determining footing sides L and B

$$L = 2 (0.25 + S) = 2 (0.25 + 3.75) = 8.0 \text{ [m]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 180 - 20 \times 1.8 = 144 \text{ [kN/m}^2\text{]}$$

Area of footing A_f is obtained from:

Solved Examples

$$A_f = \frac{R}{q_{net}} = \frac{3450}{144} = 23.96 \text{ [m}^2\text{]}$$

take $A_f = 8.0 \times 3.0 = 24.0 \text{ [m}^2\text{]}$ rectangular combined footing, as shown in Figure 71.

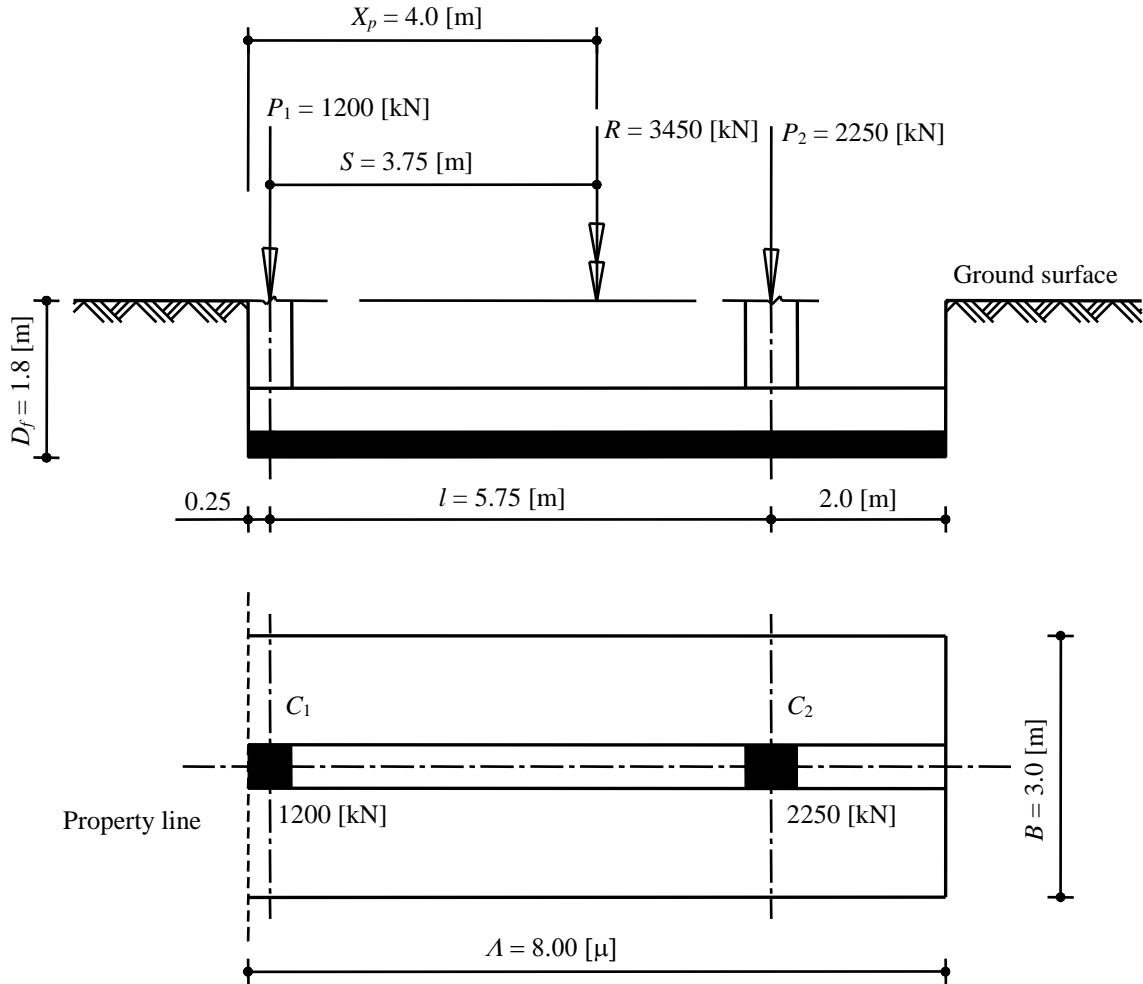


Figure 71: The combined rectangular ribbed footing with property line and column loads

11. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{3450}{24} = 143.75 \text{ [kN/m}^2\text{]} = 0.144 \text{ [MN/m}^2\text{]}$$

Now, the rectangular footing is treated as a beam footing with inverted T-section. The contact pressure per meter under the footing is given by:

$$q = q_o B = 0.144 \times 3.0 = 0.431 \text{ [MN/m]}$$

Solved Examples

12. Determining the maximum moment M_{max}

Figure 72 shows the load diagram, bending moment and shear force diagram for the beam footing.

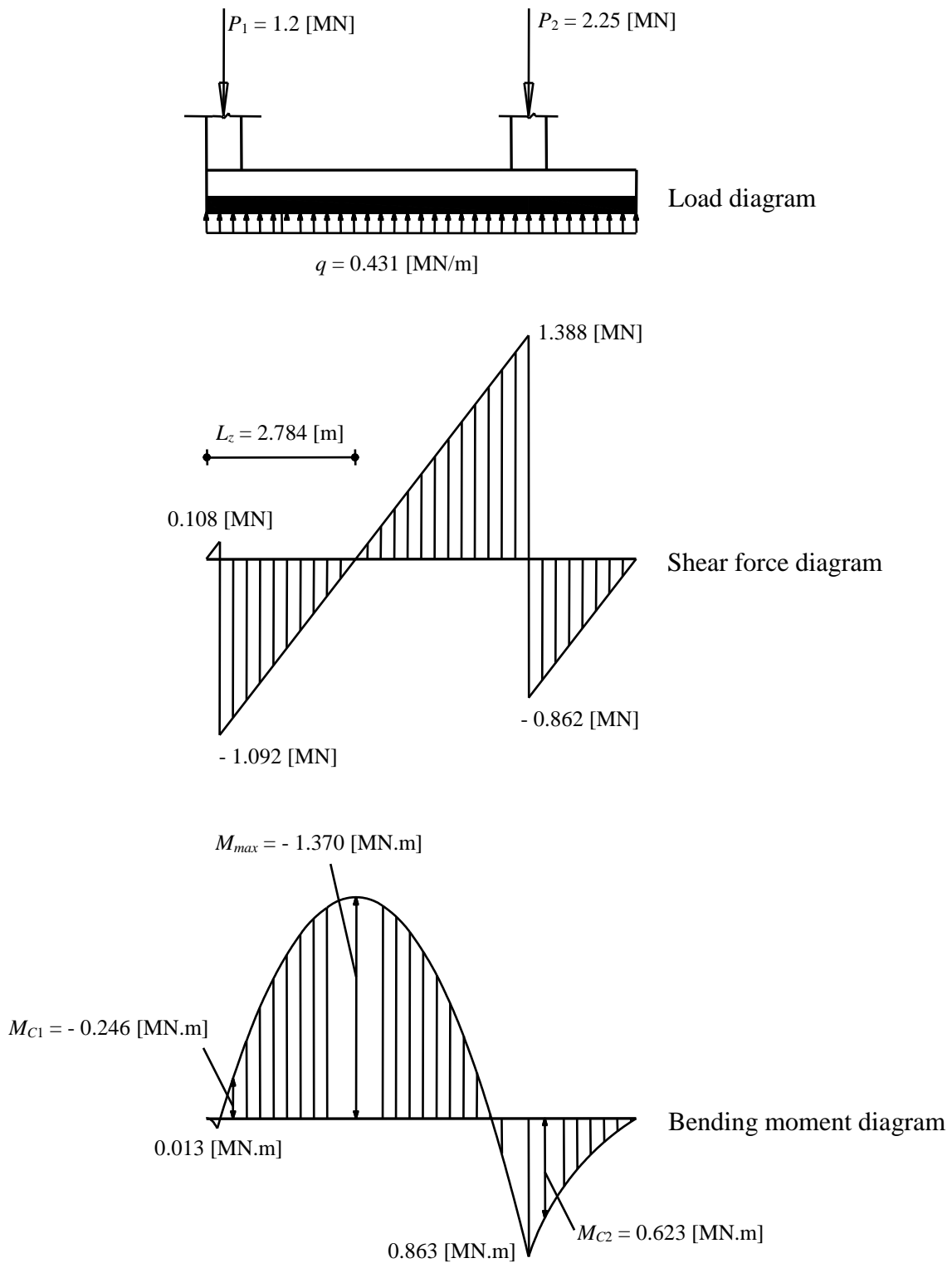


Figure 72: Load, shear force and bending moment diagrams with critical sections of the rib

Solved Examples

The critical bending moments acting on column sides are:

$$M_{C1} = 0.431 \times (0.5)^2 / 2 - 1.2 \times 0.25 = -0.246 \text{ [MN.m]}$$

$$M_{C2} = 0.431 \times (1.7)^2 / 2 = 0.623 \text{ [MN.m]}$$

The maximum bending moment occurs at the position of zero shear. Point of zero shear locates at l_z from the outer face of the footing.

$$P_1 = ql_z$$

$$1.2 = 0.431 \times l_z$$

$$l_z = 2.783 \text{ [m]}$$

$$M_{max} = q(l_z)^2 / 2 - P_1(l_z - C_1) = 0.431 \times (2.783)^2 / 2 - 1.2(2.783 - 0.25) = -1.370 \text{ [MN.m]}$$

13. Determining the depth of the rib d

Determining the depth d_m required to resist the longitudinal moment M_{max}

The section of the footing is T-section with depth d_m and breadth of flange B_T

B_T is the least of:

$$B = 3.0 \text{ [m]}$$

$$4b = 4 \times 0.5 = 2.0 \text{ [m]}$$

$$1/4 \text{ Span} = 5.2 / 4 = 1.3 \text{ [m]}$$

Take $B_T = 1.3 \text{ [m]}$

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{max}}{B_T}}$$

$$d_m = 0.766 \sqrt{\frac{1.37}{1.3}} = 0.79 \text{ [m]}$$

Take $d = 0.8 \text{ [m]} > d_m = 0.79 \text{ [m]}$ and $t = 0.85 \text{ [m]}$

Check for section capacity to resist maximum shear force Q_{max}

Solved Examples

The maximum critical shear section lies at a distance $d/2 = 0.375$ [m] from the side face of the internal column.

Effective depth of the section	$d = 0.8$ [m]
Rib width	$b = 0.65$ [m]
Main value of shear strength for concrete C250	$q_c = 0.9$ [MN/m ²]
Max.value of shear strength for concrete C250	$q_2 = 1.9$ [MN/m ²]

for maximum shear force Q_{max} :

$$Q(x) = 0.431 X - 1.2$$

$$X = 6 - 0.3 - 0.4 = 5.3 \text{ [m]}$$

$$Q_{max} = 0.431 \times 5.3 - 1.2 = 1.086 \text{ [MN]}$$

The shear stress q is given by:

$$q = \frac{Q_{max}}{b d}$$

$$q = \frac{1.086}{0.65 \times 0.8} = 2.09 \text{ [MN/m}^2\text{]} > q_2 = 1.90 \text{ [MN/m}^2\text{]}$$

Determining the depth d_s required to resist the shear force Q_{max}

$$d_s = \frac{Q_{max}}{b q_2}$$

$$d_s = \frac{1.086}{0.65 \times 1.90} = 0.88 \text{ [m]}$$

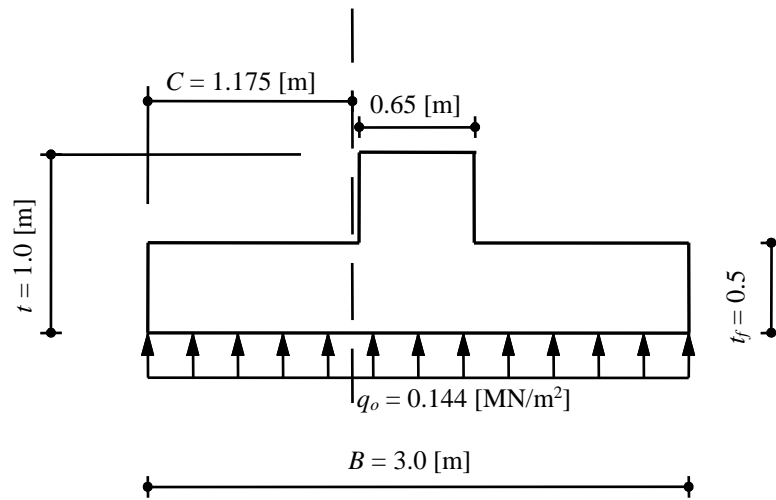
Take $d = 0.95$ [m] $> d_s = 0.88$ [m] and $t = 1.0$ [m], then the section is designed as singly reinforced T- section.

14. Determining the depth d_f required to resist the transverse moment M_T

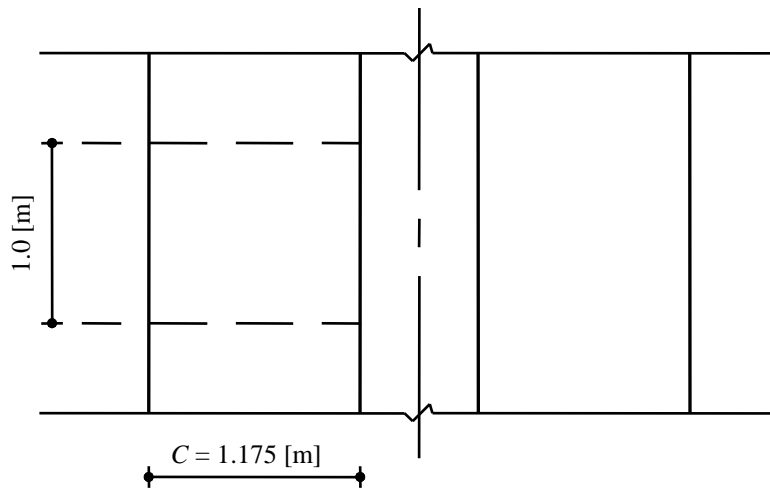
The maximum transverse bending moment M_T lies at the rib face as shown in Figure 73. M_T value is given by:

$$M_T = q_o \times 1.0 \times (c)^2 / 2 = 0.144 \times 1.0 \times (1.175)^2 / 2 = 0.099 \text{ [MN.m]}$$

Solved Examples



Cross section of the footing



Part plane of the footing

Figure 73: Critical section of transverse moment

The footing depth d_f is:

$$d_f = k_1 \sqrt{\frac{M_T}{1.0 \text{ [m]}}}$$

$$d_f = 0.766 \sqrt{\frac{0.099}{1.0}} = 0.242 \text{ [m]}$$

Take $d_f = 0.45$ [m] $>$ 0.242 [m] and $t_f = 0.5$ [m], then the section is designed as singly reinforced section.

15. Computing the area of steel reinforcement in longitudinal direction A_s

At first, we should calculate the position of the neutral axis:

Solved Examples

$$k_z = \sqrt{\frac{30}{f_c(r+1)}}$$

$$f_c = 9.5 \text{ [MN/m}^2\text{]} = 95 \text{ [kg/cm}^2\text{]}$$

$$r = f_s / f_c = 200 / 9.5 = 21 \text{ [-]}$$

$$k_z = \sqrt{\frac{30}{95(21+1)}} = 0.1198$$

$$z = k_z \sqrt{\frac{M_{\max}}{B_T}} = 0.1198 \sqrt{\frac{1.37 \times 10^7}{130}} = 38.89 \text{ [cm]} < t_f = 50 \text{ [cm]}$$

The neutral axis lies inside the slab footing, so the section is treated as rectangular section with B_T and d_m as shown in Figure 74.

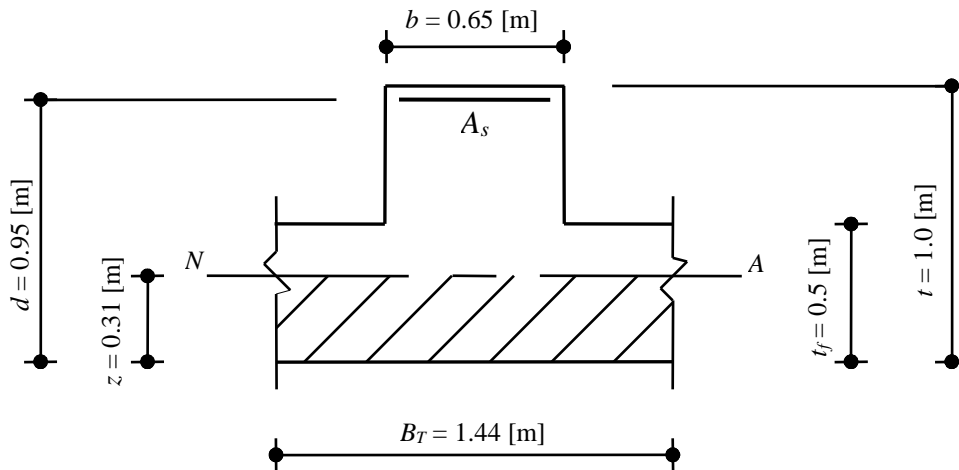


Figure 74: T- section subjected to longitudinal moment M_{\max}

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 50 \times 100 = 7.5 \text{ [cm}^2/\text{m]}$
 Take $A_{smin} = 5\Phi 16 = 10.1 \text{ [cm}^2/\text{m]}$.

The required maximum area of steel reinforcement in longitudinal direction A_{smax} is:

$$A_{smax} = \frac{M_{\max}}{k_2 d}$$

$$A_{smax} = \frac{1.37}{172 \times 0.95} = 0.00839 \text{ [m}^2\text{]} = 83.9 \text{ [cm}^2\text{]}$$

Chosen steel $18\Phi 25 = 88.2 \text{ [cm}^2\text{]}$

The required area of steel reinforcement for sec. 2 in longitudinal direction A_{sC2} is:

Solved Examples

$$A_{sc2} = \frac{M_{c2}}{k_2 d}$$

$$A_{sc2} = \frac{0.623}{172 \times 0.95} = 0.003813 \text{ [m}^2\text{]} = 38.13 \text{ [cm}^2\text{]}$$

Chosen steel 8Φ25 = 39.28 [cm²]

16. Check for shear stress

The maximum critical shear section lies at a distance $d/2 = 0.475$ [m] from the side face of the internal column.

for maximum shear force Q_{max} :

$$Q(x) = 0.431 X - 1.2$$

$$X = 6 - 0.3 - 0.475 = 5.225 \text{ [m]}$$

$$Q_{max} = 0.431 \times 5.225 - 1.2 = 1.052 \text{ [MN]}$$

The shear stress q is given by:

$$q = \frac{Q_{max}}{b d}$$

$$q = \frac{1.052}{0.65 \times 0.95} = 1.704 \text{ [MN/m}^2\text{]}$$

$$q_2 = 1.9 \text{ [MN/m}^2\text{]} > q = 1.704 \text{ [MN/m}^2\text{]} > q_c = 0.9 \text{ [MN/m}^2\text{]}$$

The shear would be resisted using vertical stirrups and bent bars.

Shear stress carried by stirrups

Using stirrups 5Φ10/ m, 4 branches @ 200 [mm]

$$q_{ss} = \frac{A_{st} f_s}{b S}$$

$$q_{ss} = \frac{(4 \times 78.5 \times 10^{-6}) \times 200}{0.65 \times 0.2} = 0.483 \text{ [MN/m}^2\text{]}$$

Shear stress carried by bent bars

$$q_{sb} = q_s - q_{ss} = 1.254 - 0.483 = 0.771 \text{ [MN/m}^2\text{]}$$

$$q_{sb} = \frac{A_{sb} f_s}{b d} (\sin \alpha + \cos \alpha)$$

Solved Examples

$$0.771 = \frac{A_{sb} \times 200}{0.65 \times 0.95} (\sin 60^\circ + \cos 60^\circ)$$

$$A_{sb} = 0.001742 \text{ [m}^2\text{]} = 17.42 \text{ [cm}^2\text{]}$$

$$\text{Use bent bars } 4\Phi 25 = 19.635 \text{ [cm}^2\text{]}$$

17. Computing the area of steel reinforcement in transverse direction A_{sT}

The required area of steel reinforcement in transverse direction A_{sT} is:

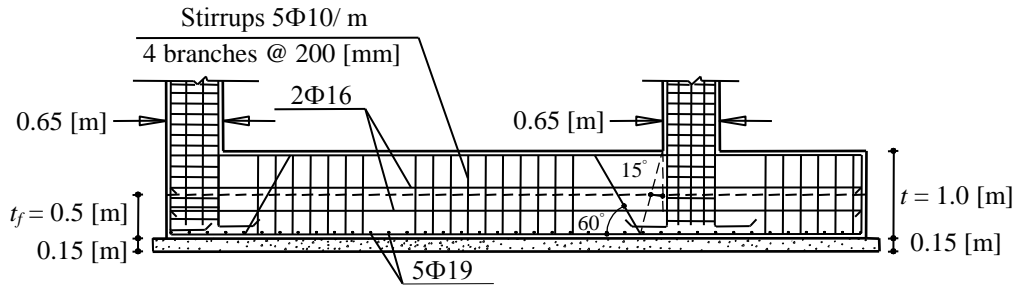
$$A_{sT} = \frac{M_T}{k_2 d_f}$$

$$A_{sT} = \frac{0.099}{172 \times 0.45} = 0.001279 \text{ [m}^2\text{/m]} = 12.79 \text{ [cm}^2\text{/m]}$$

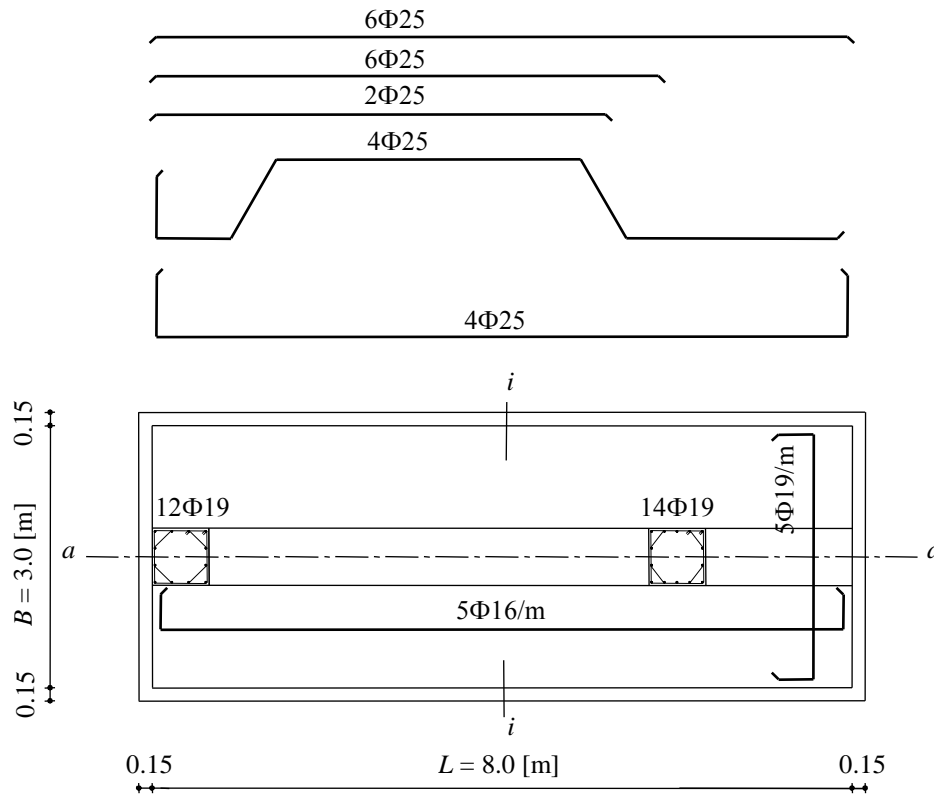
Chosen steel $5\Phi 19 = 14.2 \text{ [cm}^2\text{/m]}$

The details of reinforcement in plan and sections *a-a* and *i-i* through the footing are shown in Figure 75

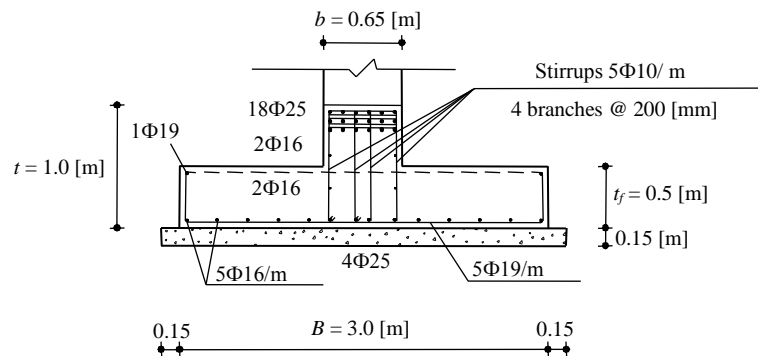
Solved Examples



a) Section *a-a*



b) Plan with transverse reinforcement



c) Cross section *i-i*

Figure 75: Footing dimensions and reinforcement

Solved Examples

Example 27: Design of a combined footing for three equal columns

Design a combined footing for three equal columns, each of 0.50×0.50 [m], reinforced by $4\Phi 19$, carrying a load of $P = 1500$ [kN] as shown in Figure 76. The distance center to center of columns is 3.0 [m]. The allowable soil pressure is $q_{all} = 210$ [kN/m²] at a depth of $D_f = 2.0$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

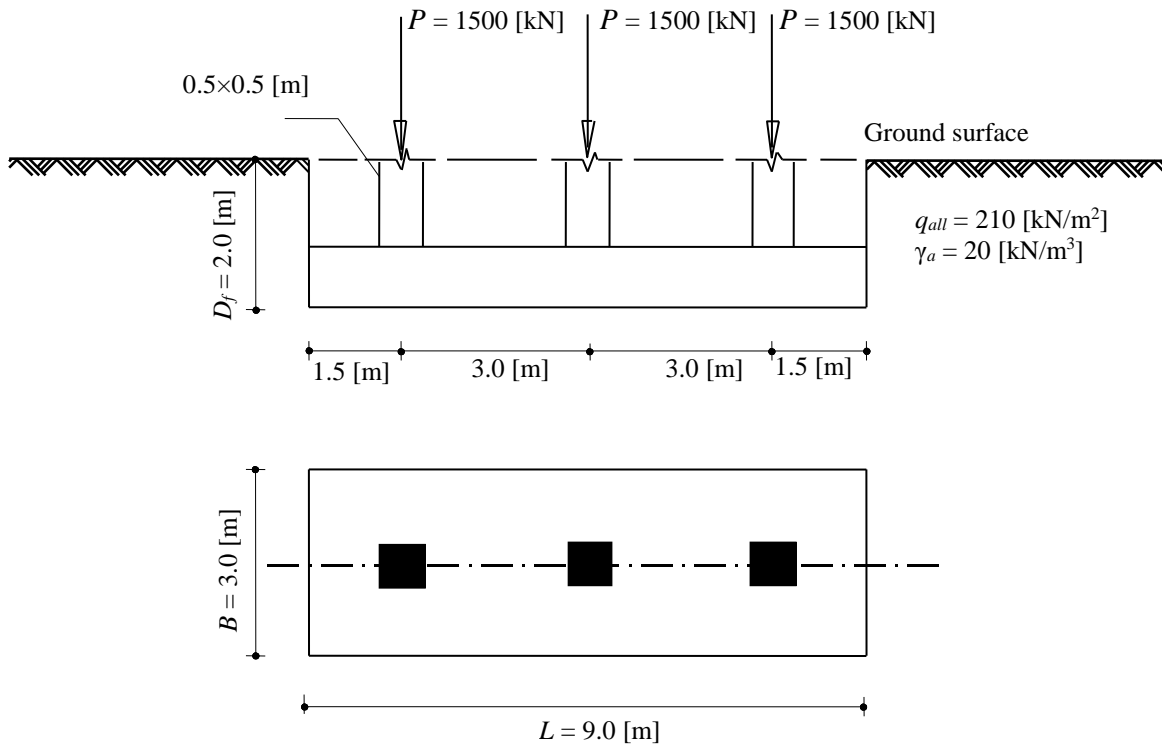


Figure 76: Combined rectangular footing for three columns

1. Determining footing sides L and B

Resultant of loads R at the ground surface level is given by:

$$R = 3 P = 3 \times 1500 = 4500 \text{ [kN]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 210 - 20 \times 2.0 = 170 \text{ [kN/m}^2\text{]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R}{q_{net}} = \frac{4500}{170} = 26.47 \text{ [m}^2\text{]}$$

take $A_f = 3.0 \times 9.0 = 27.0$ [m²] rectangular combined footing

Solved Examples

2. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{4500}{27} = 166.67 \text{ [kN/m}^2\text{]} = 0.167 \text{ [MN/m}^2\text{]}$$

Now, the rectangular footing is treated as a beam footing. The contact pressure per meter is given by:

$$q = q_o B = 0.167 \times 3.0 = 0.5 \text{ [MN/m]}$$

Figure 77 shows the load diagram and moment diagram for the beam footing.

3. Determining the maximum moment M_{max}

From Figure 77, the maximum bending moment in the longitudinal direction occurs at the middle of the footing which is:

$$M_{max} = P \frac{c^2}{2} = 0.5 \frac{1.25^2}{2} = 0.391 \text{ [MN.m]}$$

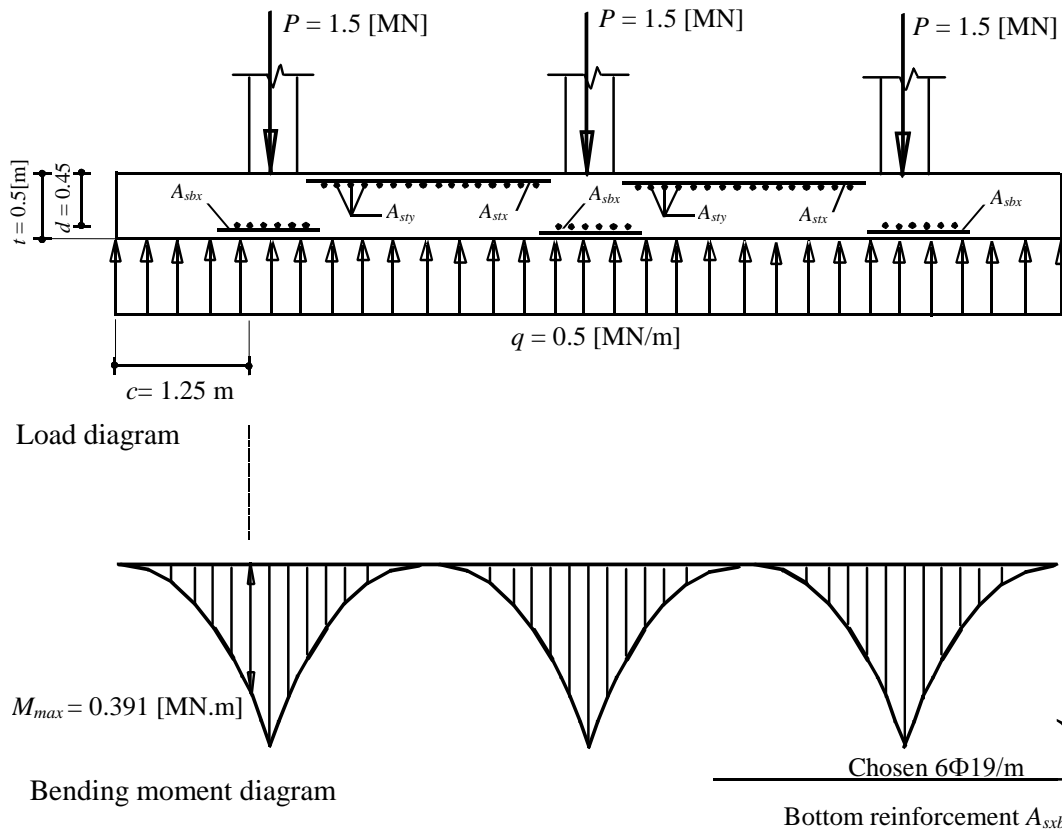


Figure 77: Contact pressure, bending moment and critical sections

Solved Examples

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5$ [MN/m²] and $f_s = 200$ [MN/m²], the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient k_2 [MN/m²] to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172$ [MN/m²].

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.391}{3.0}} = 0.28 \text{ [m]}$$

Take $d = 0.45$ [m] $> d_m = 0.28$ [m], then the section is designed as singly reinforced section.

The corresponding k_1 for $d = 0.45$ [m] is given by:

$$k_1 = 0.45 \sqrt{\frac{3.0}{0.391}} = 1.25$$

From Table 2 at $k_1 = 1.27$, $f_c = 5.0$ [MN/m²] and $k_2 = 182$

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.225$ [m] from the face of the column as shown in Figure 78.

Solved Examples

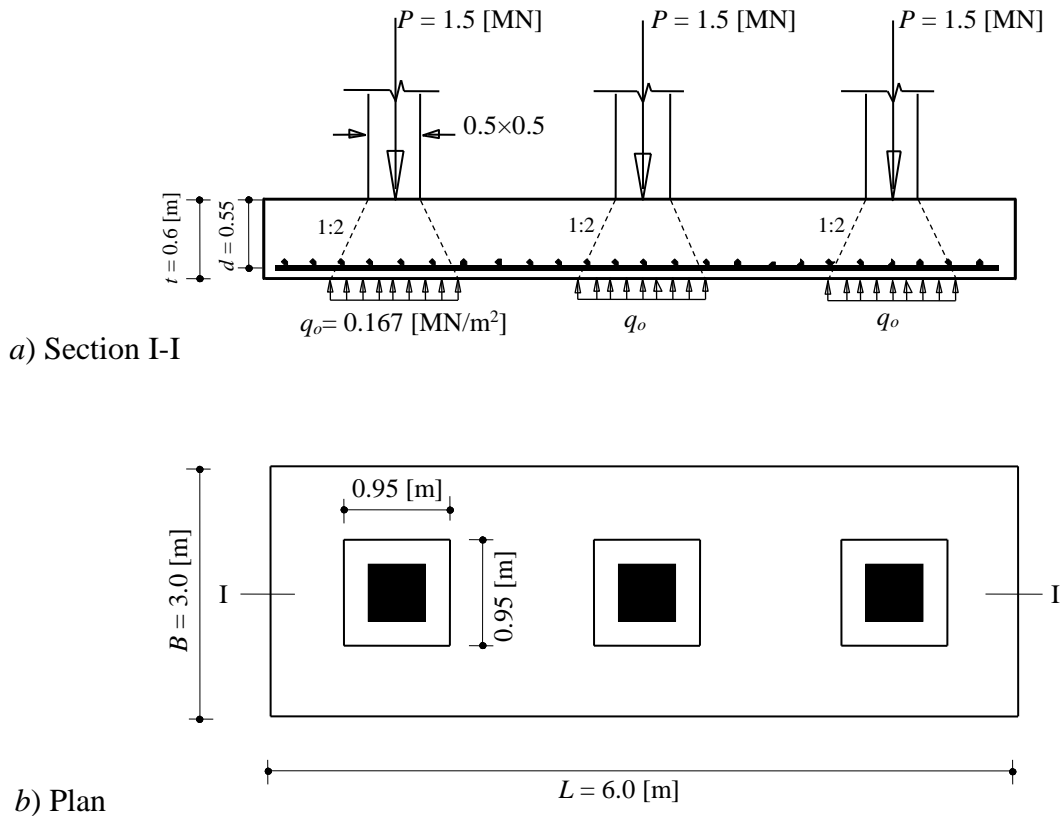


Figure 78: Critical section for punching shear

Geometry (Figure 78)

Effective depth of the section	$d = 0.45 \text{ [m]}$
Column side	$a = b = 0.5 \text{ [m]}$
Area of critical punching shear section	$A_p = (a + d)^2 = 0.9025 \text{ [m}^2\text{]}$
Perimeter of critical punching shear section	$b_o = 4(a + d) = 3.8 \text{ [m]}$

Loads and stresses

Column load	$P = 1.5 \text{ [MN]}$
Soil pressure under the column	$q_o = 0.167 \text{ [MN/m}^2\text{]}$
Main value of shear strength for concrete C 250	$q_{cp} = 0.9 \text{ [MN/m}^2\text{]}$

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.5 - 0.167 \times 0.9025 = 1.35 \text{ [MN]}$$

The punching shear stress q_p is given by:

Solved Examples

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{1.35}{3.8 \times 0.45} = 0.79 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9 \text{ [MN/m}^2\text{]} > q_p = 0.79 \text{ [MN/m}^2\text{]}$, the section is safe for punching shear.

6. Computing the area of steel reinforcement

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 50 \times 100 = 7.5 \text{ [cm}^2/\text{m]}$

Take $A_{smin} = 5\Phi 16 = 10.1 \text{ [cm}^2/\text{m]}$.

The required area of steel reinforcement A_s is:

$$A_{sxb} = \frac{M_{max}}{k_2 d}$$

$$A_{sxb} = \frac{0.391}{182 \times 0.45} = 0.004774 \text{ [m}^2/3.0 \text{ m]}$$

$$A_{sxt} = 47.74 \text{ [cm}^2/3.0 \text{ m]} = 15.91 \text{ [cm}^2/\text{m]}$$

Chosen steel $6\Phi 19/\text{m} = 17.0 \text{ [cm}^2/\text{m]}$

7. Computing the steel reinforcement in transverse direction

The transverse bending moment may be approximately determined at each column by assuming that the column load is distributed outward at 45 [°] from the face of the column over an appropriate area as shown in Figure 79. Both sides of this area should not be greater than the breadth of the footing.

Solved Examples

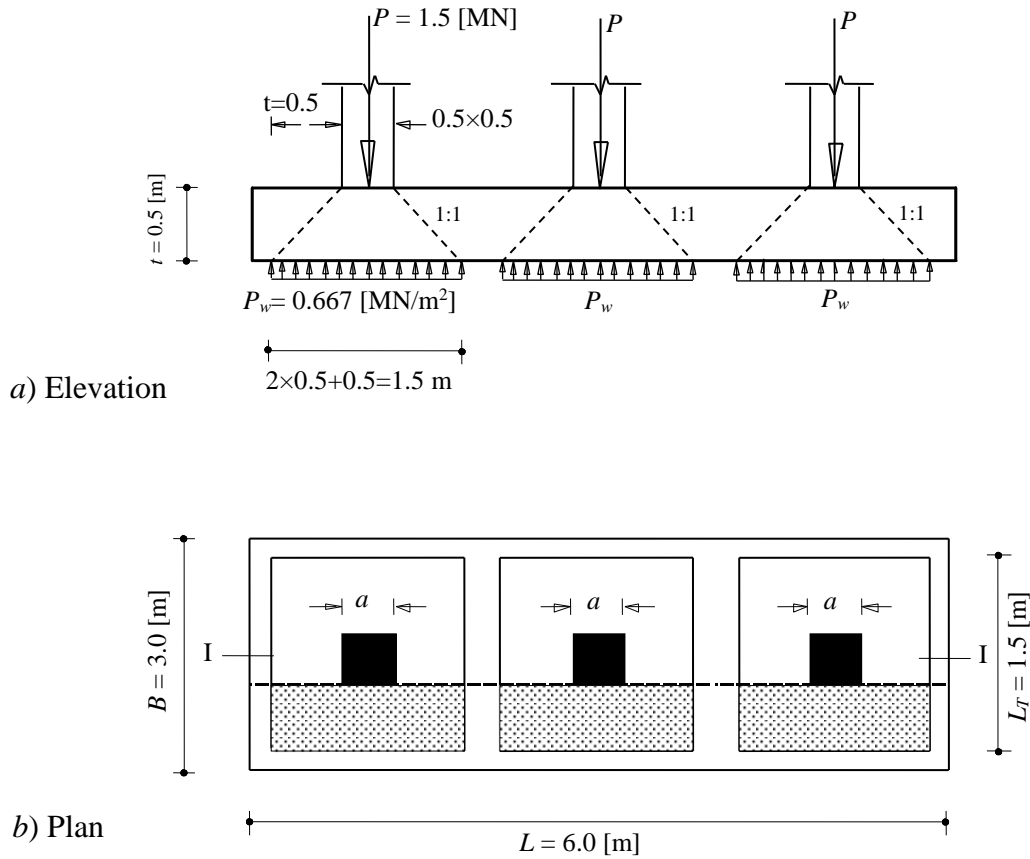


Figure 79: sections for determining transverse moments

Side length of the loaded area L_T is given by:

$$L_T = a + 2t = 0.5 + 2 \times 0.5 = 1.5 \text{ [m]}$$

The transverse bending moment M_T under the column is given by:

$$M_T = \frac{P}{8} \frac{(L_T - a)^2}{L_T} = \frac{1.5}{8} \frac{(1.5 - 0.5)^2}{1.5} = 0.125 \text{ [MN.m]}$$

The required area of steel reinforcement in transverse direction under the column A_{sT} is:

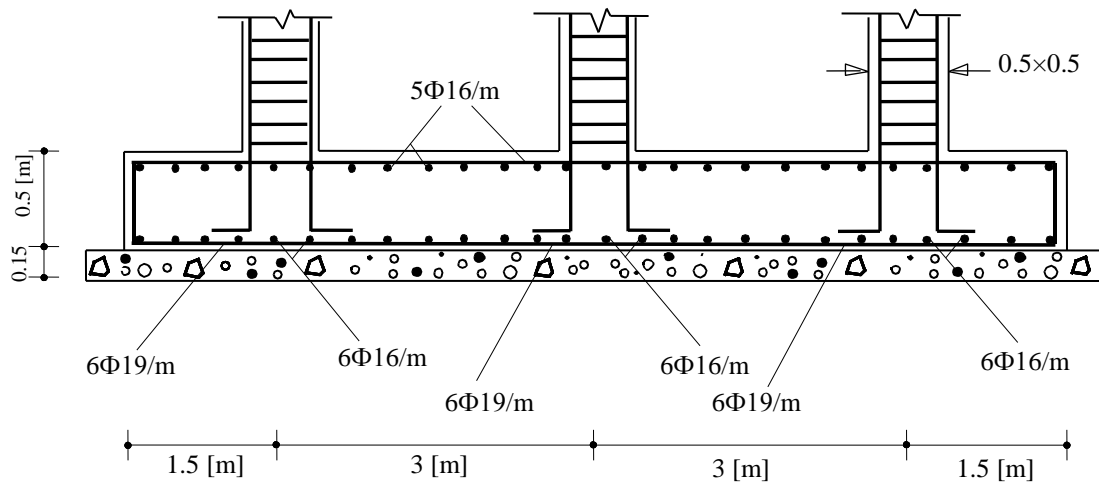
$$A_{sT} = \frac{M_T}{k_2 d}$$

$$A_{sT} = \frac{0.125}{182 \times 0.45} = 0.001526 \text{ [m}^2\text{/1.5m]}$$

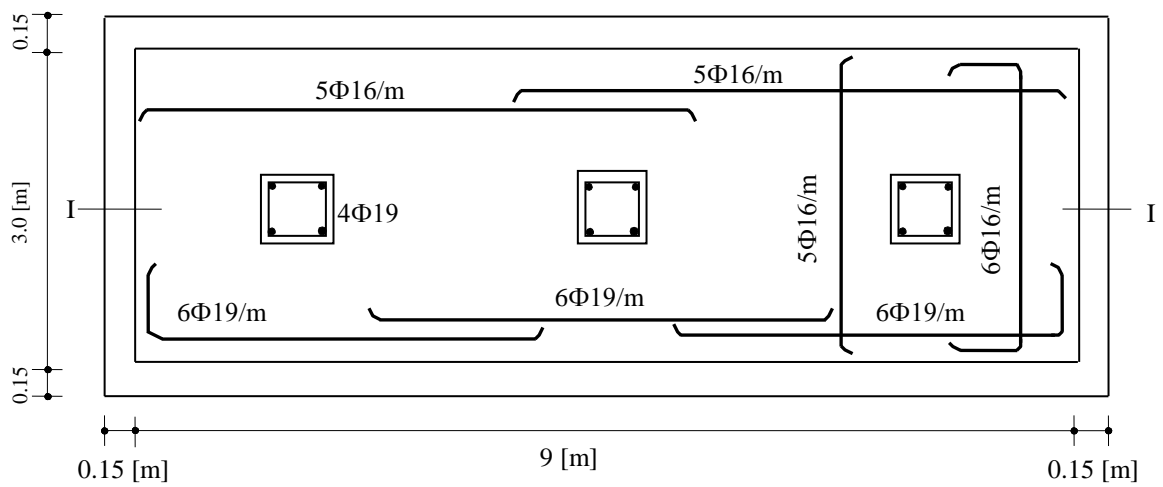
$$A_{sT} = 15.26 \text{ [cm}^2\text{/1.5m]} = 10.17 \text{ [cm}^2\text{/m]}$$

Chosen steel $6\Phi 16/\text{m} = 12.1 \text{ [cm}^2\text{/m]}$. The details of reinforcement in plan and section $a-a$ through the footing are shown Figure 80.

Solved Examples



a) Section I-I



b) Plan

Figure 80: Details of reinforcement in plan and section *a-a* through the footing

Solved Examples

Example 28: Design of a combined footing for three unequal columns

Design a combined rectangular footing for three columns. Edge columns C_1 and C_3 have dimensions of 0.3×0.5 [m²], reinforcement $6\Phi 19$ and a load of 1000 [kN], while the central column C_2 has dimensions of 0.5×0.75 [m²], reinforcement $10\Phi 25$ and a load of 3000 [kN] as shown in Figure 81. The allowable soil pressure $q_{all} = 210$ [kN/m²] at a depth of $D_f = 2.0$ [m] and the average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

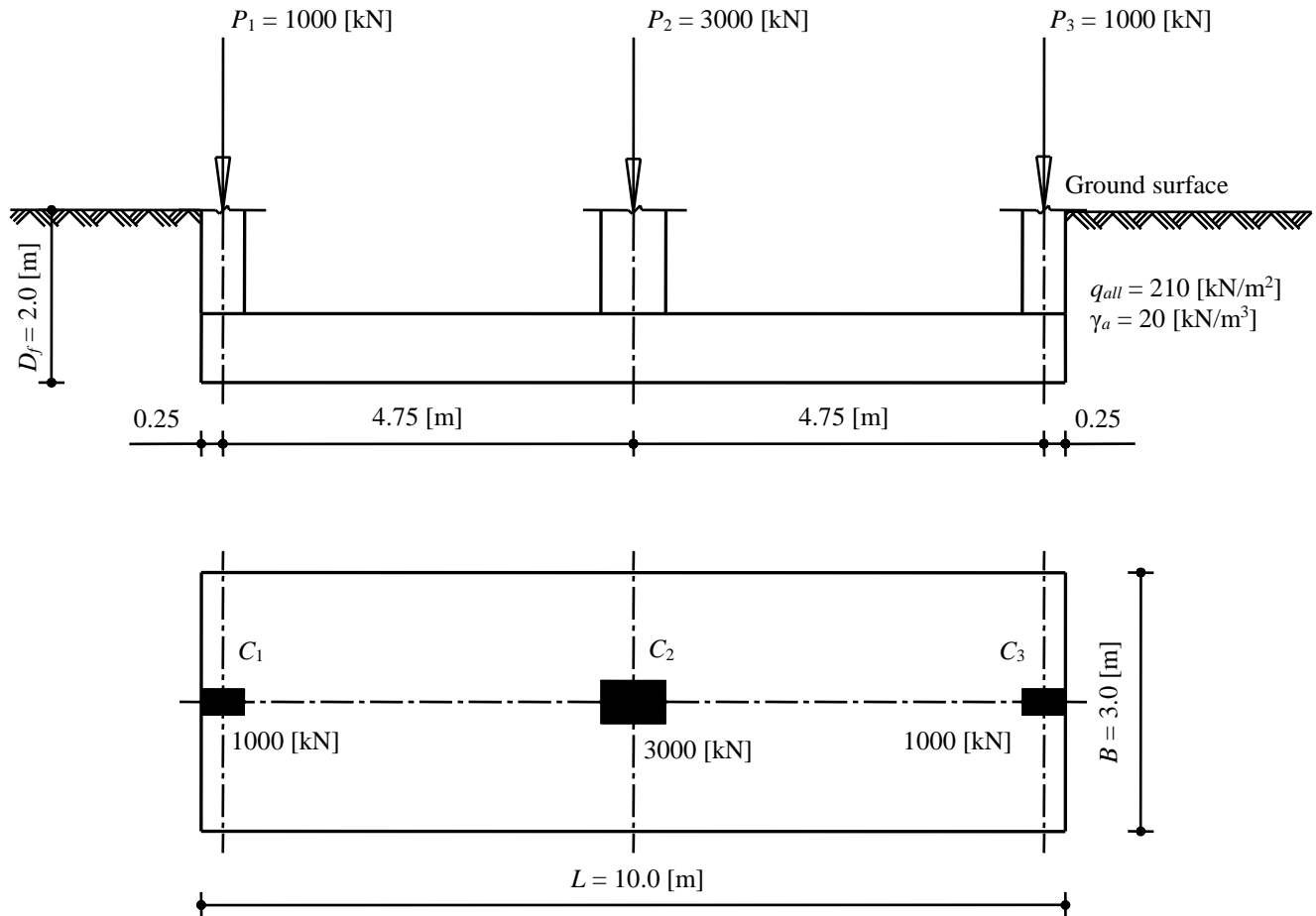


Figure 81: Combined rectangular footing for three unequal columns

1. Determining footing sides L and B

Resultant of loads R at the ground surface level is given by:

$$R = 2 P_1 + P_2 = 2 \times 1000 + 3000 = 5000 \text{ [kN]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 210 - 20 \times 2.0 = 170 \text{ [kN/m}^2\text{]}$$

Area of footing A_f is obtained from:

Solved Examples

$$A_f = \frac{R}{q_{net}} = \frac{5000}{170} = 29.41 \text{ [m}^2\text{]}$$

take $A_f = 10.0 \times 3.0 = 30.0 \text{ [m}^2\text{]}$ rectangular combined footing

2. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{5000}{30} = 166.67 \text{ [kN/m}^2\text{]} = 0.167 \text{ [MN/m}^2\text{]}$$

Now, the rectangular footing is treated as a beam footing. The contact pressure per meter is given by:

$$q = q_o B = 0.167 \times 3.0 = 0.5 \text{ [MN/m]}$$

3. Determining the critical bending moments

The maximum bending moment in the longitudinal direction of the footing occurs at the side face of the center column. Figure 82 shows the load diagram, bending moment and shear force diagram for the beam footing.

Solved Examples

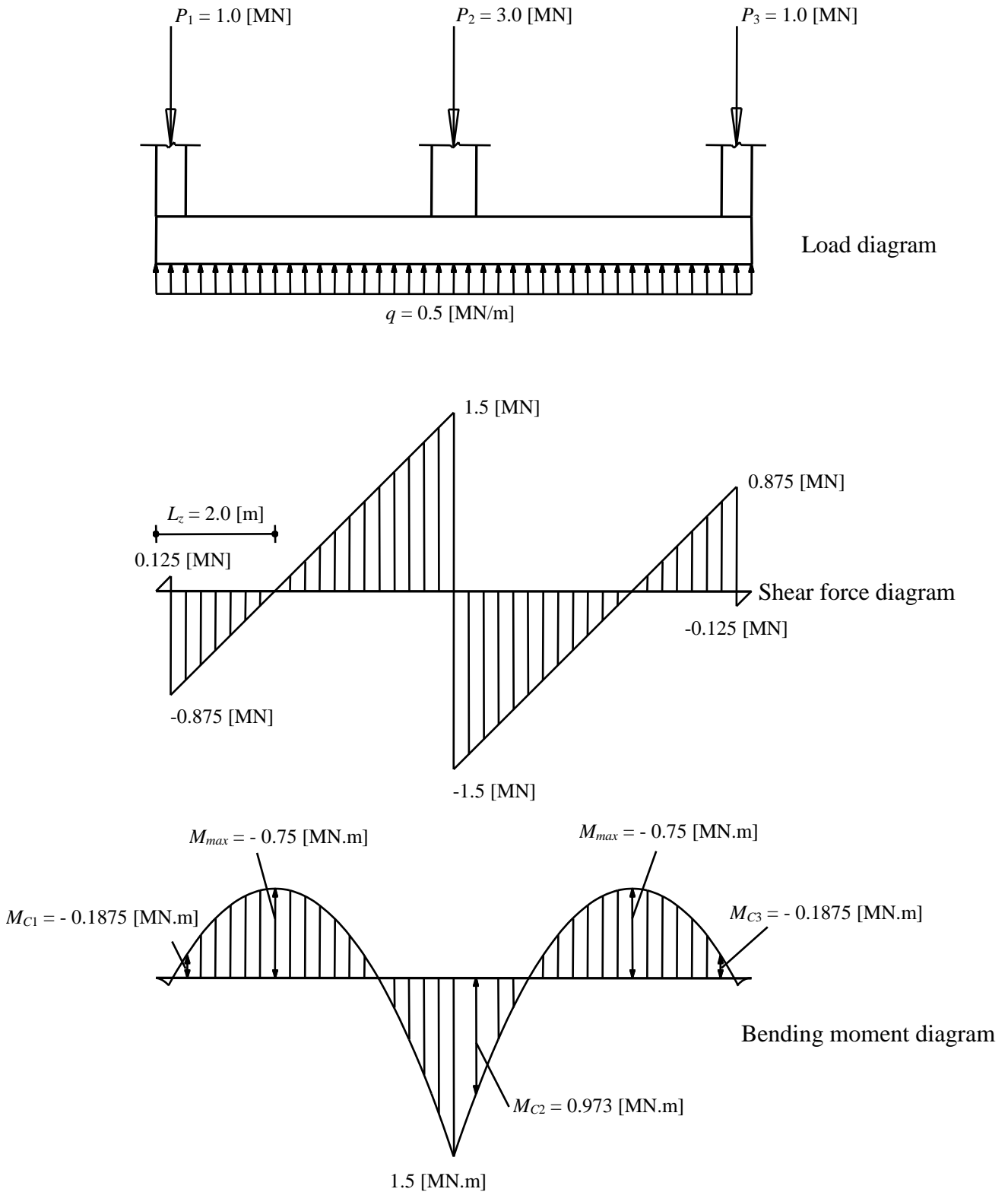


Figure 82: Load, shear force and bending moment diagrams with critical sections

The critical bending moments acting on column sides are:

Solved Examples

$$M_{C1} = M_{C3} = 0.5 \times (0.5)^2 - 1.0 \times 0.25 = -0.1875 \text{ [MN.m]}$$

$$M_{C2} = 0.5 \times (4.625)^2 - 1.0 \times 4.375 = 0.973 \text{ [MN.m]}$$

The maximum bending moment occurs at the position of zero shear. Point of zero shear locates at l_z from the outer face of the footing.

$$P_1 = q l_z$$

$$1 = 0.5 \times l_z$$

$$l_z = 2.0 \text{ [m]}$$

$$M_{max} = q (l_z)^2 / 2 - P_1 (l_z - C_1)$$

$$M_{max} = 0.5(2.0)^2 / 2 - 1(2.0 - 0.25) = -0.75 \text{ [MN.m]}$$

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.973}{3.0}} = 0.436 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.436 \text{ [m]}$, and $t = 0.5 \text{ [m]}$

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.225 \text{ [m]}$ from the face of the column as shown in Figure 83.

Column C_2

Geometry (Figure 83)

Effective depth of the section	$d = 0.45 \text{ [m]}$
Column side	$a = 0.75 \text{ [m]}$ and $b = 0.5 \text{ [m]}$
Area of critical punching shear section	$A_p = (a + d)(b + d) = 1.14 \text{ [m}^2\text{]}$
Perimeter of critical punching shear section	$b_o = 2(a + d) + (b + d) = 4.3 \text{ [m]}$

Solved Examples

Loads and stresses

Column load	$P = 3.0$ [MN]
Soil pressure under the column	$q_o = 0.167$ [MN/m ²]
Main value of shear strength for concrete C 250	$q_{cp} = 0.9$ [MN/m ²]

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 3.0 - 0.167 \times 1.14 = 2.81 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{2.81}{4.3 \times 0.45} = 1.452 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.75}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

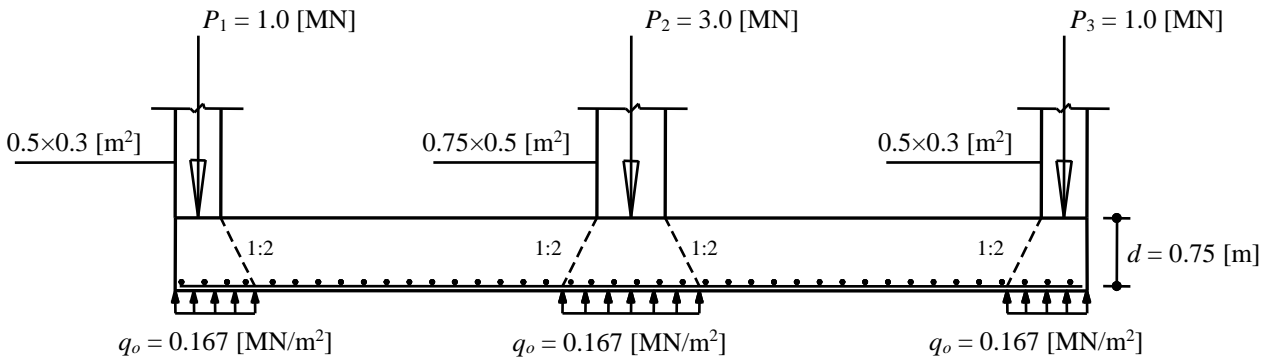
$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]} < q_p = 1.452 \text{ [MN/m}^2\text{]}$$

Determining the depth required to resist punching shear d_p

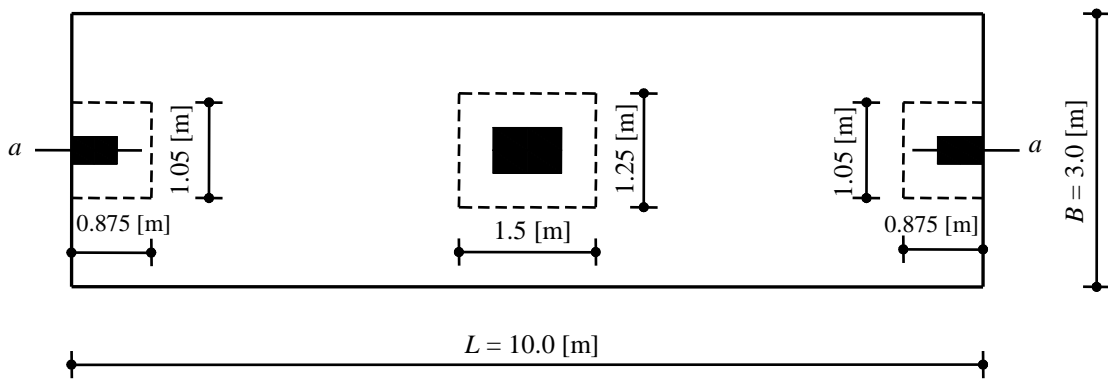
$$d_p = \frac{2.81}{4.3 \times 0.9} = 0.726 \text{ [m]}$$

Take $d = 0.75$ [m] $> d_m = 0.726$ [m] and $t = 0.8$ [m], then the section is designed as singly reinforced section.

Solved Examples



a) Section a-a



b) Plan

Figure 83: Critical section for punching shear

Column $C_1 = C_3$

Geometry (Figure 83)

Effective depth of the section	$d = 0.75[\text{m}]$
Column side	$a = 0.5 [\text{m}]$ and $b = 0.3[\text{m}]$
Area of critical punching shear section	$A_p = (a + d/2) (b + d) = 0.91875[\text{m}^2]$
Perimeter of critical punching shear section	$b_o = 2 (a + d/2) + (b + d) = 2.8 [\text{m}]$

Loads and stresses

Column load	$P = 1.0 [\text{MN}]$
Soil pressure under the column	$q_o = 0.167 [\text{MN}/\text{m}^2]$
Main value of shear strength for concrete C 250	$q_{cp} = 0.9 [\text{MN}/\text{m}^2]$

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.0 - 0.167 \times 0.91875 = 0.847 [\text{MN}]$$

Solved Examples

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{0.847}{2.8 \times 0.75} = 0.403 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.3}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9 \text{ [MN/m}^2\text{]} > q_p = 0.403 \text{ [MN/m}^2\text{]}$, the section is safe for punching shear.

6. Computing the area of steel reinforcement in longitudinal direction A_s

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 80 \times 100 = 12.0 \text{ [cm}^2/\text{m]}$

Take $A_{smin} = 5\Phi 19 = 14.176 \text{ [cm}^2/\text{m]}$.

The required area of steel reinforcement in longitudinal direction A_s is shown in Table 5:

Table 5: Determination of tension reinforcement in longitudinal direction

Position	Breadth B [m]	Moment M [MN.m]	Calculated A_s $A_s = M / (k_2.d) / B$ [cm ² / m]	Chosen reinforcement A_s
$A_{sC1} = A_{sC3}$	3.0	- 0.1875	4.845	5Φ19/ m
A_{sC2}	3.0	0.9727	25.134	9Φ19/ m
A_{smax}	3.0	- 0.7500	19.380	7Φ19/ m

7. Computing the steel reinforcement in transverse direction

The transverse bending moment may be approximately determined at each column by assuming that the column load is distributed outward at 45 [°] from the face of the column over an appropriate area as shown in Figure 84. Both sides of this area should not be greater than the breadth of the footing.

Solved Examples

Column $C_1 = C_3$

Length of the loaded area L_{T1} is given by:

$$L_{T1} = a_1 + t = 0.5 + 0.8 = 1.3 \text{ [m]}$$

Breadth of the loaded area B_{T1} is given by:

$$B_{T1} = b_1 + 2 t = 0.3 + 2 \times 0.8 = 1.9 \text{ [m]}$$

The loaded area A_{T1} is given by:

$$A_{T1} = L_{T1} \times B_{T1} = 1.3 \times 1.9 = 2.47 \text{ [m}^2\text{]}$$

The contact pressure under the loaded area P_{w1} is given by:

$$P_{w1} = \frac{P_1}{A_{T1}} = \frac{1.0}{2.47} = 0.405 \text{ [MN/m}^2\text{]}$$

The transverse bending moment M_{T1} under the column is given by:

$$M_{T1} = P_{w1} \left(\frac{B_{T1} - b_1}{2} \right)^2 L_{T1} \times \frac{1}{2} = 0.405 \times \left(\frac{1.9 - 0.3}{2} \right)^2 \times 1.3 \times \frac{1}{2} = 0.169 \text{ [MN.m]}$$

The required area of steel reinforcement in transverse direction under the column A_{sT} is:

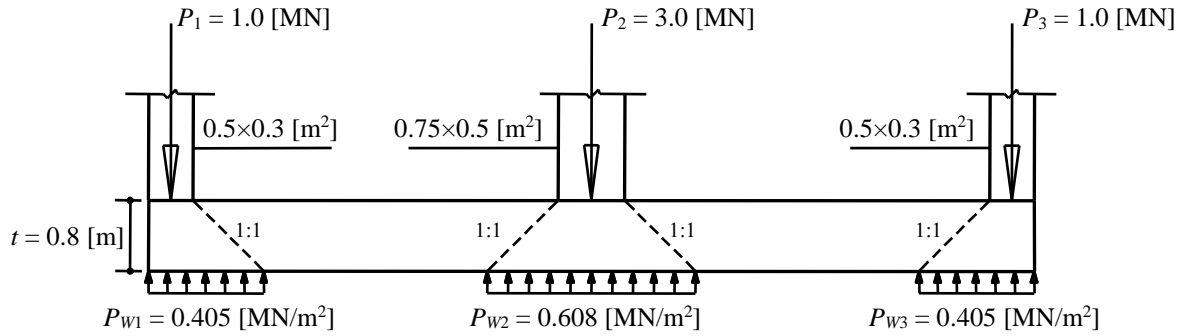
$$A_{sT1} = \frac{M_{T1}}{k_2 d}$$

$$A_{sT} = \frac{0.169}{172 \times 0.75} = 0.001310 \text{ [m}^2\text{/1.3m]}$$

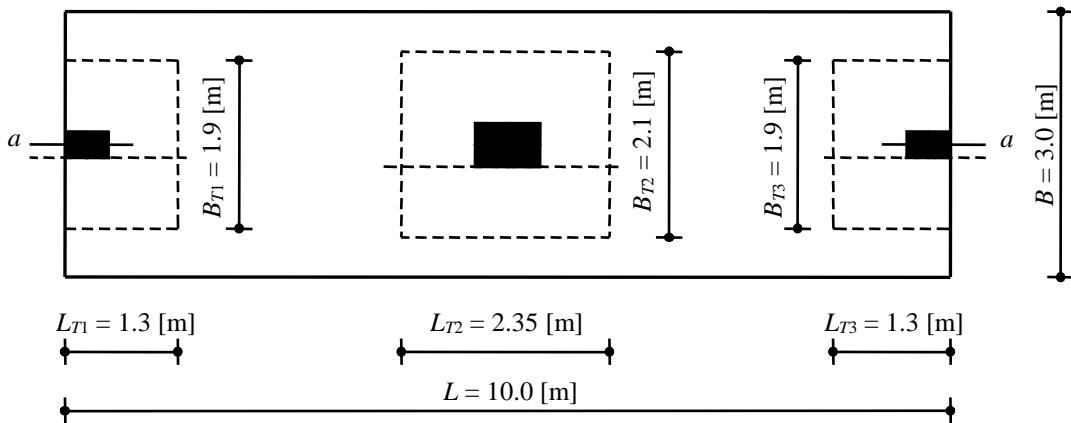
$$A_{sT} = 13.10 \text{ [cm}^2\text{/1.3m]} = 10.08 \text{ [cm}^2\text{/m]}$$

Chosen steel $5\Phi 16/\text{m} = 10.1 \text{ [cm}^2\text{/m]}$

Solved Examples



a) Section *a-a*



b) Plan

Figure 84: sections of transverse moment

Column C_2

Length of the loaded area L_{T2} is given by:

$$L_{T2} = a_2 + 2 t = 0.75 + 2 \times 0.8 = 2.35 \text{ [m]}$$

Breadth of the loaded area B_{T2} is given by:

$$B_{T2} = b_2 + 2 t = 0.5 + 2 \times 0.8 = 2.1 \text{ [m]}$$

The loaded area A_{T2} is given by:

$$A_{T2} = L_{T2} \times B_{T2} = 2.35 \times 2.1 = 4.935 \text{ [m}^2\text{]}$$

The contact pressure under the loaded area P_{w2} is given by:

$$P_{w2} = \frac{P_2}{A_{T2}} = \frac{3.0}{4.935} = 0.608 \text{ [MN/m}^2\text{]}$$

Solved Examples

The transverse bending moment M_{T2} under the column is given by:

$$M_{T2} = P_{w2} \left(\frac{B_{T2} - b_2}{2} \right)^2 L_{T2} \times \frac{1}{2} = 0.608 \times \left(\frac{2.1 - 0.5}{2} \right)^2 \times 2.35 \times \frac{1}{2} = 0.457 \text{ [MN.m]}$$

The required area of steel reinforcement in transverse direction under the column A_{sT2} is:

$$A_{sT2} = \frac{M_{T2}}{k_2 d}$$

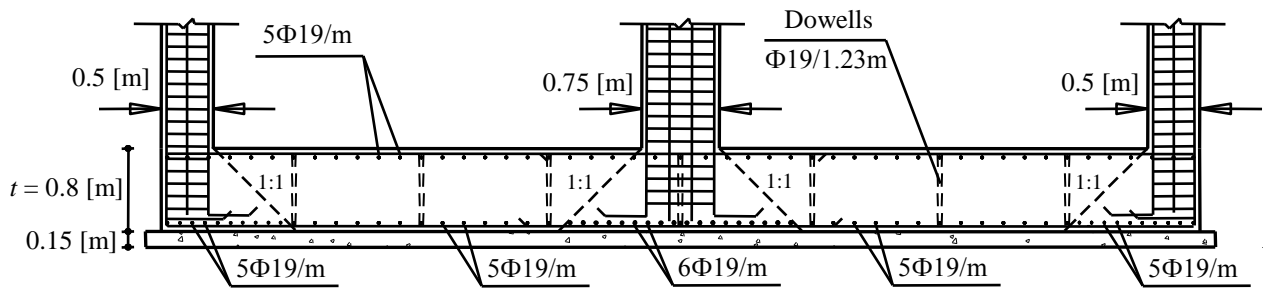
$$A_{sT2} = \frac{0.457}{172 \times 0.75} = 0.003543 \text{ [m}^2\text{/2.35m]}$$

$$A_{sT2} = 35.43 \text{ [cm}^2\text{/2.35m]} = 15.075 \text{ [cm}^2\text{/m]}$$

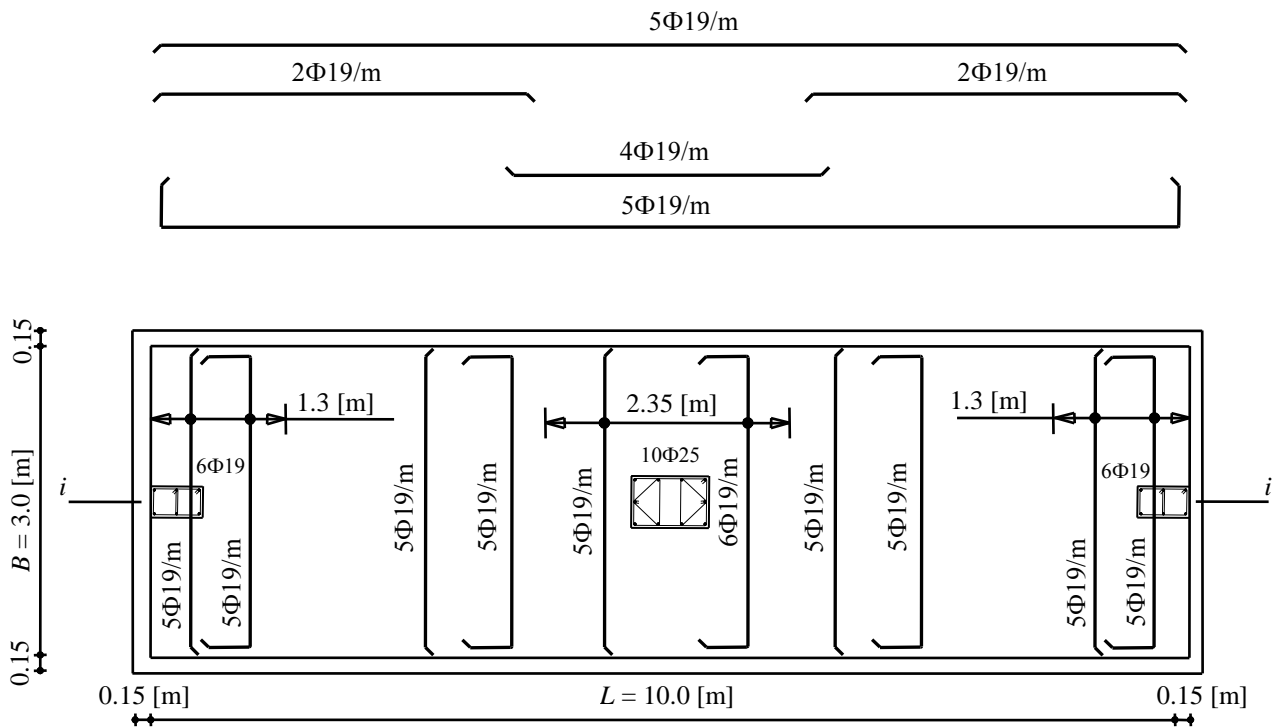
Chosen steel $6\Phi 19/\text{m} = 17.01 \text{ [cm}^2\text{/m]}$

The details of reinforcement in plan and section *i-i* through the footing are shown in Figure 85.

Solved Examples



a) Section I-I



b) Plan with transverse reinforcement

Figure 85: Footing dimensions and reinforcement

Solved Examples

Example 29: Design of a combined ribbed footing for three unequal columns

Design a combined ribbed footing for three columns. Edge columns C_1 and C_3 have dimensions of 0.5×0.7 [m²], reinforcement $10\Phi 22$ and a load of 2300 [kN], while the central column C_2 has dimensions of 0.5×0.8 [m²], reinforcement $12\Phi 22$ and a load of 3000 [kN] as shown in Figure 86. The allowable soil pressure $q_{all} = 235$ [kN/m²] at a depth of $D_f = 2.0$ [m] and the average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

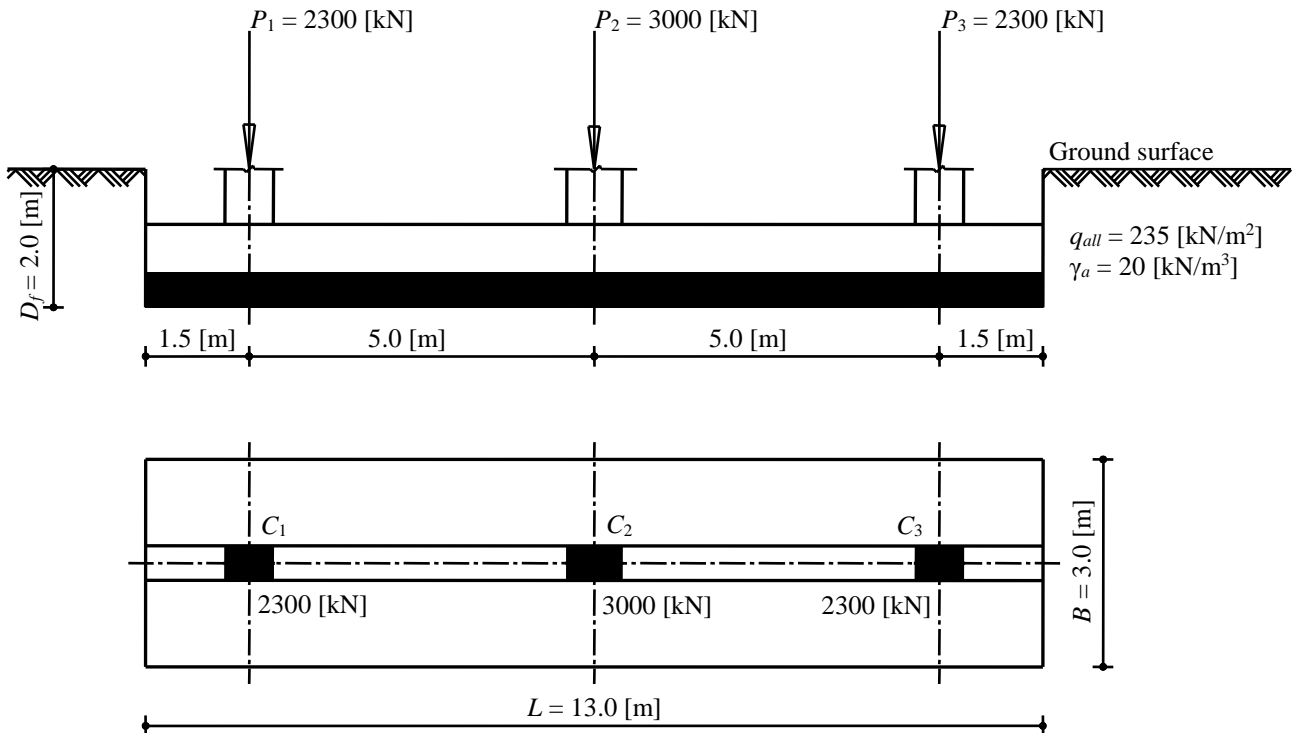


Figure 86: rectangular ribbed footing for three unequal columns

1. Determining footing sides L and B

Resultant force R equals to:

$$R = 2 P_1 + P_2 = 2 \times 2300 + 3000 = 7600 \text{ [kN]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 235 - 20 \times 2.0 = 195 \text{ [kN/m}^2\text{]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R}{q_{net}} = \frac{7600}{195} = 38.974 \text{ [m}^2\text{]}$$

take $A_f = 13.0 \times 3.0 = 39.0$ [m²] rectangular combined footing

2. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{7600}{39} = 195 \text{ [kN/m}^2\text{]} = 0.195 \text{ [MN/m}^2\text{]}$$

Now, the rectangular footing is treated as a beam footing. The contact pressure per meter is given by:

$$q = q_o B = 0.195 \times 3.0 = 0.585 \text{ [MN/m]}$$

3. Determining the critical bending moments

The maximum bending moment in the longitudinal direction of the footing occurs at the side face of the center column. Figure 87 shows the load diagram, bending moment and shear force diagram for the beam footing.

Solved Examples

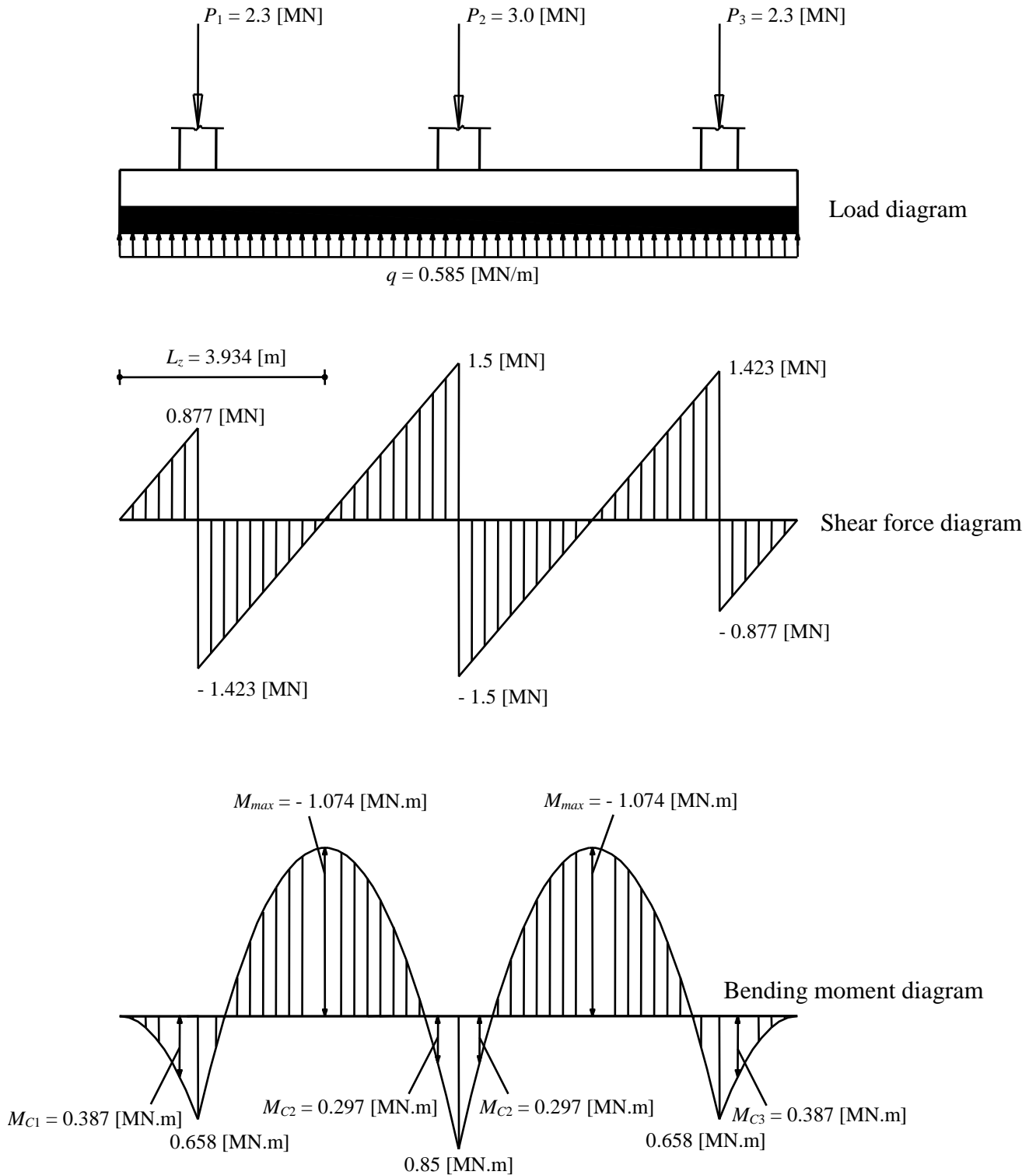


Figure 87: Load, shear force and bending moment diagrams with critical sections

The critical bending moments acting on column sides are:

$$M_{C1} = M_{C3} = 0.585 \times (1.15)^2 / 2 = 0.387 \text{ [MN.m]}$$

Solved Examples

$$M_{C2} = 0.585 \times (6.1)^2 / 2 - 2.3 \times 4.6 = 0.297 \text{ [MN.m]}$$

The maximum bending moment occurs at the position of zero shear. Point of zero shear locates at l_z from the outer face of the footing.

$$P_1 = q l_z$$

$$2.3 = 0.585 \times l_z$$

$$l_z = 3.934 \text{ [m]}$$

$$M_{max} = q (l_z)^2 / 2 - P_1 (l_z - C)$$

$$M_{max} = 0.585(3.934)^2 / 2 - 2.3(3.934 - 1.5) = - 1.074 \text{ [MN.m]}$$

4. Determining the depth of the rib d

Determining the depth d_m required to resist the longitudinal moment M_{max}

The section of the footing is T- section with depth d_m and breadth of flange B_T

B_T is the least of:

$$B = 3.0 \text{ [m]}$$

$$4b = 4 \times 0.5 = 2.0 \text{ [m]}$$

$$1/4 \text{ Span} = 5.0 / 4 = 1.25 \text{ [m]}$$

Take $B_T = 1.25 \text{ [m]}$

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{max}}{B_T}}$$

$$d_m = 0.766 \sqrt{\frac{1.074}{1.25}} = 0.71 \text{ [m]}$$

Take $d = 0.75 \text{ [m]} > d_m = 0.71 \text{ [m]}$, and $t = 0.8 \text{ [m]}$

Solved Examples

Check for section capacity to resist maximum shear force Q_{max}

The maximum critical shear section lies at a distance $d/2 = 0.375$ [m] from the side face of the internal column.

Effective depth of the section	$d = 0.75$ [m]
Rib width	$b = 0.5$ [m]
Main value of shear strength for concrete C250	$q_c = 0.9$ [MN/m ²]
Max. value of shear strength for concrete C250	$q_2 = 1.9$ [MN/m ²]

for maximum shear force Q_{max} :

$$Q(x) = 0.585 X - 2.3$$

$$X = 6.5 - 0.4 - 0.375 = 5.725 \text{ [m]}$$

$$Q_{max} = 0.585 \times 5.725 - 2.3 = 1.047 \text{ [MN]}$$

The shear stress q is given by:

$$q = \frac{Q_{max}}{b d}$$

$$q = \frac{1.047}{0.5 \times 0.75} = 2.792 \text{ [MN/m}^2\text{]} > q_2 = 1.90 \text{ [MN/m}^2\text{]}$$

Determining the depth d_s required to resist the shear force Q_{max}

$$d_s = \frac{Q_{max}}{b q_2}$$

$$d_s = \frac{1.047}{0.5 \times 1.90} = 1.102 \text{ [m]}$$

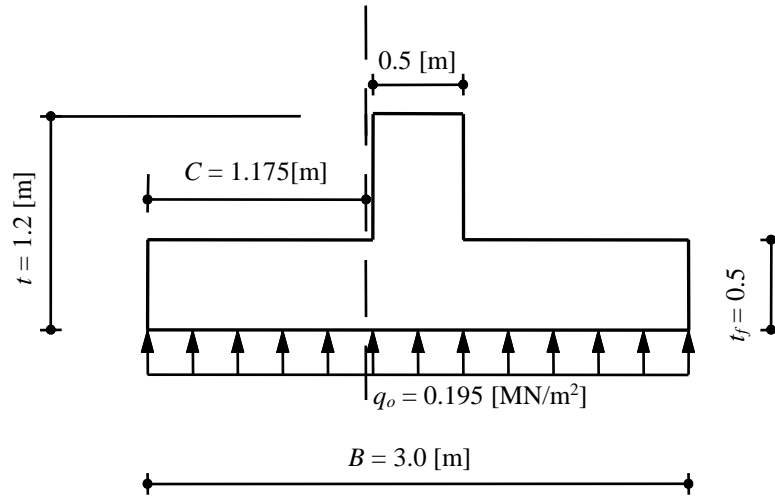
Take $d = 1.15$ [m] $> d_s = 1.104$ [m] and $t = 1.2$ [m], then the section is designed as singly reinforced T- section.

5. Determining the depth d_f required to resist the transverse moment M_T

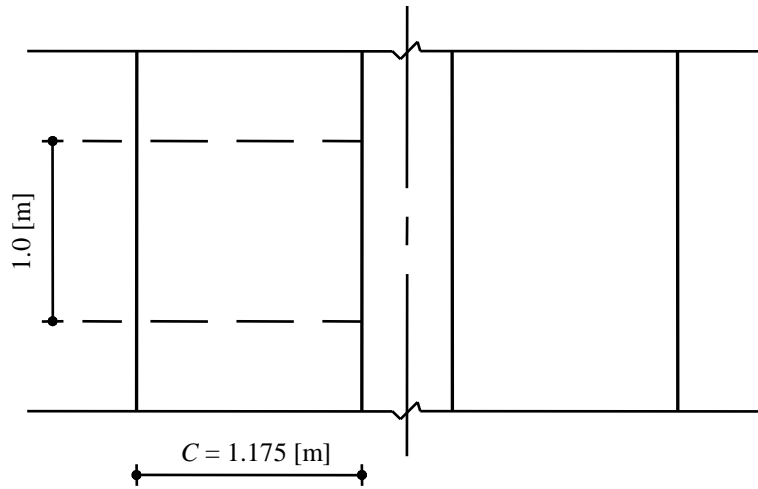
The maximum transverse bending moment M_T lies at the rib face as shown in Figure 88. M_T value is given by:

$$M_T = q_o \times 1.0 \times (c)^2 / 2 = 0.195 \times 1.0 \times (1.25)^2 / 2 = 0.152 \text{ [MN.m]}$$

Solved Examples



Cross section of the footing



Part plane of the footing

Figure 88: Critical section of transverse moment

The footing depth d_f is:

$$d_f = k_1 \sqrt{\frac{M_T}{1.0 \text{ [m]}}}$$

$$d_f = 0.766 \sqrt{\frac{0.152}{1.0}} = 0.299 \text{ [m]}$$

Take $d_f = 0.45$ [m] $>$ 0.299 [m] and $t_f = 0.5$ [m], then the section is designed as singly reinforced section.

6. Computing the area of steel reinforcement in longitudinal direction A_s

At first we should calculate the position of the neutral axis:

Solved Examples

$$k_z = \sqrt{\frac{30}{f_c(r+1)}}$$

$$f_c = 9.5 \text{ [MN/m}^2\text{]} = 95 \text{ [kg/cm}^2\text{]}$$

$$r = f_s / f_c = 200 / 9.5 = 21 \text{ [-]}$$

$$k_z = \sqrt{\frac{30}{95(21+1)}} = 0.1198$$

$$z = k_z \sqrt{\frac{M_{\max}}{B_T}} = 0.1198 \sqrt{\frac{1.074 \times 10^7}{125}} = 35.12 \text{ [cm]} < t_f = 50 \text{ [cm]}$$

The neutral axis lies inside the slab footing, so the section is treated as rectangular section with B_T and d_m as shown in Figure 89.

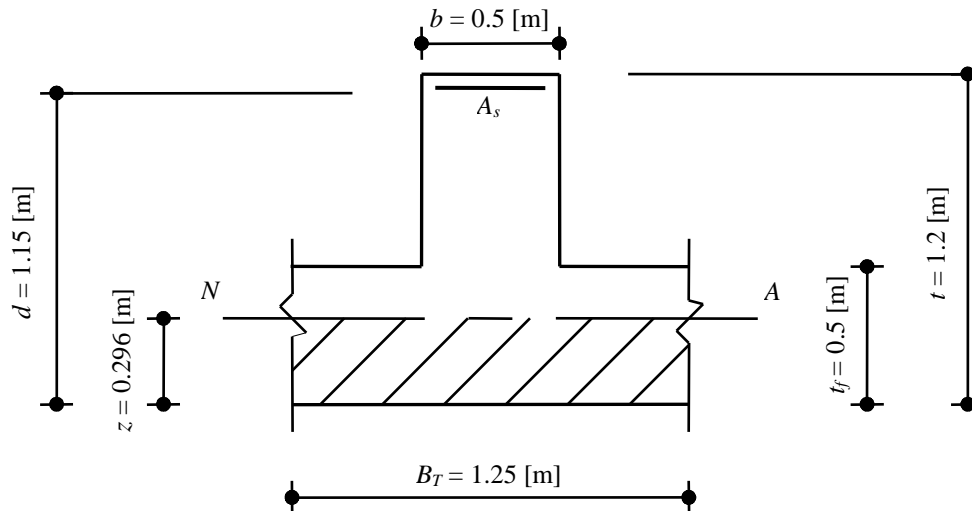


Figure 89: T- section subjected to longitudinal moment M_{\max}

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 50 \times 100 = 7.5 \text{ [cm}^2\text{/ m]}$

Take $A_{smin} = 5\Phi 16 = 10.1 \text{ [cm}^2\text{/ m]}$.

The required area of steel reinforcement in longitudinal direction A_s is shown in Table 6:

Table 6: Determination of tension reinforcement in longitudinal direction

Position	Moment $M \text{ [MN.m]}$	Calculated A_s $A_s = M / (k_2.d) \text{ [cm}^2\text{]}$	Chosen reinforcement A_s
$A_{sC1} = A_{sC3}$	0.387	19.544	3Φ22 + 4Φ19
A_{sC2}	0.297	15.015	6Φ22
A_{smax}	- 1.074	54.297	15Φ22

Solved Examples

7. Check for shear stress

The maximum critical shear section lies at a distance $d/2 = 0.575$ [m] from the face of the internal column.

for maximum shear force Q_{max} :

$$Q(x) = 0.585 X - 2.3$$

$$X = 6.5 - 0.4 - 0.575 = 5.525 \text{ [m]}$$

$$Q_{max} = 0.585 \times 5.525 - 2.3 = 0.93 \text{ [MN]}$$

The shear stress q is given by:

$$q = \frac{Q_{max}}{b d}$$
$$q = \frac{0.93}{0.5 \times 1.15} = 1.617 \text{ [MN/m}^2\text{]}$$

$$q_2 = 1.9 \text{ [MN/m}^2\text{]} > q = 1.617 \text{ [MN/m}^2\text{]} > q_c = 0.9 \text{ [MN/m}^2\text{]}$$

$$q_s = q - q_c / 2 = 1.617 - 0.9/2 = 1.167 \text{ [MN/m}^2\text{]}$$

The shear would be resisted using vertical stirrups and bent bars.

Shear stress carried by stirrups

Using stirrups $5\phi 10/$ m, 4 branches @ 200 [mm]

$$q_{ss} = \frac{A_{st} f_s}{b S}$$
$$q_{ss} = \frac{(4 \times 78.5 \times 10^{-6}) \times 200}{0.5 \times 0.2} = 0.628 \text{ [MN/m}^2\text{]}$$

Shear stress carried by bent bars

$$q_{sb} = q_s - q_{ss} = 1.167 - 0.628 = 0.539 \text{ [MN/m}^2\text{]}$$

$$q_{sb} = \frac{A_{sb} f_s}{b d} (\sin \alpha + \cos \alpha)$$
$$0.539 = \frac{A_{sb} \times 200}{0.5 \times 1.15} (\sin 60^\circ + \cos 60^\circ)$$

$$A_{sb} = 0.0011344 \text{ [m}^2\text{]} = 11.344 \text{ [cm}^2\text{]}$$

Solved Examples

Use bent bars $3\Phi 22 = 19.0 \text{ [cm}^2\text{]}$

8. Computing the area of steel reinforcement in transverse direction A_{sT}

The required area of steel reinforcement in transverse direction A_{sT} is:

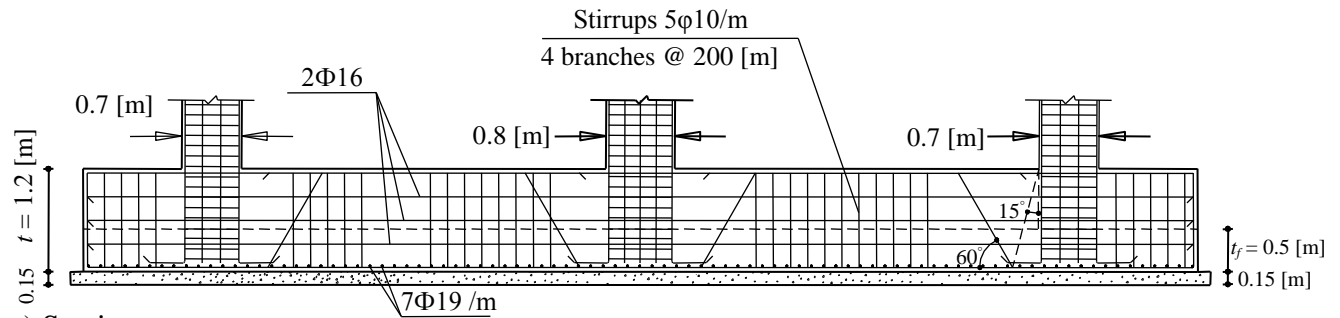
$$A_{sT} = \frac{M_T}{k_2 d_f}$$

$$A_{sT} = \frac{0.152}{172 \times 0.45} = 0.001964 \text{ [m}^2\text{/m]} = 19.64 \text{ [cm}^2\text{/m]}$$

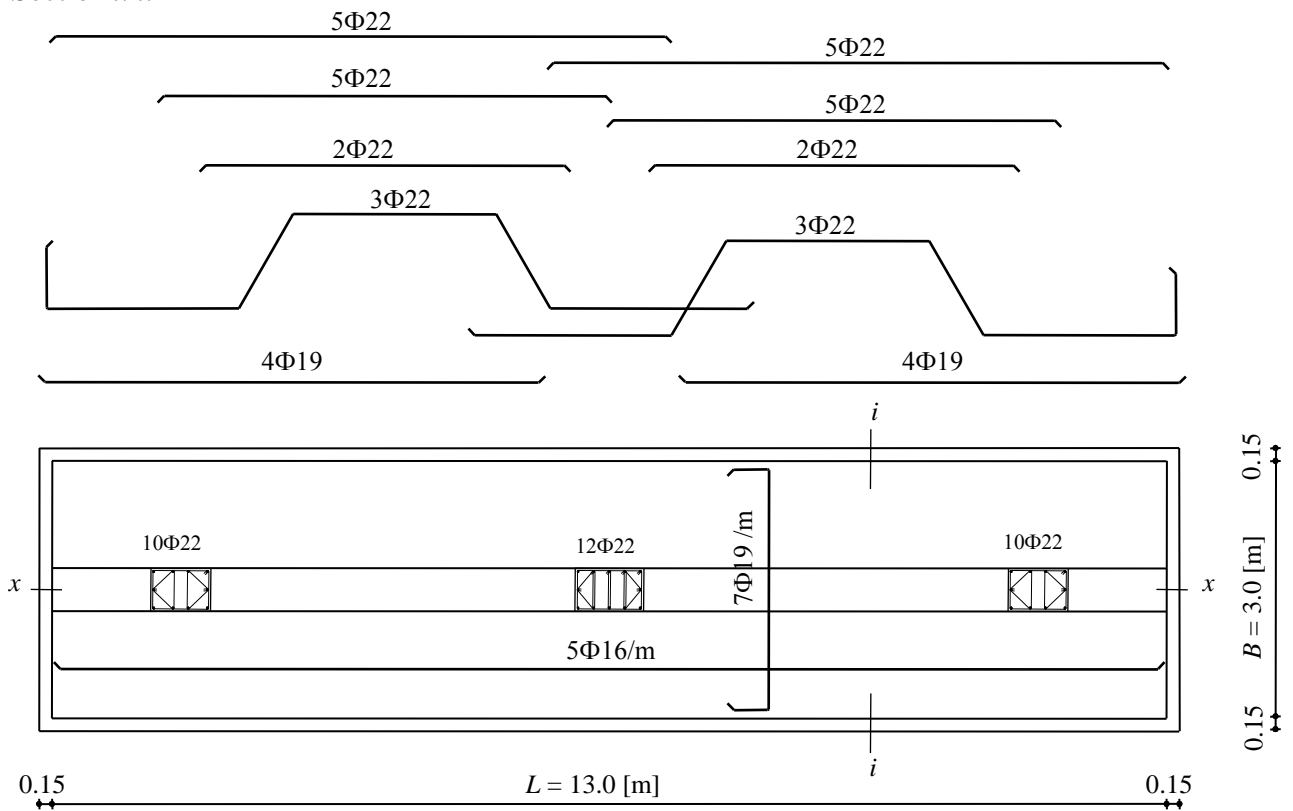
Chosen steel $7\Phi 19 = 19.847 \text{ [cm}^2\text{/m]}$

The details of reinforcement in plan and sections $a-a$ and $i-i$ through the footing are shown in Figure 90.

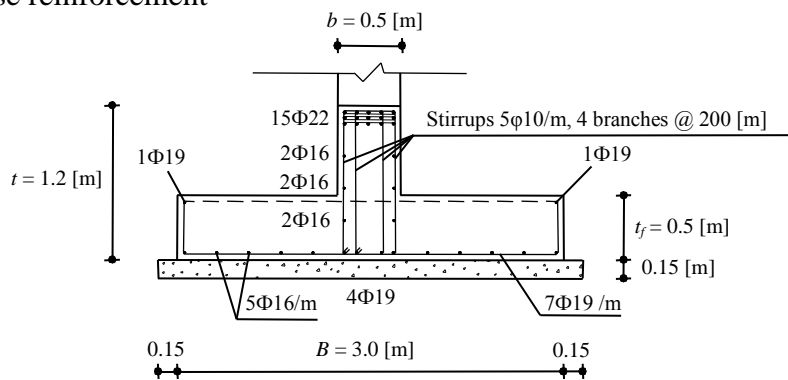
Solved Examples



a) Section $x-x$



b) Plan with transverse reinforcement



c) Cross section $i-i$

Figure 90: Footing dimensions and reinforcement

5- Design of Combined Trapezoidal and T-Section Footings

Solved Examples

Example 30: Design of a trapezoidal combined footing

It is required to design the trapezoidal combined footing shown in Figure 91. The footing is support to two columns C_1 and C_2 spaced at $l_2 = 4.8$ [m] apart. Due to the site conditions, the projections of the footing beyond the centers of columns C_1 and C_2 are limited to $l_1 = 0.9$ [m] and $l_3 = 1.3$ [m], respectively. Column C_1 is 0.5 [m] \times 0.5 [m], reinforced by $8\Phi 16$ [mm] and carries a load of $P_1 = 1200$ [kN]. Column C_2 is 0.6 [m] \times 0.6 [m], reinforced by $12\Phi 19$ [mm] and carries a load of $P_2 = 2000$ [kN]. The allowable soil pressure is $q_{all} = 265$ [kN/m²] at a depth of $D_f = 1.25$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

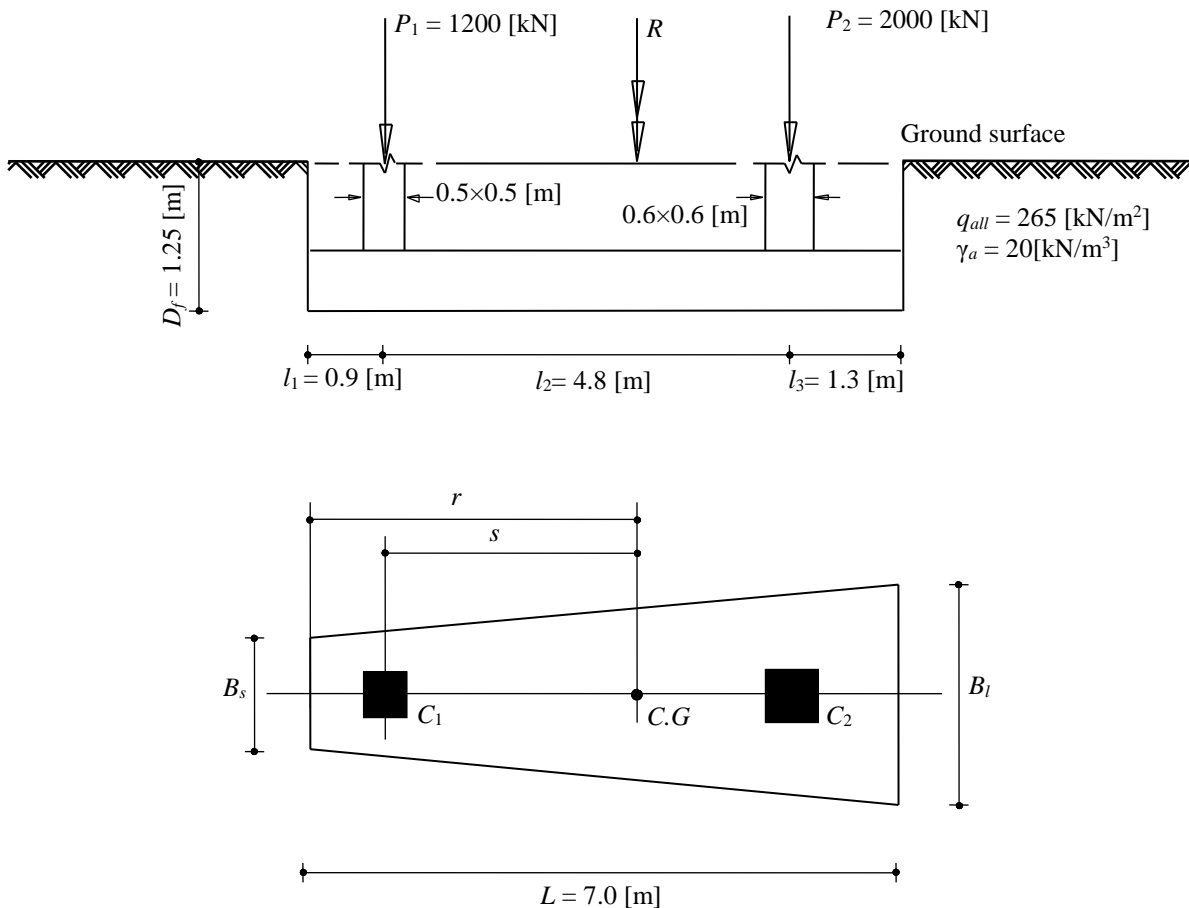


Figure 91: Combined trapezoidal footing

1. Determining footing sides B_s and B_l

The primary design required to establish the area of footing so that the center of footing area coincides with the center of gravity of the resultant. This will be conducted as follows:

Resultant of loads R at the ground surface level is given by:

$$R = P_1 + P_2 = 1200 + 2000 = 3200 \text{ [kN]}$$

Resultant of total loads R_T at the foundation level is given by:

Solved Examples

$$R_T = \frac{R}{\left(1 - \frac{\gamma_a D_f}{q_{all}}\right)}$$

$$R_T = \frac{3200}{\left(1 - \frac{20 \times 1.25}{265}\right)} = 3533.33 \text{ [kN]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R_T}{q_{all}} = \frac{3533.33}{265} = 13.33 \text{ [m}^2\text{]}$$

Referring to Figure 91, area of the footing A_f is given by:

$$A_f = \frac{L}{2}(B_s + B_l)$$

$$13.33 = \frac{7.0}{2}(B_s + B_l)$$

Simplifying,

$$B_s + B_l = 3.81 \quad (1)$$

Taking the moment of column loads about the center of the column C_1 , the distance s between the point of application of the resultant and the center of column C_1 is obtained from:

$$s \times R = P_2 \times l_2$$

$$s \times 3200 = 2000 \times 4.8$$

$$s = 3.0 \text{ [m]}$$

Hence, the point of application of the resultant is also the centroid of the footing area. Therefore, it can be shown from the geometry of the footing that the distance r from the short side B_s to the center of area is given by:

$$r = \frac{L}{3} \frac{B_s + 2B_l}{B_s + B_l}$$

$$l_1 + s = \frac{L}{3} \frac{B_s + 2B_l}{B_s + B_l}$$

Solved Examples

$$0.9 + 3.0 = \frac{7.0}{3} \frac{B_s + 2B_l}{B_s + B_l}$$

Simplifying,

$$2.04B_s - B_l = 0.0 \quad (2)$$

Solving Equations (1) and (2) yields to the required dimensions of B_s and B_l as follows:

$$B_s = 1.25 \text{ [m]} \text{ and } B_l = 2.56 \text{ [m]}$$

Chosen dimensions of B_s and B_l are:

$$B_s = 1.3 \text{ [m]} \text{ and } B_l = 2.6 \text{ [m]}$$

2. Computing the contact pressure and internal forces

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{3200}{0.5 \times (1.3 + 2.6) \times 7.0} = 234 \text{ [kN/m}^2\text{]} = 0.234 \text{ [MN/m}^2\text{]}$$

Now, the trapezoidal footing is treated as a beam. In this case the contact pressure under the base of the footing per meter will be linear. The contact pressure per meter at the short side is given by:

$$q_s = q_o B_s = 0.234 \times 1.3 = 0.304 \text{ [MN/m]}$$

While the contact pressure per meter at the long side is given by:

$$q_l = q_o B_l = 0.234 \times 2.6 = 0.608 \text{ [MN/m]}$$

To draw the contact pressure, shear force and bending moment diagrams for the beam footing, general expressions for these parameters are required. General expression for the contact pressure per meter $q(x)$ at a distance x from the short side is given by:

$$q(x) = q_s + (q_l - q_s) \frac{X}{L} = 0.304 + (0.608 - 0.304) \frac{X}{7.0}$$

$$q(x) = 0.304 + 0.043 X$$

Shear force $Q(x)$ due to linear contact pressure is given by:

$$Q(x) = \int q(x) dx$$

General expression for the shear force $Q(x)$ at a distance x from the short side is given by:

$$Q(x) = \int (0.304 + 0.043X) dx$$

Solved Examples

$$Q(x) = 0.304X + 0.022X^2$$

Moment $M(x)$ due to linear contact pressure is given by:

$$M(x) = \int Q(x) dx$$

General expression for the moment $M(x)$ at a distance x from the short side is given by:

$$M(x) = \int (0.304X + 0.022X^2) dx$$

$$M(x) = 0.152X^2 + 0.007X^3$$

Figure 92 shows the contact pressure, shear force and moment diagrams for the beam footing using the above general expressions.

3. Determining the maximum moment M_{max}

The maximum negative bending moment in the longitudinal direction occurs at the point of zero shear between columns. Equating the shear forces to zero at that point gives the distance x of maximum bending moment from the short side. At point of zero shear is:

$$Q(x) - P_1 = 0.0$$

$$0.304X + 0.022X^2 - 1.2 = 0.0$$

Solving this equation gives:

$$X = 3.22 \text{ [m]}$$

The maximum bending moment at the point of zero shear due to contact pressure and P_1 is given by:

$$M_{max} = M(x) - P_1(X - l_1)$$

$$M_{max} = 0.152X^2 + 0.007X^3 - P_1(X - l_1)$$

$$M_{max} = 0.152 \times 3.22^2 + 0.007 \times 3.22^3 - 1.2(3.22 - 0.9)$$

$$M_{max} = -0.974 \text{ [MN.m]}$$

Footing breadth B at any section is given by:

$$B = B_s + (B_l - B_s) \frac{X}{L} = 1.3 + (2.6 - 1.3) \frac{X}{7.0}$$

Simplifying,

$$B = 1.3 + 0.186 X$$

Solved Examples

Footing breadth B at the section of maximum moment is:

$$B = 1.3 + 0.186 X$$

$$B = 1.3 + 0.186 \times 3.22 = 1.9 \text{ [m]}$$

4. Determining the moment at critical sections M_{C1} and M_{C2}

The critical sections of positive moments in the longitudinal direction lie at the faces of the columns as shown in Figure 92.

The moment at the face of column C_1 is given by:

$$M_{c1} = 0.152X^2 + 0.007X^3$$

$$M_{C1} = 0.152 \times 0.65^2 + 0.007 \times 0.65^3$$

$$M_{C1} = 0.066 \text{ [MN.m]}$$

Footing breadth B_1 at the face of column C_1 is:

$$B_1 = 1.3 + 0.186 X$$

$$B_1 = 1.3 + 0.186 \times 0.65 = 1.42 \text{ [m]}$$

The moment at the face of column C_2 is given by:

$$M_{C2} = 0.152 (l_1 + l_2 + 0.3)^2 + 0.007 (l_1 + l_2 + 0.3)^3 - P_1 (l_2 + 0.3) - (0.3 P_2)$$

$$M_{C2} = 0.152 \times 6^2 + 0.007 \times 6^3 - 1.2 \times 5.1 - 0.3 \times 2$$

$$M_{C2} = 0.264 \text{ [MN.m]}$$

Footing breadth B_2 at the face of column C_2 is:

$$B_2 = 1.3 + 0.186 X$$

$$B_2 = 1.3 + 0.186 \times 6 = 2.42 \text{ [m]}$$

Solved Examples

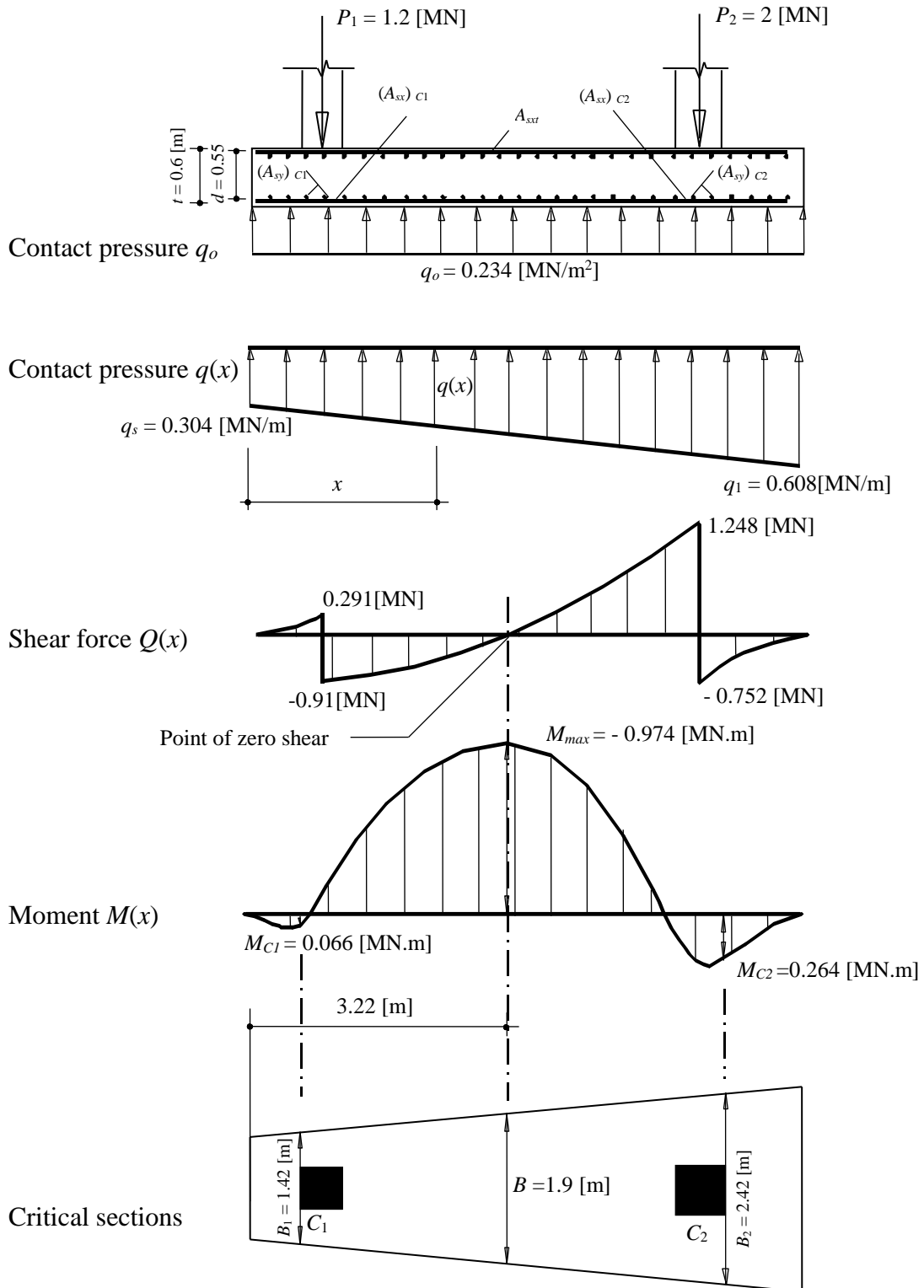


Figure 92: Internal forces and critical sections

Solved Examples

5. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5$ [MN/m²] and $f_s = 200$ [MN/m²], the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient k_2 [MN/m²] to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172$ [MN/m²].

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.974}{1.9}} = 0.55 \text{ [m]}$$

6. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.275$ [m] from the face of the column as shown in Figure 93. The check for punching shear under columns C_1 and C_2 is shown in Table 7.

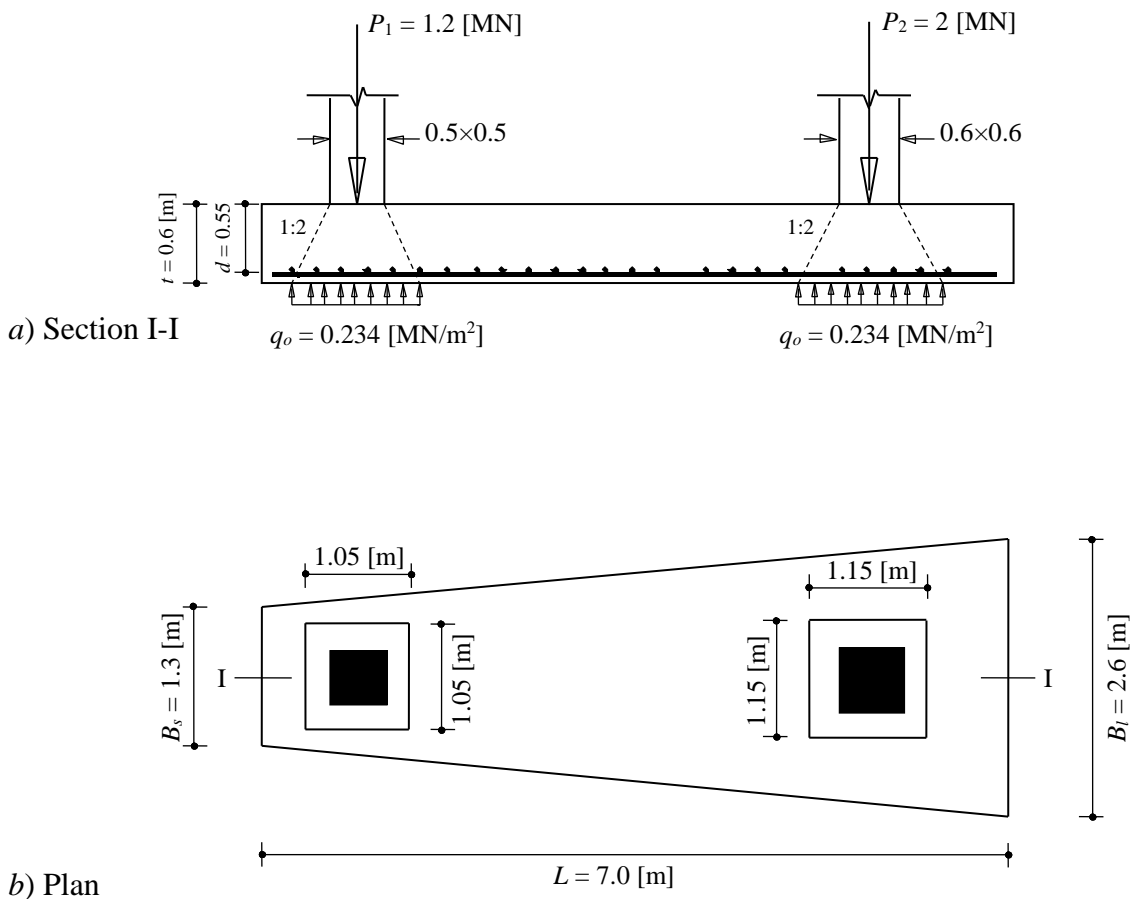


Figure 93: Critical section for punching shear

Solved Examples

Table 7: Check for punching shear

Load, stress and geometry	Column C ₁	Column C ₂
Column load P [MN]	1.2	2.0
Contact pressure q_o [MN/m ²]	0.234	0.234
Column sides $a \times b$ [m ²]	0.5×0.5	0.6×0.6
Footing thickness d [m]	0.55	0.55
Critical perimeter $b_o = 4(a + d)$ [m]	4.2	4.6
Critical area $A_p = (a + d)^2$ [m ²]	1.1025	1.3225
Punching load $Q_p = P - q_o A_p$ [MN]	0.942	1.69
Punching shear stress $q_p = Q_p / (b_o d)$ [MN/m ²]	0.408	0.67

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = (0.5 + 1) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

For both columns $q_{pall} > q_p$, the footing section is safe for punching shear.

7. Computing the steel reinforcement in longitudinal direction

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 60 \times 100 = 9.0$ [cm²/m]

Take $A_{smin} = 5\Phi 16 = 10.1$ [cm²/m].

Determining the required area of steel reinforcement in both x -directions and y -directions is shown in Table 8:

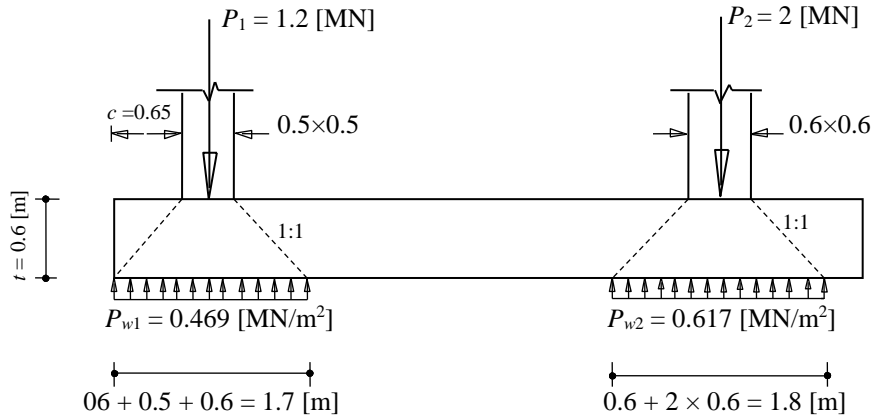
Table 8: Determination of tension reinforcement for x -direction

Position	Breadth B [m]	Moment M [MN.m]	Calculated A_s $A_s = M / (k_2.d) / B$ [cm ² / m]	Chosen reinforcement A_s
A_{sx1}	1.9	0.974	54.19	12Φ25/m
$(A_{sxb}) C_1$	1.42	0.066	4.91	5Φ16/m = A_{smin}
$(A_{sxb}) C_2$	2.42	0.264	11.53	5Φ19/m

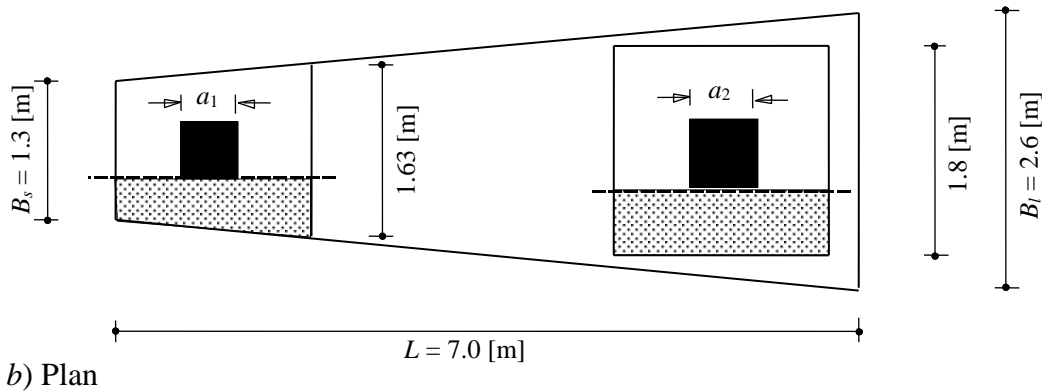
Solved Examples

8. Computing the steel reinforcement in transverse direction

The transverse bending moment may be approximately determined at each column by assuming that the column load is distributed outward at 45 [°] from the face of the column over an appropriate area as shown in Figure 94. Both sides of this area should not be greater than the breadth of the footing.



a) Elevation



b) Plan

Figure 94: sections of transverse moment

Column C₁

Length of the loaded area L_{T1} is given by:

$$L_{T1} = c + a_1 + t = 0.65 + 0.5 + 0.6 = 1.75 \text{ [m]}$$

Width of the loaded area B_{T1} at distance 1.7 [m] is:

$$B_{T1} = 1.3 + 0.186 \times L_{T1} = 1.3 + 0.186 \times 1.75 = 1.63 \text{ [m]}$$

Assuming for simplicity that the loaded area is a square area where its side L_a is determined from:

$$L_a = \sqrt{\frac{B_s + B_T}{2} L_{T1}} = \sqrt{\frac{1.3 + 1.63}{2}} 1.75 = 1.6 \text{ [m]}$$

Solved Examples

The transverse bending moment M_{T1} under the column C_1 is given by:

$$M_{T1} = \frac{P_1 (L_a - a_1)^2}{8 L_a} = \frac{1.2 (1.6 - 0.5)^2}{8 \times 1.6} = 0.113 \text{ [MN.m]}$$

Column C_2

The loaded area under the column C_2 is square of side L_{T2} :

$$L_{T2} = a_2 + 2 t = 0.6 + 2 \times 0.6 = 1.8 \text{ [m]}$$

The transverse bending moment M_{T2} under the column C_2 is given by:

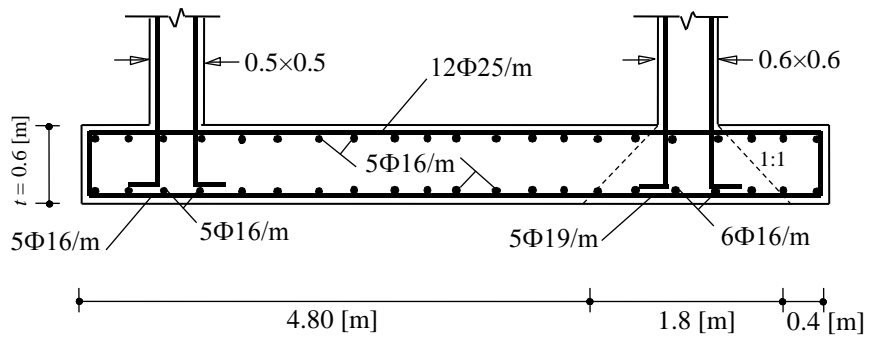
$$M_{T2} = \frac{P_2 (L_{T2} - a_2)^2}{8 L_{T2}} = \frac{2 (1.8 - 0.6)^2}{8 \times 1.8} = 0.2 \text{ [MN.m]}$$

Determining the required area of steel reinforcement in transverse directions is shown in Table 9. The details of reinforcement in plan and section $a-a$ through the footing are shown in Figure 95.

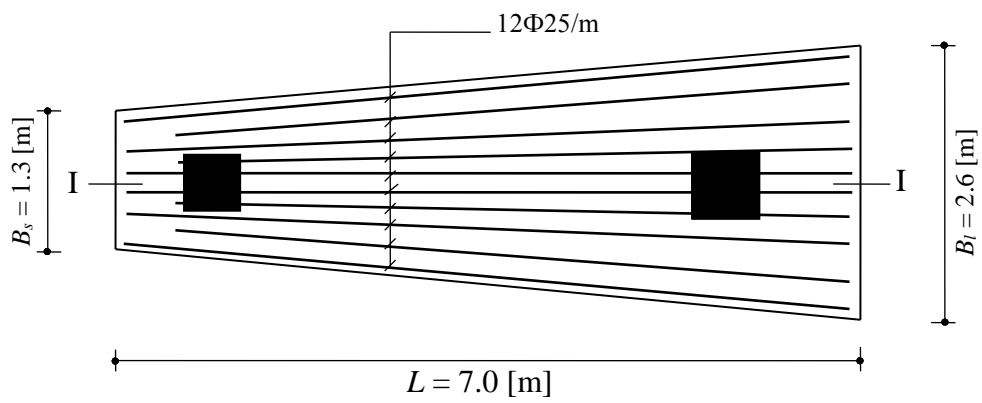
Table 9: Determination of reinforcement for transverse direction

Position	Breadth L_T [m]	Moment M [MN.m]	Calculated A_s $A_s = M / (k_2.d) / B$ [cm ² / m]	Chosen reinforcement A_s
$(A_{syb}) C_1$	1.6	0.113	7.47	$5\Phi 16/\text{m} = A_{smin}$
$(A_{syb}) C_2$	1.8	0.2	11.75	$6\Phi 16/\text{m}$

Solved Examples



a) Section I-I



b) Plan with top reinforcement

Figure 95: Details of reinforcement in plan and section *a-a* through the footing

Solved Examples

Example 31: Design of a combined T-shape footing

Design a combined T-shape footing of two columns, each of $0.5 \times 0.5 \text{ [m}^2\text{]}$, reinforced by $4\Phi 19$. Load of column C_1 is $P_1 = 1800 \text{ [kN]}$ while that of column C_2 is $P_2 = 1200 \text{ [kN]}$ as shown in Figure 96. The distance center to center of columns is $l = 3.0 \text{ [m]}$.

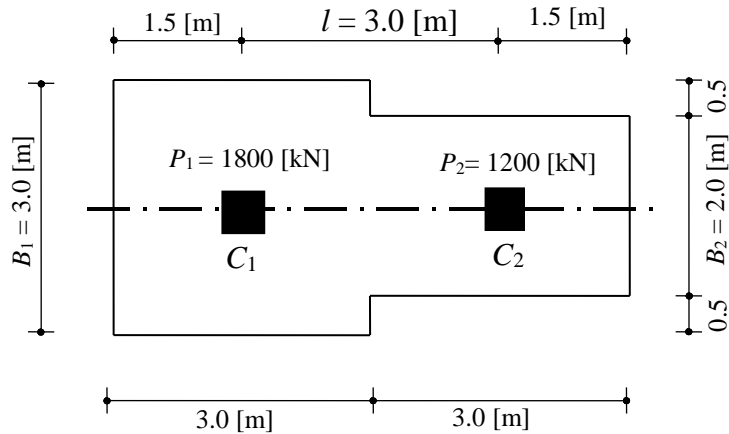


Figure 96: Combined T-section footing

1. Check for uniform contact pressure

To determine the center of gravity of the footing area from the left side, take moments of areas about the left side (Figure 97):

$$\bar{x}_f A_f = \sum x_i A_i$$

$$\bar{x}_f (3.0 \times 3.0 + 3.0 \times 2.0) = (1.5 \times 3.0 \times 3.0 + 4.5 \times 3.0 \times 2.0)$$

$$\bar{x}_f = 2.7 \text{ [m]}$$

Point of application of the resultant force from column C_1 :

$$R = P_1 + P_2 = 1800 + 1200 = 3000 \text{ [kN]}$$

$$R C = P_2 l$$

$$3000 C = 1200 \times 3.0$$

$$C = 1.2 \text{ [m]}$$

Point of application of the resultant force from the left side:

$$\bar{x}_R = 1.5 + 1.2 = 2.7 \text{ [m]}$$

The point of application of the resultant force coincides with the center of gravity of the footing area. Then the contact pressure under the footing will be uniform.

Solved Examples

2. Computing the contact pressure

The contact pressure per meter square under the base of the footing is given by:

$$q_o = \frac{R}{A_f} = \frac{3200}{3.0 \times 3.0 + 3.0 \times 2.0} = 200 \text{ [kN/m}^2\text{]} = 0.2 \text{ [MN/m}^2\text{]}$$

Now, the T-section footing is treated as a beam footing. The contact pressure per meter at the left side is given by:

$$q_1 = q_o B_1 = 0.2 \times 3.0 = 0.6 \text{ [MN/m]}$$

While the contact pressure per meter at the right side is given by:

$$q_2 = q_o B_2 = 0.2 \times 2.0 = 0.4 \text{ [MN/m]}$$

Figure 97 shows the load and moment diagrams for the beam footing.

3. Determining the maximum moment M_{max}

From Figure 97, the maximum bending moments in the longitudinal direction occur at faces of columns which are:

$$M_1 = q_1 \frac{l_1^2}{2} = 0.6 \frac{1.25^2}{2} = 0.469 \text{ [MN.m]}$$

$$M_2 = q_2 \frac{l_2^2}{2} = 0.4 \frac{1.25^2}{2} = 0.313 \text{ [MN.m]}$$

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{max}}{B}}$$

$$d_{m1} = 0.766 \sqrt{\frac{0.469}{3.0}} = 0.3 \text{ [m]}$$

$$d_{m2} = 0.766 \sqrt{\frac{0.313}{2.0}} = 0.3 \text{ [m]}$$

Solved Examples

Take $d = 0.45 \text{ [m]} > d_m = 0.3 \text{ [m]}$.

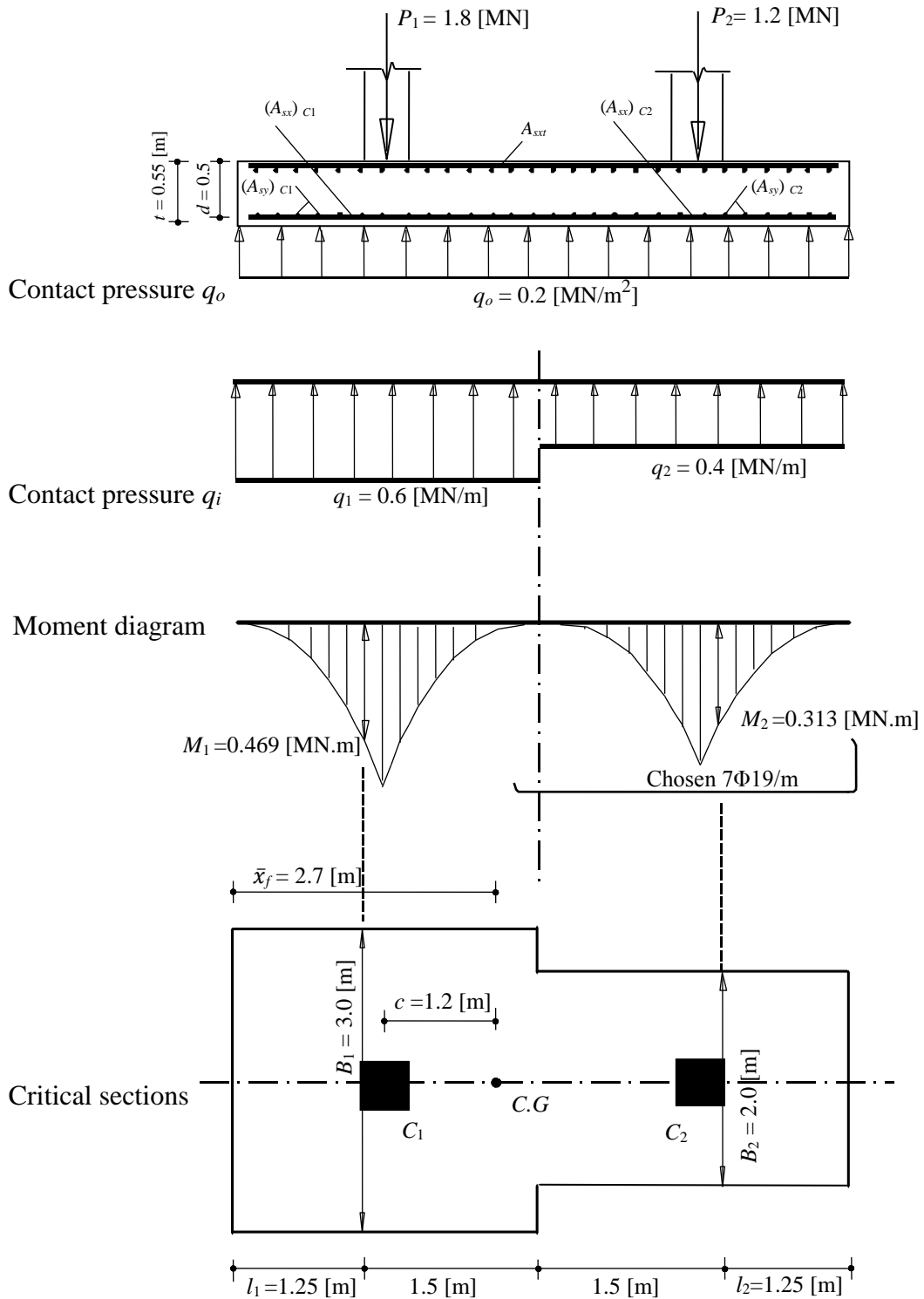


Figure 97: Internal forces and critical sections

Solved Examples

5. Determining the depth required to resist the punching shear d_p

The critical punching shear section on a perimeter at a distance $d/2 = 0.225$ [m] from the face of the column as shown in Figure 98. The check for punching shear under columns C_1 and C_2 is shown in Table 10.

Table 10: Check for punching shear

Load, stress and geometry	Column C_1	Column C_2
Column load P [MN]	1.8	1.2
Contact pressure q_o [MN/m ²]	0.2	0.2
Column sides $a \times b$ [m ²]	0.5×0.5	0.5×0.5
Footing thickness d [m]	0.45	0.45
Critical perimeter $b_o = 4(a + d)$ [m]	3.8	3.8
Critical area $A_p = (a + d)^2$ [m ²]	0.9025	0.9025
Punching load $Q_p = P - q_o A_p$ [MN]	1.62	1.02
Punching shear stress $q_p = Q_p / (b_o d)$ [MN/m ²]	0.947	0.596

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = (0.5 + 1) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

For column C_1 $q_p > q_{pall}$, the footing section is unsafe for punching shear. Then, the required thickness to resist punching shear is:

$$d_p = \frac{Q_p}{b_o q_{pall}} = \frac{1.62}{3.8 \times 0.9} = 0.47 \text{ [m]}$$

Take $d = 0.5$ [m] $> d_p = 0.47$ [m] and $t = 0.55$ [m].

Solved Examples

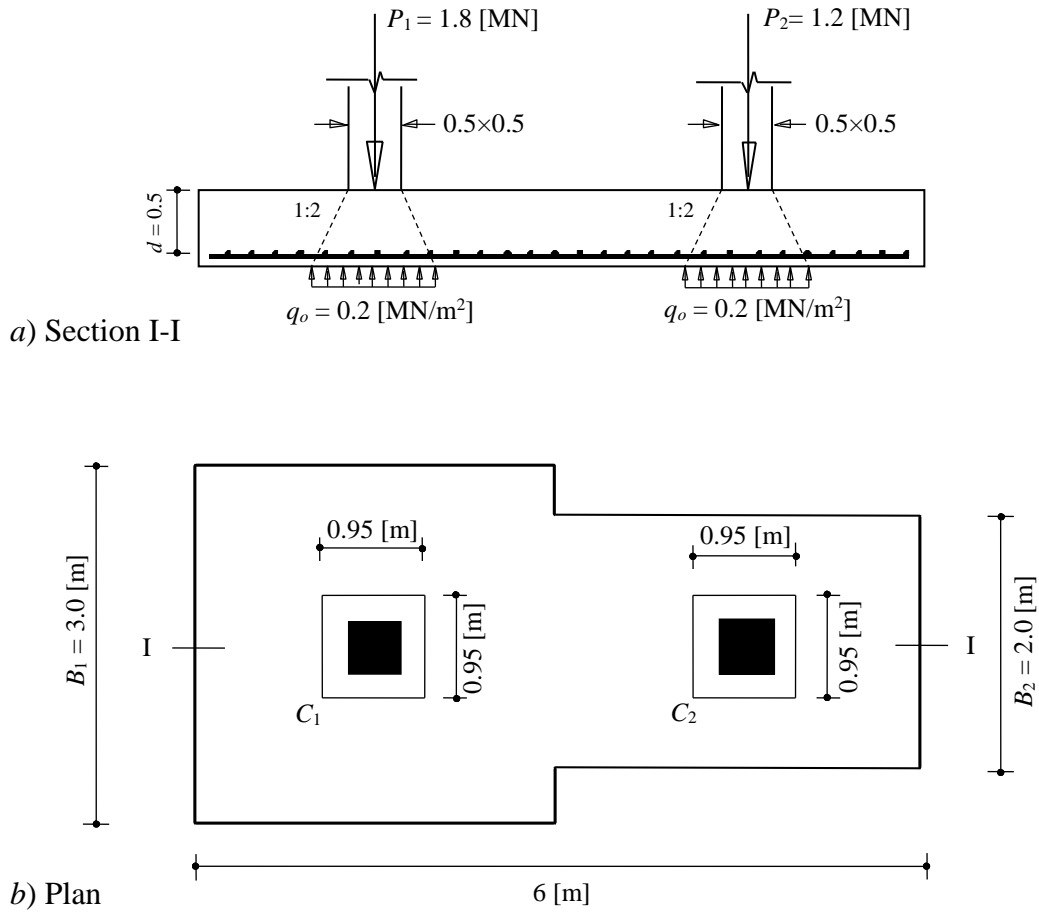


Figure 98: Critical section for punching shear

6. Computing the steel reinforcement in longitudinal direction

The section is designed as singly reinforced section, the corresponding k_1 for $d = 0.5$ [m] is given by:

$$k_1 = 0.5 \sqrt{\frac{3.0}{0.469}} = 1.26$$

From Table 2 at $k_1 = 1.27$, $f_c = 5.0$ [MN/m²] and $k_2 = 182$

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 55 \times 100 = 8.25$ [cm²/m]

Take $A_{smin} = 5\Phi 16 = 10.1$ [cm²/m].

Determining the required area of steel reinforcement in both x -directions and y -directions is shown in Table 11:

Solved Examples

Table 11: Determination of tension reinforcement for x -direction

Position	Breathth B [m]	Moment M [MN.m]	Calculated A_s $A_s = M / (k_2.d) / B$ [cm ² / m]	Chosen reinforcement A_s
$(A_{sxb}) C_1$	3.0	0.469	17.18	7 Φ 19/m
$(A_{sxb}) C_2$	2.0	0.313	17.18	7 Φ 19/m

7. Computing the steel reinforcement in transverse direction

The transverse bending moment may be approximately determined at each column by assuming that the column load is distributed outward at 45 [°] from the face of the column over an appropriate area as shown in Figure 99. Both sides of this area should not be greater than the breadth of the footing.

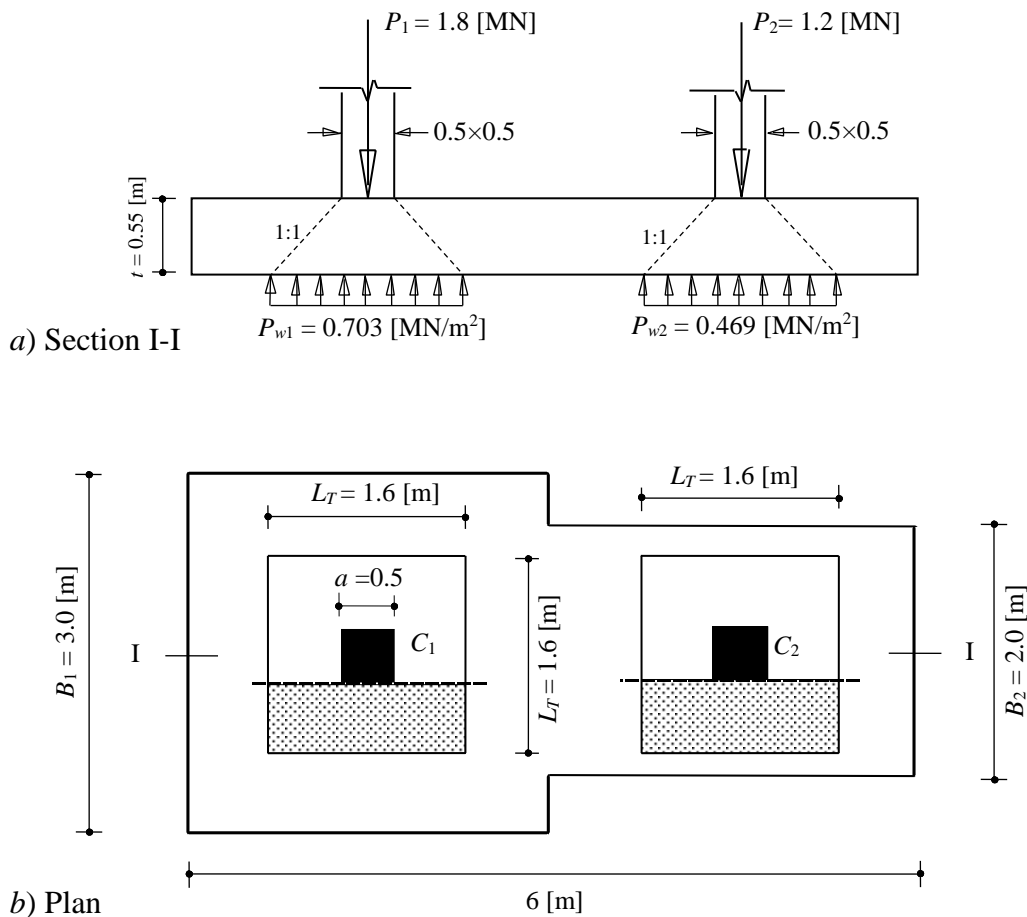


Figure 99: sections for determining transverse moments

Length of the loaded area L_T is given by:

$$L_T = a + 2 t = 0.5 + 2 \times 0.55 = 1.6 \text{ [m]}$$

The transverse bending moment M_{T1} under the column C_1 is given by:

Solved Examples

$$M_{T1} = \frac{P_1}{8} \frac{(L_T - a)^2}{L_T} = \frac{1.8}{8} \frac{(1.6 - 0.5)^2}{1.6} = 0.17 \text{ [MN.m]}$$

The transverse bending moment M_{T2} under the column C_2 is given by:

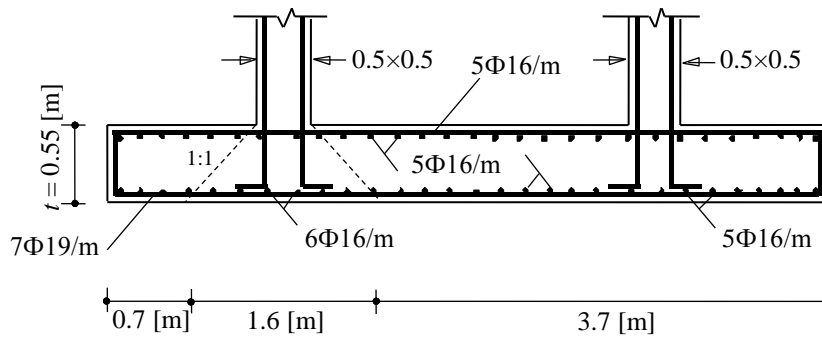
$$M_{T2} = \frac{P_2}{8} \frac{(L_T - a)^2}{L_T} = \frac{1.2}{8} \frac{(1.6 - 0.5)^2}{1.6} = 0.11 \text{ [MN.m]}$$

Determining the required area of steel reinforcement in transverse directions is shown in Table 12. The details of reinforcement in plan and section $a-a$ through the footing are shown in Figure 100.

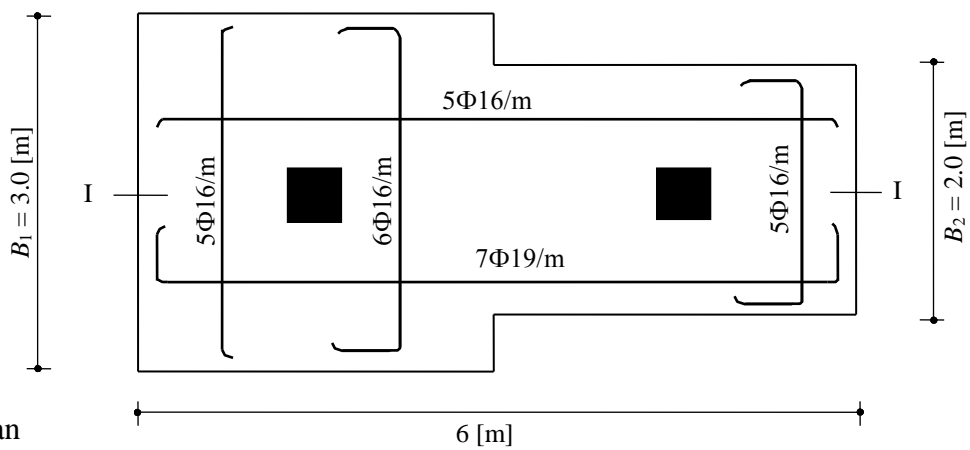
Table 12: Determination of reinforcement for transverse direction

Position	Breadth L_T [m]	Moment M [MN.m]	Calculated A_s $A_s = M / (k_2.d) / B$ [cm ² / m]	Chosen reinforcement A_s
$(A_{syb}) C_1$	1.6	0.17	12.06	6Φ16/m
$(A_{syb}) C_2$	1.6	0.11	7.55	5Φ16/m = A_{smin}

Solved Examples



a) Section I-I



b) Plan

Figure 100: details of reinforcement in plan and section *a-a* through the footing

6- Design of Strap Footings

Solved Examples

Example 32: Design of a strap footing for two equal interior columns

Design a strap footing for two equal interior columns, each of 0.5×0.5 [m²], reinforced by $4\Phi 19$, carrying a load of $P = 1500$ [kN] as shown in Figure 101. The distance center to center of columns is 5.0 [m]. The allowable soil pressure is $q_{all} = 210$ [kN/m²] at a depth of $D_f = 2.0$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

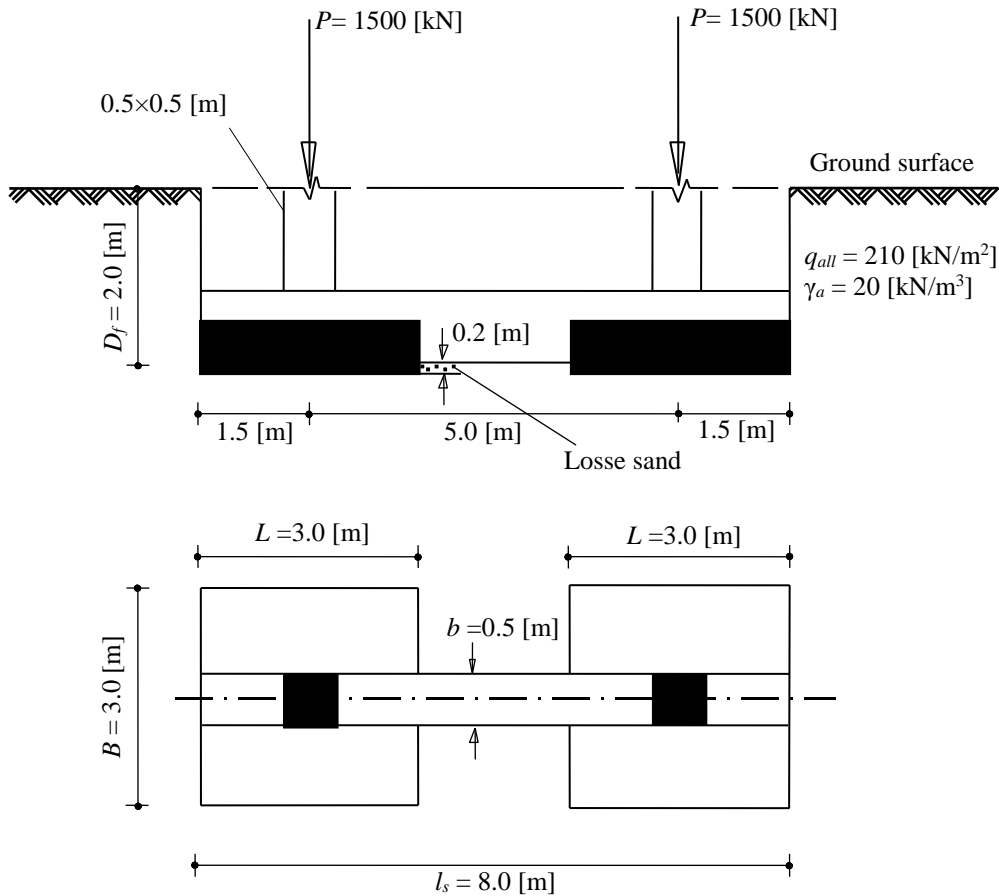


Figure 101: Combined strap footing

1. Determining footing sides

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 210 - 20 \times 2.0 = 170 \text{ [kN/m}^2\text{]}$$

Area of each footing A_f is obtained from:

$$A_f = \frac{P}{q_{net}} = \frac{1500}{170} = 8.82 \text{ [m}^2\text{]}$$

Take $A_f = 3.0 \times 3.0 = 9.0$ [m²] square footing

Solved Examples

2. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{1500}{9.0} = 166.67 \text{ [kN/m}^2\text{]} = 0.167 \text{ [MN/m}^2\text{]}$$

Now, the strap footing is treated as a beam supporting on two equal footings. The contact pressure per meter is given by:

$$q = q_o B_s = 0.167 \times 3.0 = 0.5 \text{ [MN/m]}$$

Figure 102 shows the load, shear force and moment diagrams for the strap footing.

3. Design of the strap beam

From Figure 102, the maximum bending moment in the strap beam occurs at the face of the column, which is:

$$M_{\max} = q \frac{c^2}{2} = 0.5 \frac{1.25^2}{2} = 0.391 \text{ [MN.m]}$$

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\max}}{b}}$$

$$d_m = 0.766 \sqrt{\frac{0.391}{0.5}} = 0.68 \text{ [m]}$$

Take $d = 0.70 \text{ [m]} > d_m = 0.68 \text{ [m]}$ and $t = 0.75 \text{ [m]}$, then the section is designed as singly reinforced section.

The required area of steel reinforcement A_s is:

$$A_s = \frac{M_{\max}}{k_2 d}$$

$$A_s = \frac{0.391}{172 \times 0.7} = 0.003248 \text{ [m}^2\text{]}$$

$$A_s = 32.48 \text{ [cm}^2\text{]}$$

Solved Examples

Chosen steel $9\Phi 22 = 34.2 \text{ [cm}^2\text{]}$ at bottom. Take $A_s' = 0.25 A_s = 4\Phi 19 = 11.3 \text{ [cm}^2\text{]}$

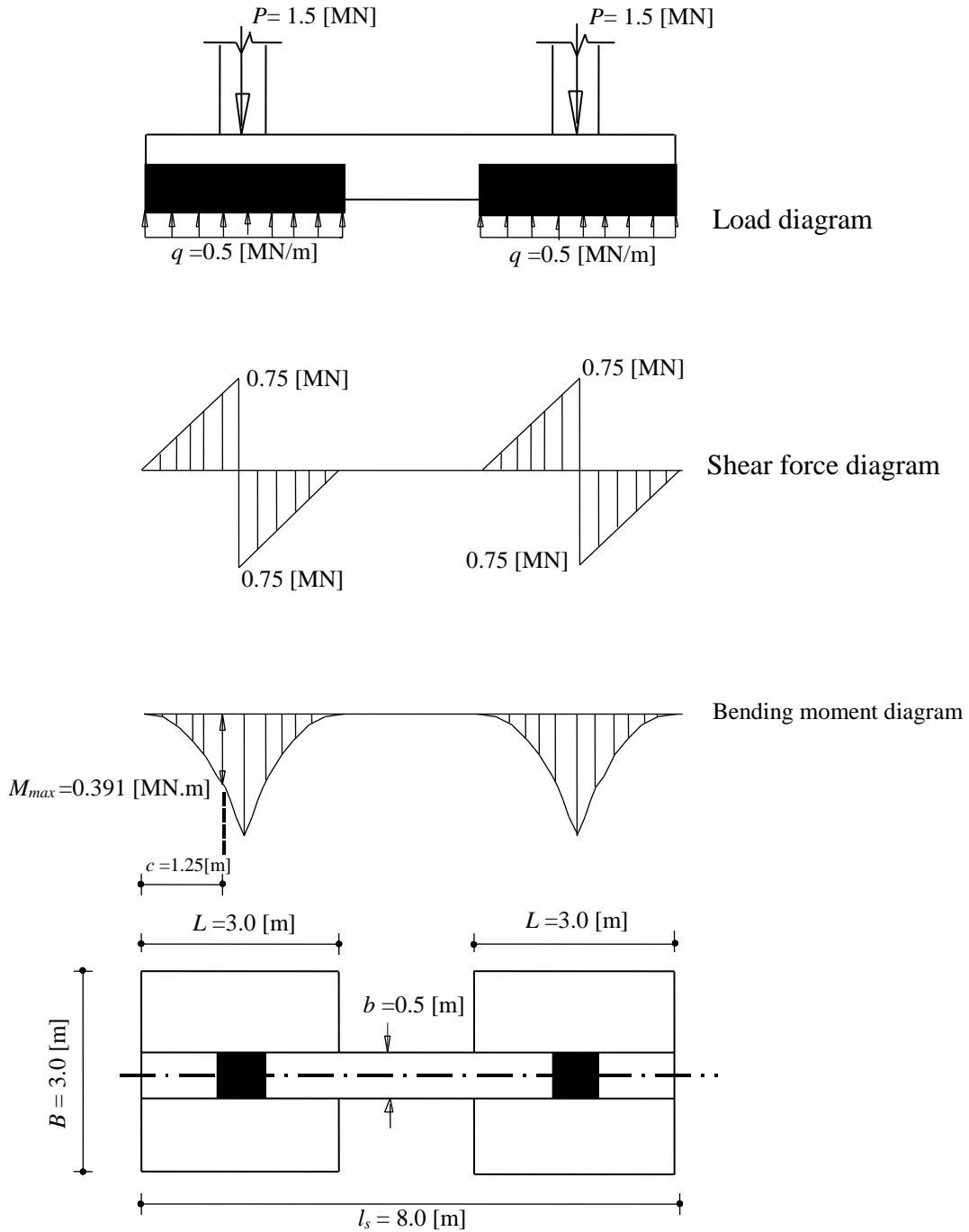


Figure 102: Load, shear force and moment diagrams with critical section of the strap beam

4. Design of the footing slab

The maximum bending moment for the footing slab lies at the strap face as shown in Figure 103. The transverse bending moment M_T at the strap face is given by:

Solved Examples

$$M_T = \frac{q_o}{8} L (B - b)^2 = \frac{0.167}{8} 3.0 (3.0 - 0.5)^2 = 0.391 \text{ [MN.m]}$$

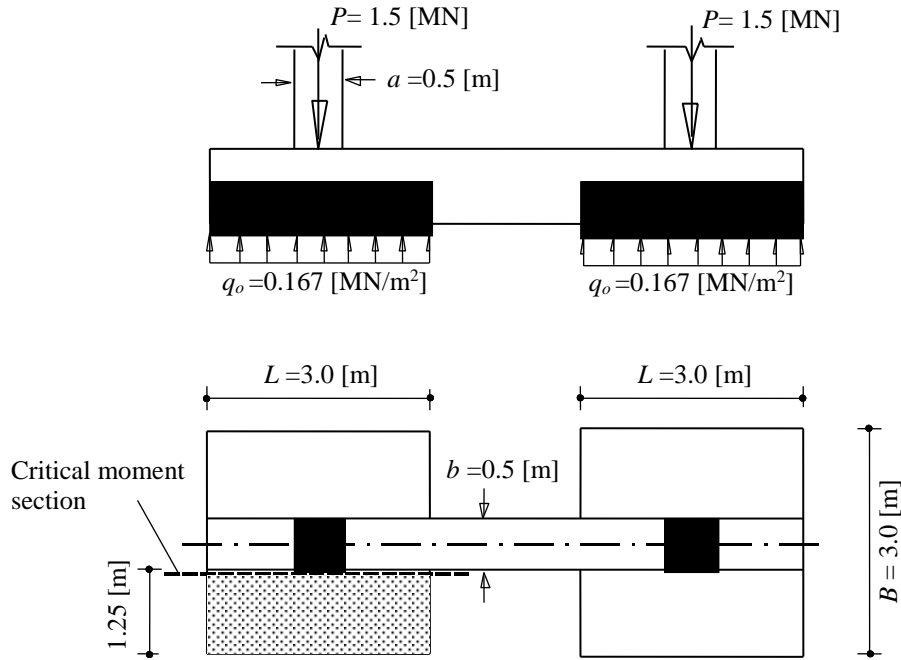


Figure 103: Section for determining footing reinforcement

The footing depth d_f is:

$$d_f = k_1 \sqrt{\frac{M_T}{L}}$$

$$d_f = 0.766 \sqrt{\frac{0.391}{3.0}} = 0.28 \text{ [m]}$$

Take $d_f = 0.45 \text{ [m]} > 0.28 \text{ [m]}$ and $t_f = 0.5 \text{ [m]}$, then the section is designed as singly reinforced section.

The corresponding k_1 for $d_f = 0.45 \text{ [m]}$ is given by:

$$k_1 = 0.45 \sqrt{\frac{3.0}{0.391}} = 1.25$$

From Table 2 at $k_1 = 1.27$, $f_c = 5.0 \text{ [MN/m}^2\text{]}$ and $k_2 = 182$

The required area of steel reinforcement in transverse direction A_{sT} is:

$$A_{sT} = \frac{M_T}{k_2 d}$$

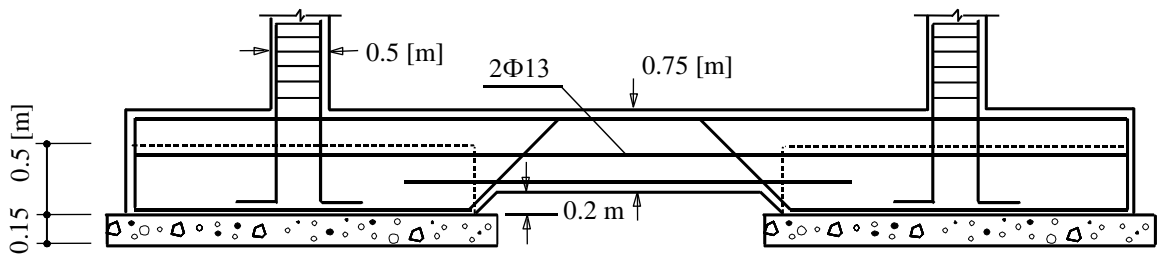
Solved Examples

$$A_s = \frac{0.391}{182 \times 0.45} = 0.004774 \text{ [m}^2\text{/3.0 m]}$$

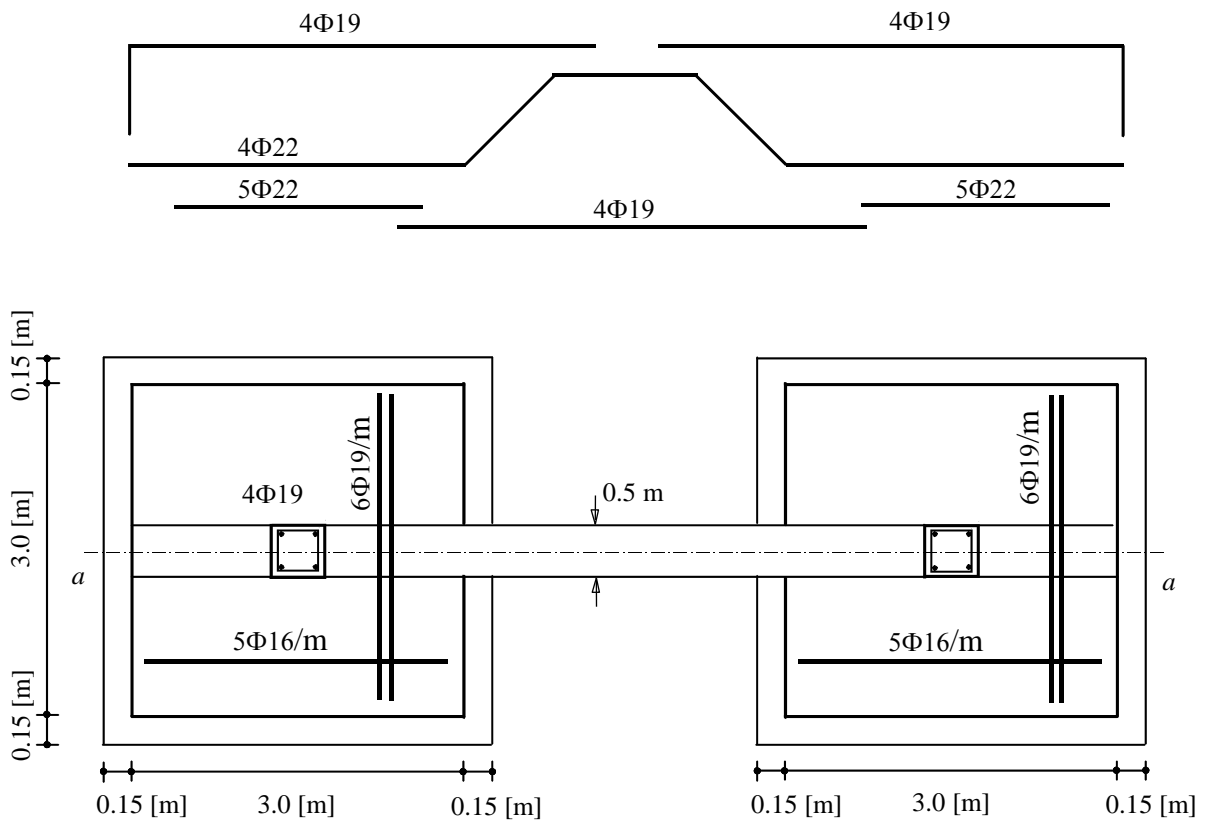
$$A_{sT} = 47.74 \text{ [cm}^2\text{/3.0 m]} = 15.91 \text{ [cm}^2\text{/m]}$$

Chosen steel $6\Phi 19/\text{m} = 17.0 \text{ [cm}^2\text{/m]}$

The details of reinforcement in plan and section *a-a* through the footing are shown in Figure 104.



a) Section I-I



b) Plan

Figure 104: Footing dimensions and reinforcement

Solved Examples

Example 33: Design of a strap footing for two equal edge columns

Design a strap footing for two equal edge columns, each of 0.5×0.5 [m²], reinforced by $4\Phi 19$, carrying a load of $P = 1500$ [kN] as shown in Figure 105. The distance center to center of columns is 7.5 [m]. The allowable soil pressure is $q_{all} = 210$ [kN/m²] at a depth of $D_f = 2.0$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

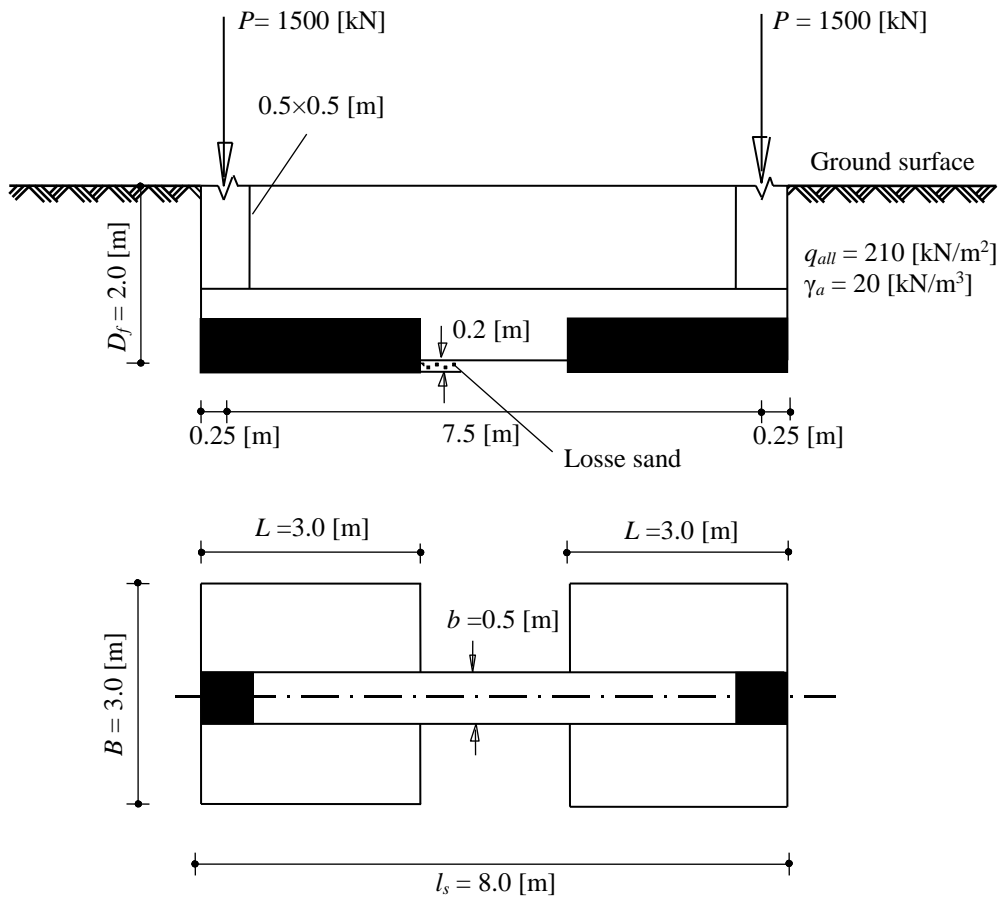


Figure 105: Combined strap footing

1. Determining footing sides

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 210 - 20 \times 2.0 = 170 \text{ [kN/m}^2\text{]}$$

Area of each footing A_f is obtained from:

$$A_f = \frac{P}{q_{net}} = \frac{1500}{170} = 8.82 \text{ [m}^2\text{]}$$

Take $A_f = 3.0 \times 3.0 = 9.0$ [m²] square footing

Solved Examples

2. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{1500}{9.0} = 166.67 \text{ [kN/m}^2\text{]} = 0.167 \text{ [MN/m}^2\text{]}$$

Now, the strap footing is treated as a beam supporting on two equal footings. The contact pressure per meter is given by:

$$q = q_o B = 0.167 \times 3.0 = 0.5 \text{ [MN/m]}$$

Figure 106 shows the load, shear force and moment diagrams for the strap footing.

3. Design of the strap beam

From Figure 106, the maximum bending moment in the strap beam occurs at the face of the column, which is:

$$M_{\max} = p c - q \frac{L^2}{2} = 1.5 \times 2.75 - 0.5 \frac{3.0^2}{2} = 1.875 \text{ [MN.m]}$$

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\max}}{b}}$$

$$d_m = 0.766 \sqrt{\frac{1.875}{0.5}} = 1.48 \text{ [m]}$$

Take $d = 1.5 \text{ [m]} > d_m = 1.48 \text{ [m]}$ and $t = 1.55 \text{ [m]}$, then the section is designed as singly reinforced section.

The required area of steel reinforcement A_s is:

$$A_s = \frac{M_{\max}}{k_2 d}$$

$$A_s = \frac{1.875}{172 \times 1.5} = 0.007267 \text{ [m}^2\text{]}$$

$$A_s = 72.67 \text{ [cm}^2\text{]}$$

Solved Examples

Chosen steel $20\Phi 22 = 76.0 \text{ [cm}^2\text{]}$ at top. Take $A_s' = 0.25 A_s = 5\Phi 22 = 19.0 \text{ [cm}^2\text{]}$

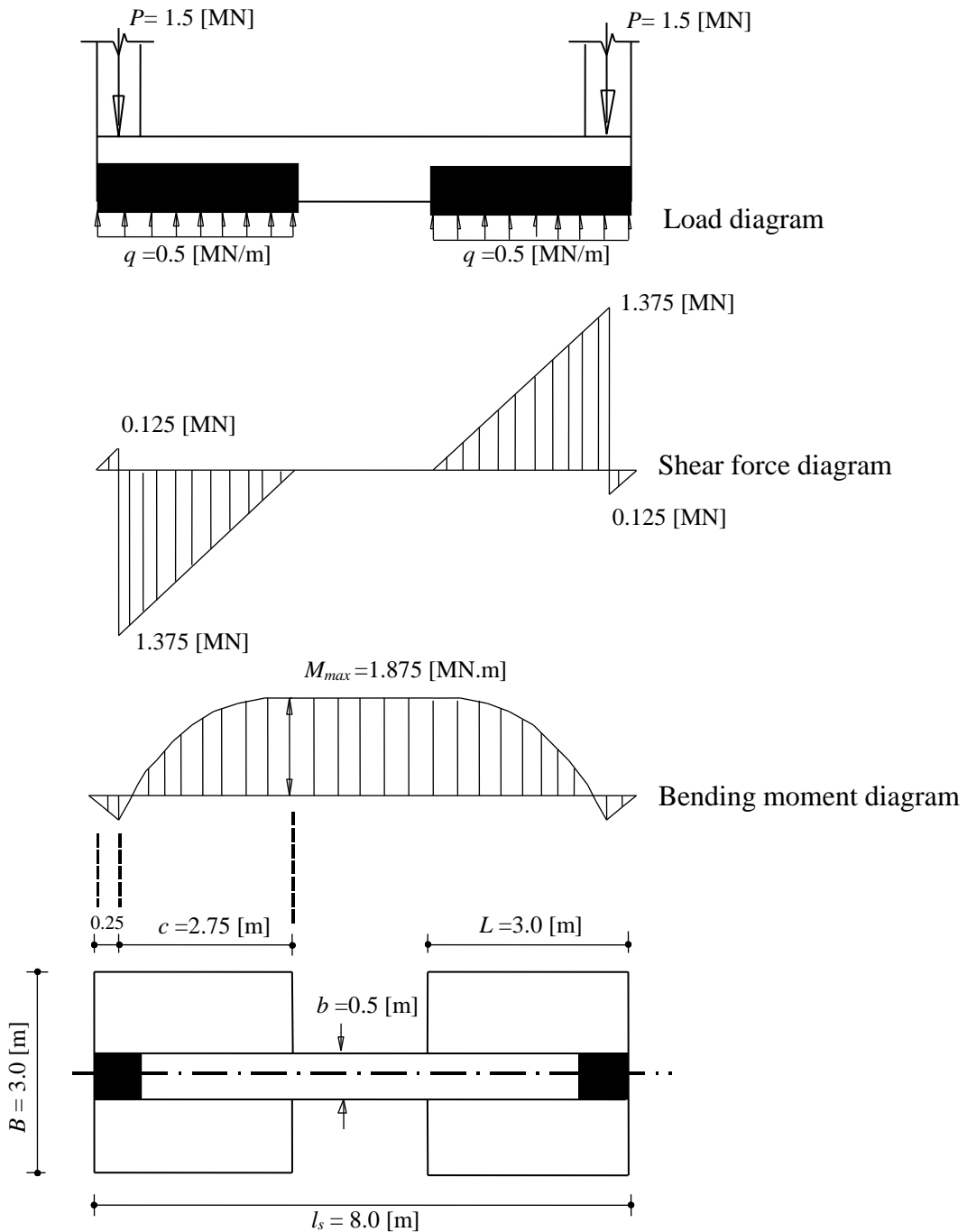


Figure 106: Load, shear force and moment diagrams with critical section of the strap beam

4. Design of the footing slab

The maximum bending moment for the footing slab lies at the strap face as shown in Figure 107. The transverse bending moment M_T at the strap face is given by:

Solved Examples

$$M_T = \frac{q_o}{8} L (B - b)^2 = \frac{0.167}{8} 3.0 (3.0 - 0.5)^2 = 0.391 \text{ [MN.m]}$$

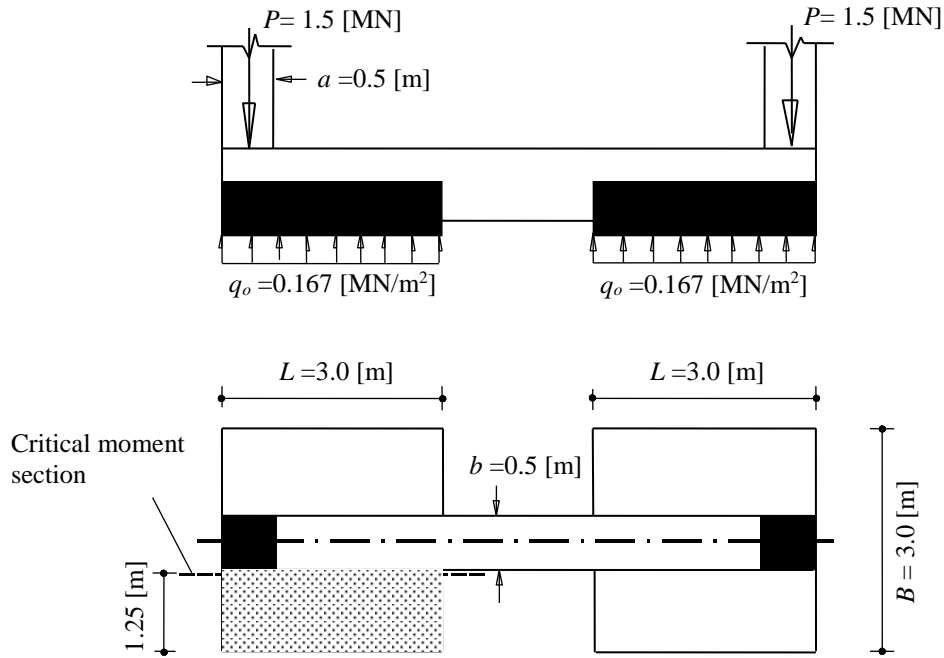


Figure 107: Section for determining footing reinforcement

The footing depth d_f is:

$$d_f = k_1 \sqrt{\frac{M_T}{L}}$$

$$d_f = 0.766 \sqrt{\frac{0.391}{3.0}} = 0.28 \text{ [m]}$$

Take $d_f = 0.45 \text{ [m]} > 0.28 \text{ [m]}$ and $t_f = 0.5 \text{ [m]}$, then the section is designed as singly reinforced section.

The corresponding k_1 for $d_f = 0.45 \text{ [m]}$ is given by:

$$k_1 = 0.45 \sqrt{\frac{3.0}{0.391}} = 1.25$$

From Table 2 at $k_1 = 1.27$, $f_c = 5.0 \text{ [MN/m}^2\text{]}$ and $k_2 = 182$

The required area of steel reinforcement in transverse direction A_{sT} is:

$$A_{sT} = \frac{M_T}{k_2 d}$$

Solved Examples

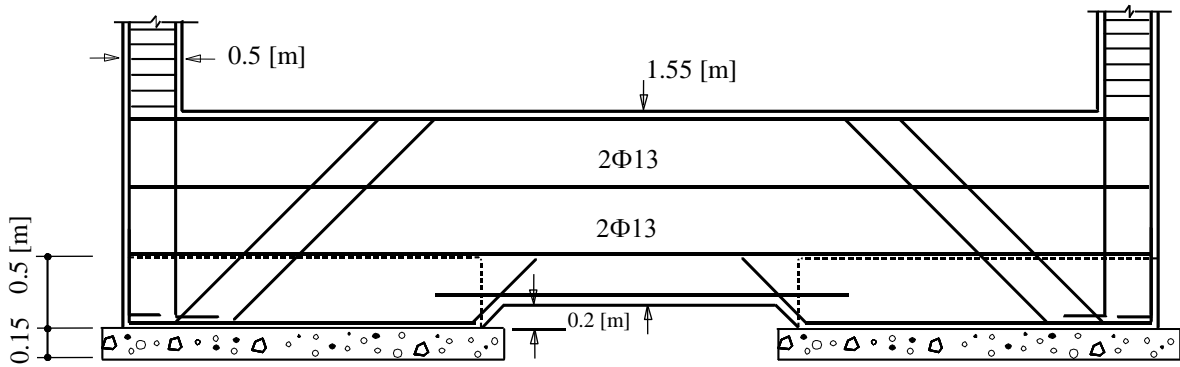
$$A_s = \frac{0.391}{182 \times 0.45} = 0.004774 \text{ [m}^2\text{/3.0 m]}$$

$$A_{sT} = 47.74 \text{ [cm}^2\text{/3.0 m]} = 15.91 \text{ [cm}^2\text{/m]}$$

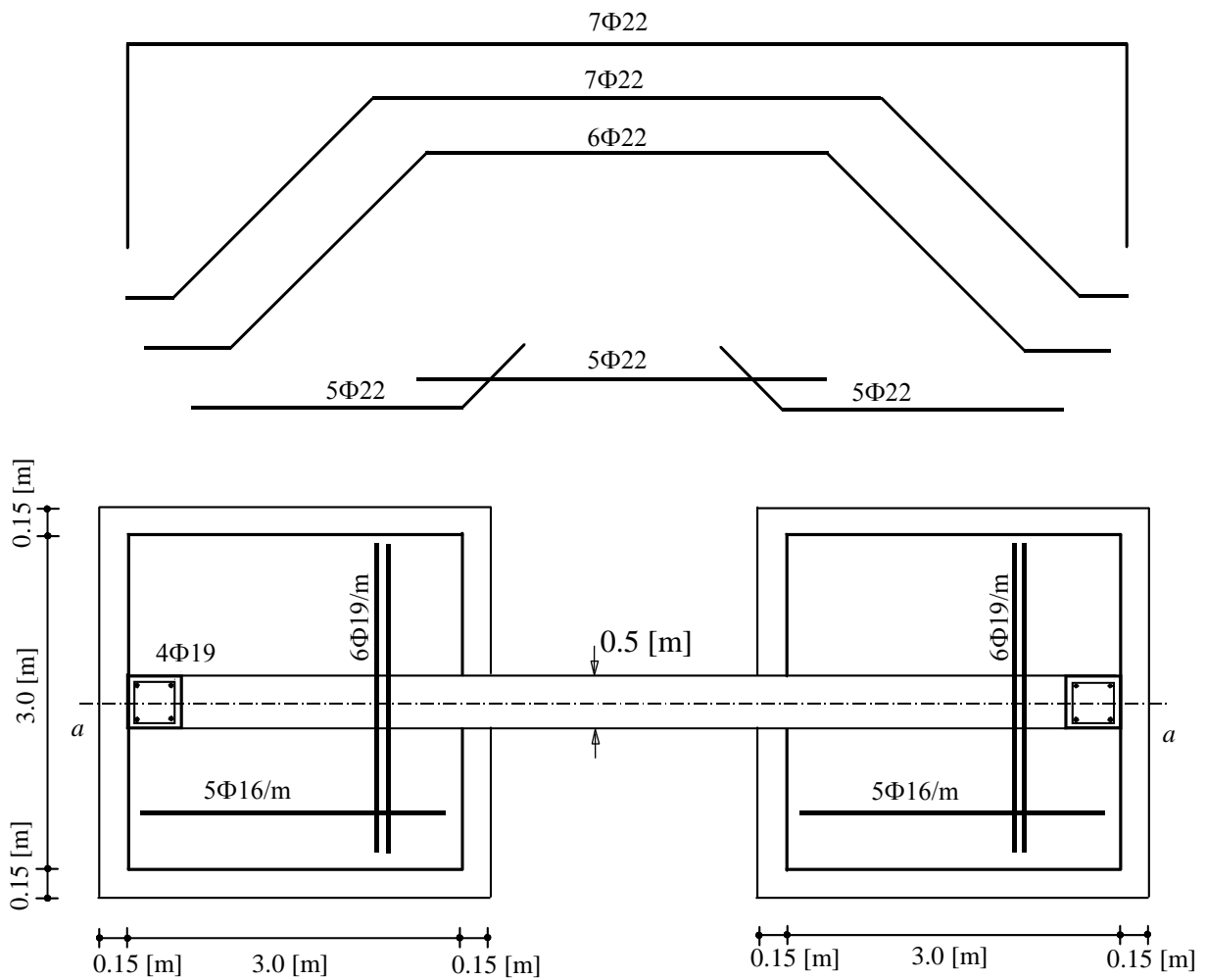
Chosen steel $6\Phi 19/\text{m} = 17.0 \text{ [cm}^2\text{/m]}$

The details of reinforcement in plan and section *a-a* through the footing are shown in Figure 108.

Solved Examples



a) Section I-I



b) Plan

Figure 108: Footing dimensions and reinforcement

Solved Examples

Example 34: Design of a strap footing for two unequal columns

Design a strap footing for an exterior column load of $P_1 = 1200$ [kN] and an interior column with a load of $P_2 = 1500$ [kN] as shown in Figure 109. Allowable bearing capacity of the soil is $q_{all} = 200$ [kN/m²] at a depth of $D_f = 1.6$ [m] and the average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

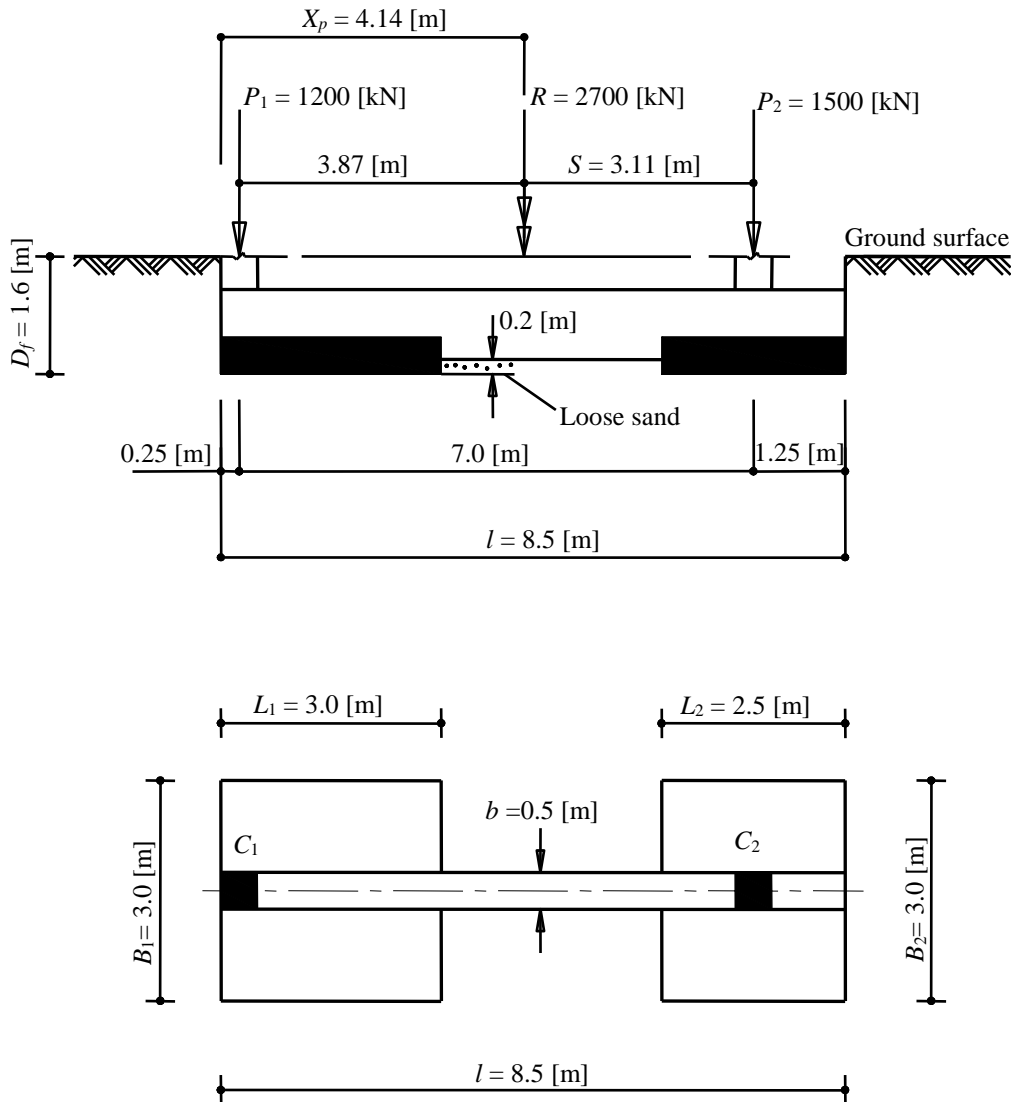


Figure 109: Combined strap footing

1. Determining point of application of the resultant force

Resultant force R equals:

$$R = P_1 + P_2 = 1200 + 1500 = 2700 \text{ [kN]}$$

Take the moment about C_2 to get the distance S :

$$P_1 l = RS$$

Solved Examples

$$1200 \times 7.0 = 2700 \times S$$

$$S = 3.11 \text{ [m]}$$

Point of application from the property line X_p :

$$X_p = 7.0 - 3.11 + 0.25 = 4.14 \text{ [m]}$$

2. Determining footing sides

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 200 - 20 \times 1.6 = 168 \text{ [kN/m}^2\text{]}$$

Trail (1)

$$A_{f1} = \frac{P_1}{q_{net}} = \frac{1200}{168} = 7.14 \text{ [m}^2\text{]}$$

Take $A_{f1} = 3.0 \times 2.5 = 7.5 \text{ [m}^2\text{]}$ Rectangular footing

$$A_{f2} = \frac{P_2}{q_{net}} = \frac{1500}{168} = 8.92 \text{ [m}^2\text{]}$$

Take $A_{f2} = 3.0 \times 3.0 = 9.0 \text{ [m}^2\text{]}$ Square footing

Total area:

Take moment of areas about the property line, to get the distance X_s :

$$A_{f1}X_1 + A_{f2}X_2 = A_t X_s$$

$$(7.5 \times 1.5) + (9.0 \times 7.25) = 16.5 X_s$$

$$X_s = 4.64 \text{ [m]} > X_p$$

$X_s > X_p$, there for A_{f2} must be reduced and A_{f1} must be exceed.

Trail (2)

$$A_{f1} = 3.0 \times 3.0 = 9.0 \text{ [m}^2\text{]} \text{ Square footing}$$

$$A_{f2} = 2.5 \times 3.0 = 7.5 \text{ [m}^2\text{]} \text{ Rectangular footing}$$

$$A_t = 9.0 + 7.5 = 16.5 \text{ [m}^2\text{]}$$

$$X_s = \frac{(9.0 \times 1.5) + (7.5 \times 7.25)}{16.5} = 4.11 \text{ [m]} \cong X_p$$

Solved Examples

3. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{2700}{16.5} = 163.64 \text{ [kN/m}^2\text{]} = 0.164 \text{ [MN/m}^2\text{]}$$

Now, the strap footing is treated as a beam supporting on two unequal footings. The contact pressure per meter square under the footing (1) is given by:

$$q_1 = q_o B_1 = 0.164 \times 3.0 = 0.491 \text{ [MN/m]}$$

While the contact pressure under the footing (2) is given by:

$$q_2 = q_o B_2 = 0.164 \times 3.0 = 0.491 \text{ [MN/m]}$$

4. Determining the critical bending moments

The maximum bending moment in the longitudinal direction of the strap beam occurs at the interior edge of the left footing. Figure 110 shows the load diagram, Bending moment and shear force diagram for the strap beam.

The critical bending moments are:

$$M_{C1} = 0.491 \times (0.5)^2 / 2 - 1.2 \times 0.25 = -0.239 \text{ [MN.m]}$$

$$M_{\text{Design}} = 0.491 \times (3.0)^2 / 2 - 1.2 \times 2.75 = -1.091 \text{ [MN.m]}$$

$$M_{C2} = 0.491 \times (1.5)^2 / 2 = 0.246 \text{ [MN.m]}$$

5. Determining the depth of the strap beam d

Determining the depth d_m required to resist the longitudinal moment M_{max}

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$. Take the strap breadth $b = 0.5 \text{ [m]}$. The maximum depth d_m for the strap as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\text{max}}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{1.09}{0.5}} = 1.13 \text{ [m]}$$

Take $d = 1.15 \text{ [m]} > d_m = 1.13 \text{ [m]}$ and $t = 1.2 \text{ [m]}$.

Solved Examples

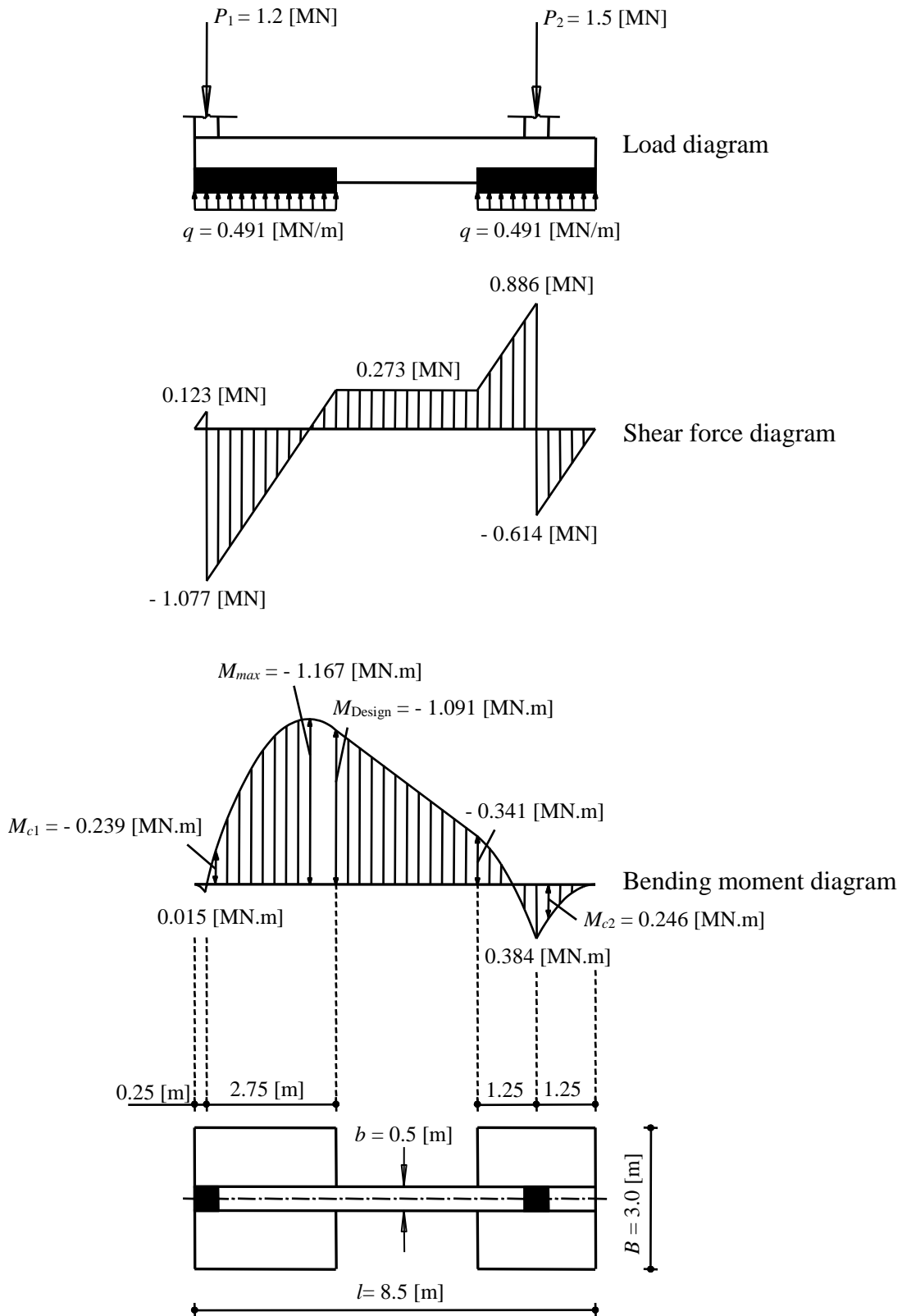


Figure 110: Load, shear force and moment diagrams with critical section

Solved Examples

Check for section capacity to resist maximum shear force Q_{max}

The maximum critical shear section lies at a distance $d/2 = 0.575$ [m] from the side face of the internal column.

Effective depth of the section	$d = 1.15$ [m]
Strap beam width	$b = 0.5$ [m]
Main value of shear strength for concrete C250	$q_c = 0.9$ [MN/m ²]
Max. value of shear strength for concrete C250	$q_2 = 1.9$ [MN/m ²]

for maximum shear force Q_{max} :

$$Q(x) = 0.491 X - 1.2$$

$$X = 0.5 + 0.575 = 1.075 \text{ [m]}$$

$$Q_{max} = 0.491 \times 1.075 - 1.2 = -0.672 \text{ [MN]}$$

The shear stress q is given by:

$$q = \frac{Q_{max}}{b d}$$

$$q = \frac{0.672}{0.5 \times 1.15} = 1.169 \text{ [MN/m}^2\text{]} < q_2 = 1.9 \text{ [MN/m}^2\text{]}$$

Take $d = 1.15$ [m] $> d_s = 1.13$ [m] and $t = 1.2$ [m], then the section is designed as singly reinforced section.

6. Computing the area of steel reinforcement in longitudinal direction A_s

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 50 \times 100 = 7.5$ [cm²/ m]

Take $A_{smin} = 5\Phi 16 = 10.1$ [cm²/ m]. The required area of steel reinforcement in longitudinal direction A_s is shown in Table 13:

Table 13: Determination of tension reinforcement in longitudinal direction

Position	Moment M [MN.m]	Calculated A_s $A_s = M / (k_2 \cdot d) / B$ [cm ² / m]	Chosen reinforcement A_s
A_{sC1}	- 0.239	12.083	2Φ22 + 4Φ19
A_{sC2}	0.246	12.437	4Φ22
A_{smax}	- 1.901	55.157	15Φ22

Solved Examples

7. Check for shear stress

The maximum critical shear section lies at a distance $d/2 = 0.575$ [m] from the side face of the internal column.

for maximum shear force Q_{max} :

$$Q(x) = 0.491 X - 1.2$$

$$X = 0.5 + 0.575 = 1.075 \text{ [m]}$$

$$Q_{max} = 0.491 \times 1.075 - 1.2 = -0.672 \text{ [MN]}$$

The shear stress q is given by:

$$q = \frac{Q_{max}}{b d}$$
$$q = \frac{0.672}{0.5 \times 1.15} = 1.169 \text{ [MN/m}^2\text{]}$$

$$q_2 = 1.9 \text{ [MN/m}^2\text{]} > q = 1.169 \text{ [MN/m}^2\text{]} > q_c = 0.9 \text{ [MN/m}^2\text{]}$$

$$q_s = q - q_c/2 = 1.169 - 0.9/2 = 0.719 \text{ [MN/m}^2\text{]}$$

The shear would be resisted using vertical stirrups and bent bars.

Shear stress carried by stirrups

Using stirrups $5\phi 8/ \text{ m}$, 4 branches @ 200 [mm]

$$q_{ss} = \frac{A_{st} f_s}{b S}$$

$$q_{ss} = \frac{(4 \times 50.27 \times 10^{-6}) \times 200}{0.5 \times 0.2} = 0.402 \text{ [MN/m}^2\text{]}$$

Shear stress carried by bent bars

$$q_{sb} = q_s - q_{ss} = 0.719 - 0.402 = 0.317 \text{ [MN/m}^2\text{]}$$

$$q_{sb} = \frac{A_{sb} f_s}{b d} (\sin \alpha + \cos \alpha)$$

$$0.317 = \frac{A_{sb} \times 200}{0.5 \times 1.15} (\sin 60^\circ + \cos 60^\circ)$$

$$A_{sb} = 0.0006672 \text{ [m}^2\text{]} = 6.672 \text{ [cm}^2\text{]}$$

Solved Examples

Use bent bars $2\Phi 22 = 7.6 \text{ [cm}^2\text{]}$

8. Design of the footing slab

The maximum bending moment for the footing slab lies at the first strap face as shown in Figure 111. The transverse bending moment M_T at the strap face is given by:

$$M_T = f_n \times 3.0 \times (1.25)^2 / 2 = 0.384 \text{ [MN.m]}$$

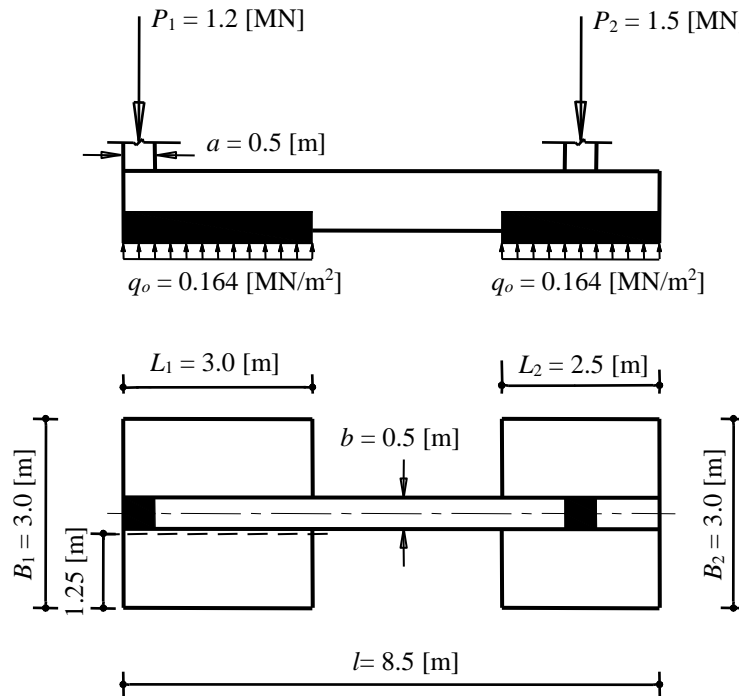


Figure 111: Section for determining footing reinforcement

The footing depth d_f is:

$$d_f = k_1 \sqrt{\frac{M_T}{L}}$$

$$d_f = 0.766 \sqrt{\frac{0.384}{3.0}} = 0.274 \text{ [m]}$$

Take $d_f = 0.45 \text{ [m]} > 0.274 \text{ [m]}$ and $t_f = 0.5 \text{ [m]}$, then the section is designed as singly reinforced section.

The required area of steel reinforcement in transverse direction A_{sT} is:

$$A_{sT} = \frac{M_T}{k_2 d_f}$$

Solved Examples

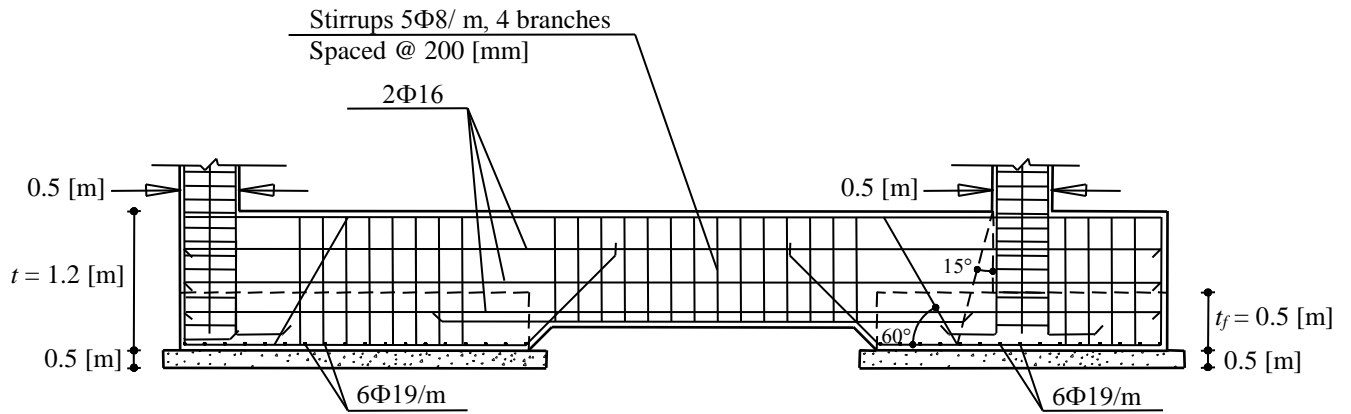
$$A_s = \frac{0.384}{172 \times 0.45} = 0.004961 \text{ [m}^2\text{/3.0 m]}$$

$$A_{sT} = 49.61 \text{ [cm}^2\text{/3.0 m]} = 16.54 \text{ [cm}^2\text{/m]}$$

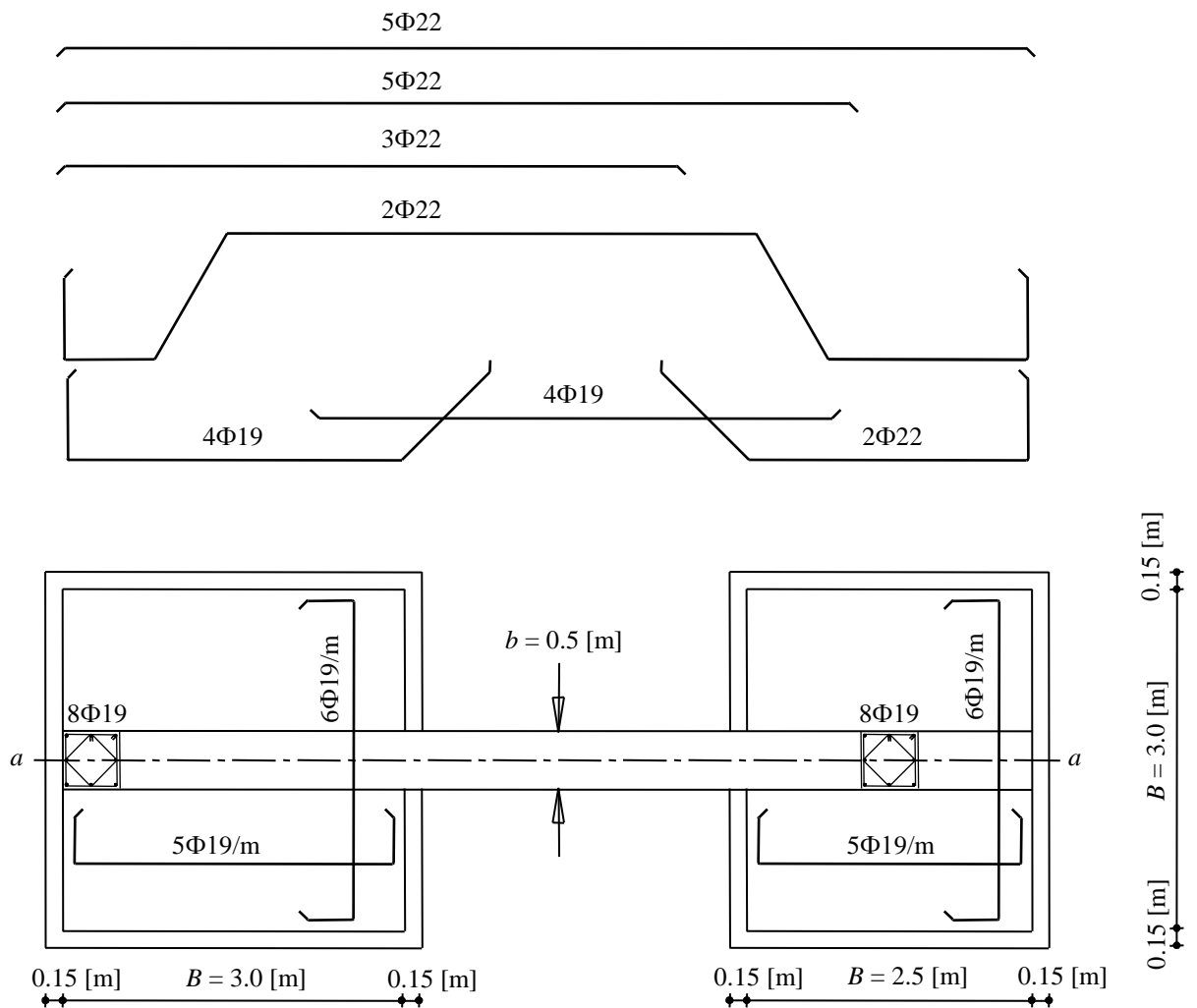
Chosen steel $6\Phi 19/\text{m} = 17.0 \text{ [cm}^2\text{/m]}$

The details of reinforcement in plan and section *a-a* through the footing are shown in Figure 112.

Solved Examples



a) Section *a-a*



b) Plan

Figure 112: Footing dimensions and reinforcement

7- Design of Rafts

Example 35: Design of a circular raft for a cylindrical core

It is required to design a foundation for a cylindrical core of a building, Figure 113. The diameter of the core wall is $d = 8.0$ [m], while the width of the wall is $t_w = 0.3$ [m]. The core lies in the center of the building and it is not subject to any significant lateral applied loading. Therefore, the core wall carries only a vertical load of $p = 300$ [kN/m]. The base of the cylindrical core wall is chosen to be a circular raft of $D = 10.0$ [m] diameter with 1.0 [m] ring cantilever as shown in Figure 113. Poisson's ratio of concrete is $\nu_c = 0.20$ [-].

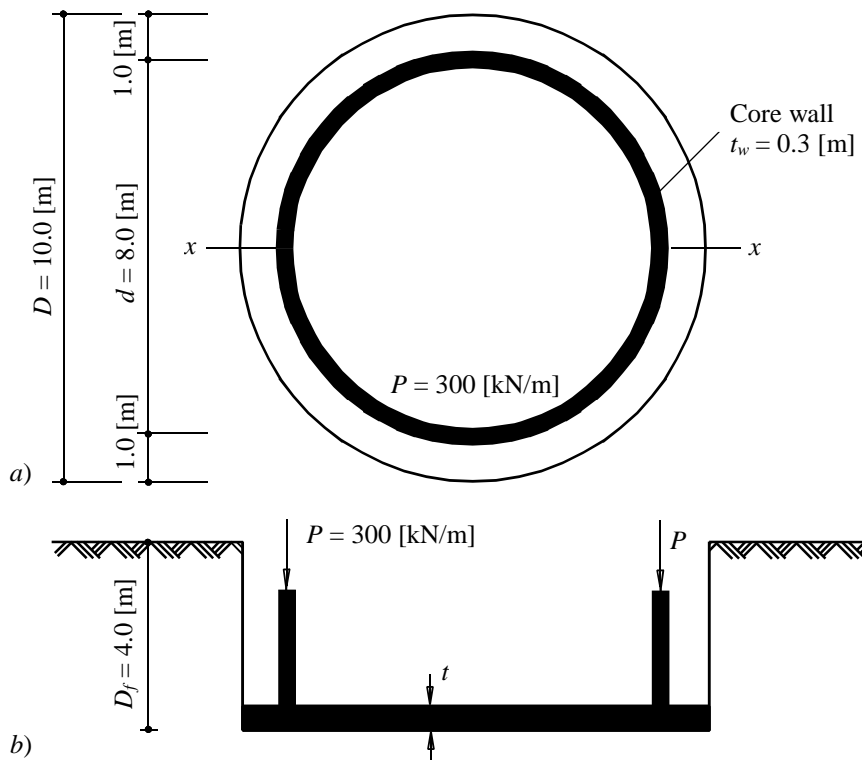


Figure 113: a) Plan of the raft with wall load and dimensions
b) Section x - x through the raft

1. Computing the contact pressure under the raft q

Total load on the raft is given by:

$$N = P \pi d = 300 \times \pi \times 8.0 = 7540 \text{ [kN]} = 7.54 \text{ [MN]}$$

The contact pressure at the base of the raft is:

$$q = \frac{N}{A_f} = \frac{7540}{\frac{\pi 10^2}{4}} = 96.0 \text{ [kN/m}^2\text{]} = 0.096 \text{ [MN/m}^2\text{]}$$

2. Computing moments at critical sections M_1 , M_2 and M_3

Critical sections of moments are shown in Figure 114.

Solved Examples

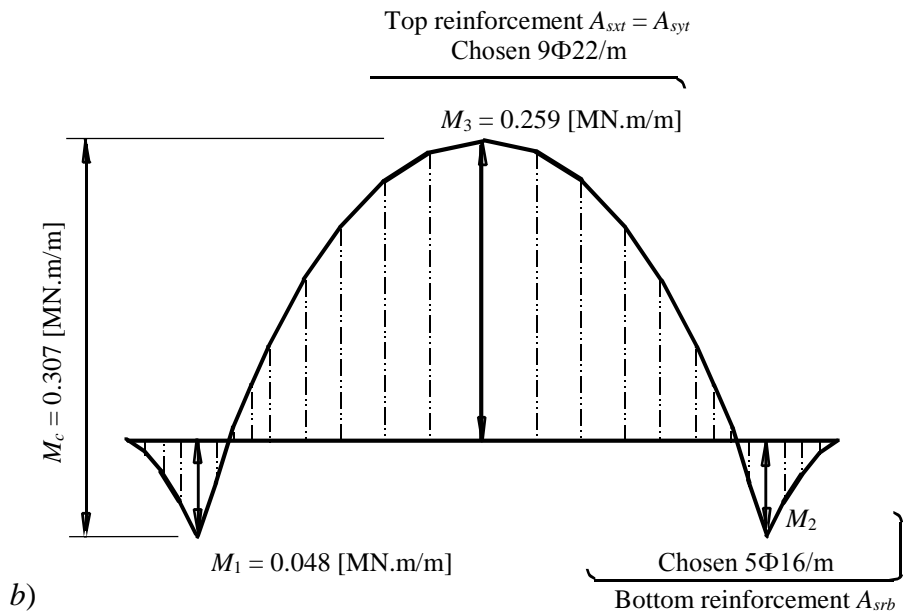
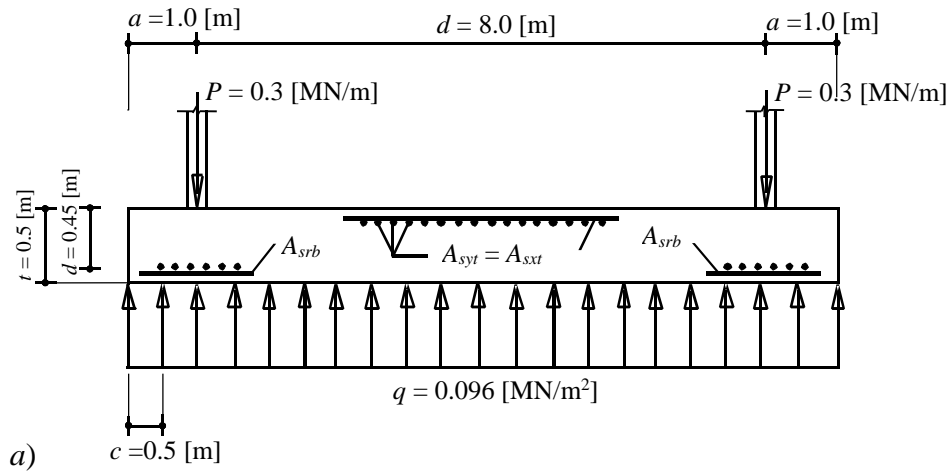


Figure 114: a) Section $x-x$ through the raft
b) Moments at section $x-x$ with main reinforcement A_s at critical sections

Contact force R :

$$R = q \frac{\pi d^2}{4} = 0.096 \frac{\pi 8.0^2}{4} = 4.83 \text{ [MN]}$$

Moment M_c :

$$M_c = \frac{3 + \nu_c}{16 \pi} R = \frac{3.0 + 0.2}{16 \pi} 4.83 = 0.307 \text{ [MN.m/m]}$$

Moment under the wall $M_1 = M_2$:

Solved Examples

$$M_1 = M_2 = q_o c a = 0.096 \times 0.5 \times 1.0 = 0.048 \text{ [MN.m/m]}$$

Moment at the center of the raft M_3 :

$$M_3 = M_c - M_1 = 0.307 - 0.048 = 0.259 \text{ [MN.m/m]}$$

3. Determining the depth required to resist the moment d_m

The maximum moment is at the raft center M_3 . From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

Take one meter strip $B = 1.0 \text{ [m]}$. The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_3}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.259}{1.0}} = 0.39 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.39 \text{ [m]}$, then the section is designed as singly reinforced section.

The corresponding k_1 for $d = 0.45 \text{ [m]}$ is given by:

$$k_1 = 0.45 \sqrt{\frac{1.0}{0.2591}} = 0.884$$

From Table 2 at $k_1 = 0.873$, $f_c = 8.0 \text{ [MN/m}^2\text{]}$ and $k_2 = 175$

4. Computing the area of steel reinforcement

The bottom reinforcement is chosen to be in radial and tangential directions while the top reinforcement is chosen to be in x -directions and y -directions.

Top reinforcement in x -directions and y -directions $A_{sxt} = A_{syt}$:

$$A_{sxt} = \frac{M_3}{k_2 d}$$

$$A_{sxt} = \frac{0.259}{175 \times 0.45} = 0.003289 \text{ [m}^2\text{/m]}$$

$$A_{sxt} = 32.89 \text{ [cm}^2\text{/m]}$$

Chosen steel $9\Phi 22 = 34.2 \text{ [cm}^2\text{/m]}$ in both directions.

Solved Examples

Bottom reinforcement in radial direction A_{srb} :

$$A_{srb} = \frac{M_1}{k_2 d}$$

$$A_{srb} = \frac{0.048}{175 \times 0.45} = 0.00061 \text{ [m}^2\text{/m]}$$

$$A_{srb} = 6.1 \text{ [cm}^2\text{/m]}$$

Chosen steel $5\Phi 16 = 10.1 \text{ [cm}^2\text{/m]}$.

The raft is reinforced by a square mesh $9\Phi 22 \text{ [mm/m]}$ in the upper surface, while the lower surface is reinforced by $5\Phi 16 \text{ [mm/m]}$ in radial and tangential directions. In addition, an upper radial and tangential reinforcement $9\Phi 19 \text{ [mm/m]}$ are used at the cantilever ring. A small square mesh $5\Phi 16 \text{ [mm/m]}$, each side is 1.0 [m] is used at the center of the raft to connect the bottom radial reinforcement.

The details of reinforcement of the raft are shown in Figure 115.

Solved Examples

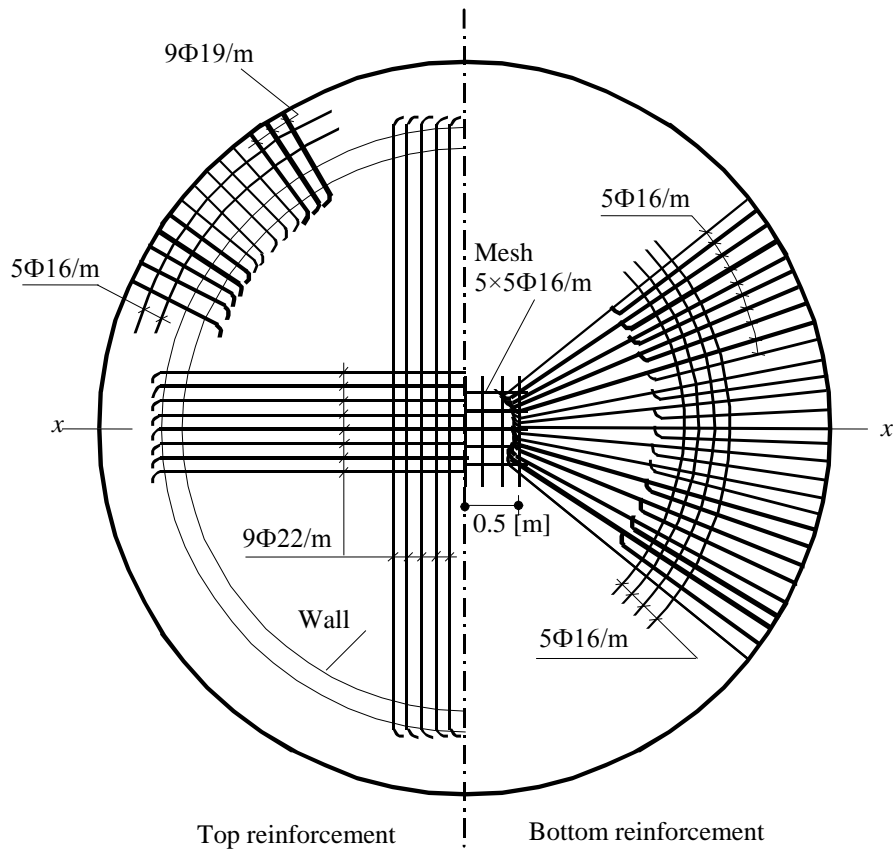
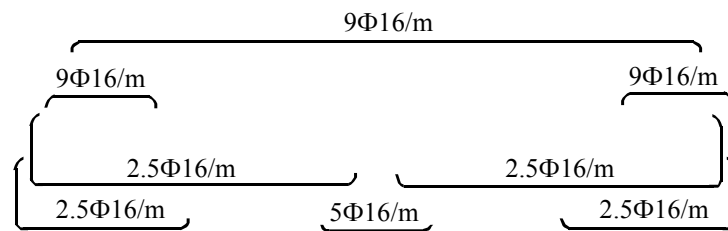
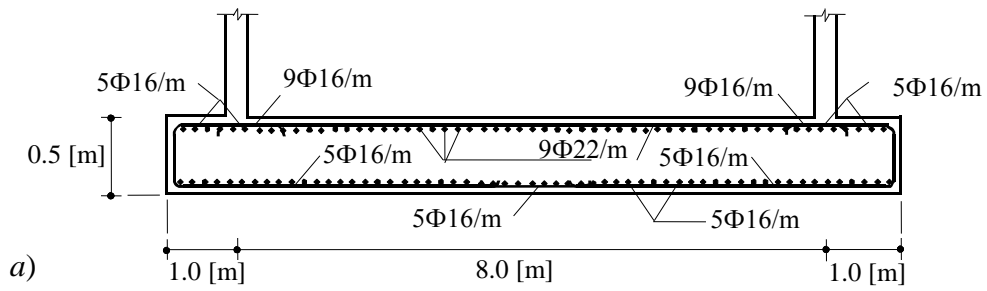


Figure 115: a) Section $x-x$ through the raft with reinforcement
 b) Reinforcement of the raft in plan

Solved Examples

Example 36: Design of a circular raft subjected to moment due to lateral loading

It is required to design a foundation for a cylindrical elevated tank, Figure 116. The diameter of the tank wall is $d = 5.6$ [m], while the width of the wall is $t_w = 0.3$ [m]. The tank is subject to a moment due to lateral loading $M_{lt} = 3000$ [kN.m] and a vertical load of $P = 454.729$ [kN/m] as shown in Figure 116. The allowable net soil pressure is $q_{net} = 190$ [kN/m²] at a depth $D_f = 1.7$ [m] and Poisson's ratio of concrete is $\nu_c = 0.20$ [-].

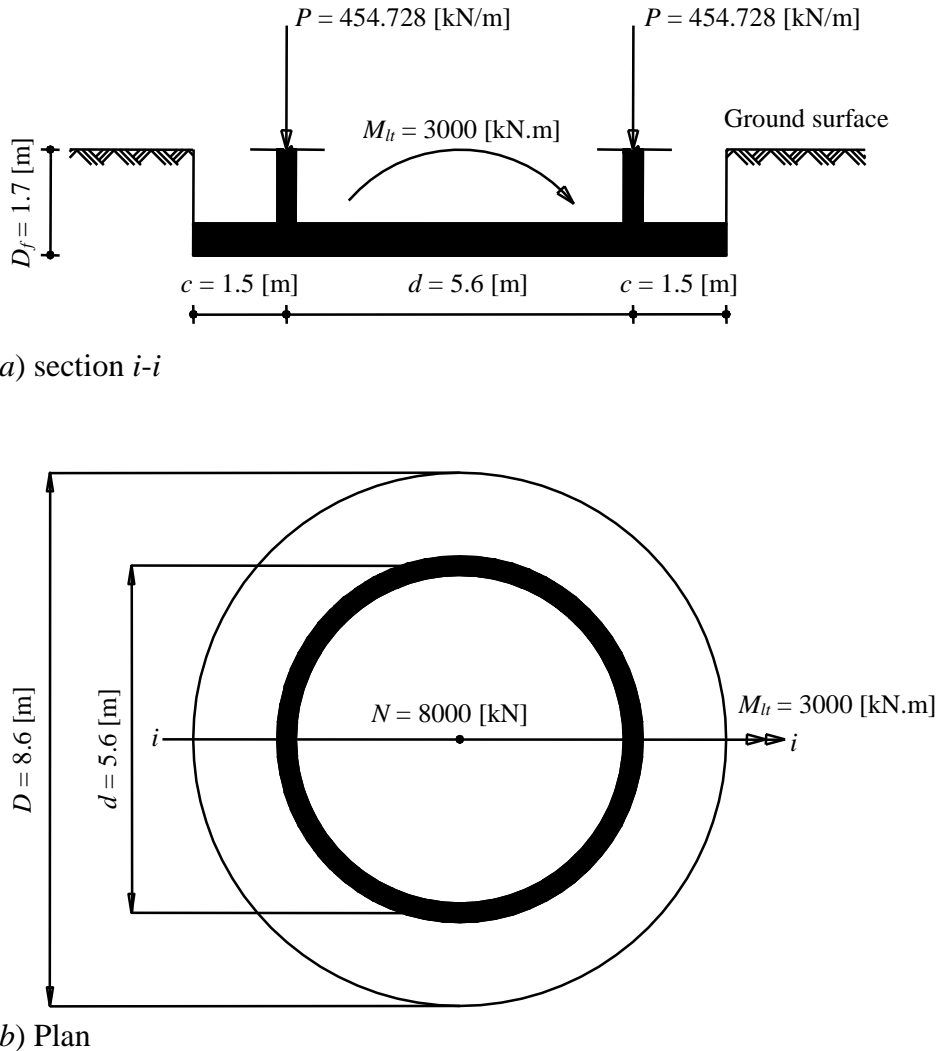


Figure 116: Circular raft for a circular tank

1. Determining the raft diameter D

Total load on the raft is given by:

$$N = P \pi d = 454.729 \times \pi \times 5.6 = 8000 \text{ [kN]}$$

The diameter of the raft D is obtained from:

Solved Examples

$$q = \frac{N}{A_r} \pm \frac{M_{lt}}{z_r}$$

$$190 = \frac{8000}{\frac{\pi}{4} D^2} \pm \frac{3000}{\frac{\pi}{32} D^3}$$

$$D^3 - 53.61 D - 160.83 = 0.0$$

Solving this equation, leads to $D = 8.515$ [m]. Take $D = 8.6$ [m]

2. Computing the contact pressure under the raft q

The contact pressure under the raft is:

$$q_{1,2} = \frac{8000}{\frac{\pi}{4} 8.6^2} \pm \frac{3000}{\frac{\pi}{32} 8.6^3}$$

$$q_{1,2} = 137.72 \pm 48.04$$

$$q_1 = 185.76 \text{ [kN/m}^2\text{]} < 190 \text{ [kN/m}^2\text{]} \quad (\text{O.K.})$$

$$q_2 = 89.68 \text{ [kN/m}^2\text{]}$$

$$q_3 = q_2 + \frac{c}{D}(q_1 - q_2)$$

$$q_3 = 89.68 + \frac{1.5}{8.6}(185.76 - 89.68)$$

$$q_3 = 106.438 \text{ [kN/m}^2\text{]}$$

$$q_4 = q_2 + \frac{c+d}{D}(q_1 - q_2)$$

$$q_4 = 89.68 + \frac{1.5+5.6}{8.6}(185.76 - 89.68)$$

$$q_4 = 169 \text{ [kN/m}^2\text{]}$$

3. Computing moments at critical sections

Critical sections of moments are shown in Figure 117.

Solved Examples

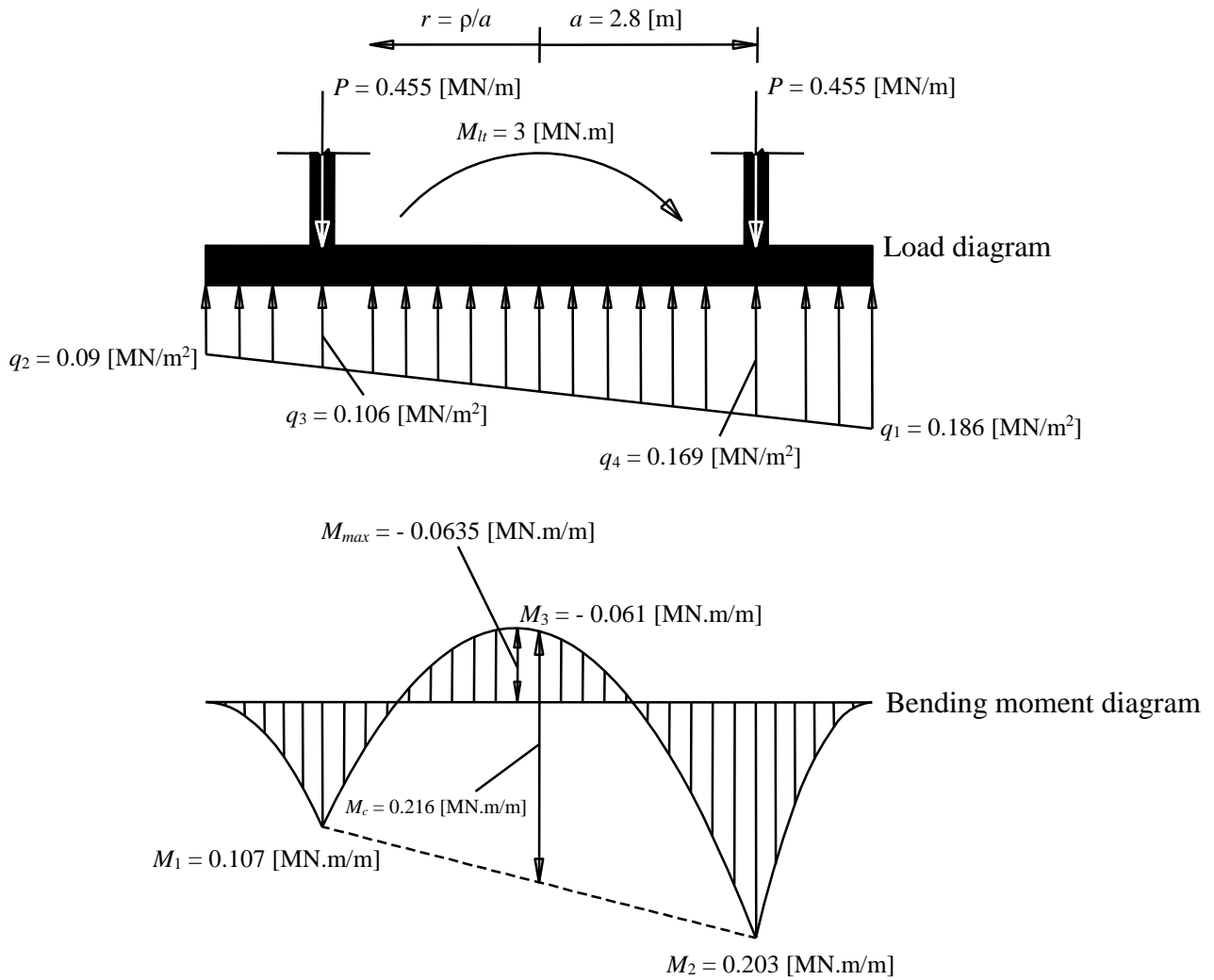


Figure 117: Load and bending moment acting on the raft

Computing the moment M_1 :

$$M_1 = (1/2 \times (q_3 - q_2) \times c^2/3) + (q_2 \times c^2/2)$$

$$M_1 = (1/2 \times (0.106 - 0.09) \times 1.5^2/3) + (0.09 \times 1.5^2/2)$$

$$M_1 = 0.107 \text{ [MN.m/m]}$$

Computing the moment M_2 :

$$M_2 = (1/2 \times (q_1 - q_4) \times 2c^2/3) + (q_4 \times 2c^2/2)$$

$$M_2 = (1/2 \times (0.186 - 0.169) \times 2 \times 1.5^2/3) + (0.169 \times 2 \times 1.5^2/2)$$

$$M_2 = 0.203 \text{ [MN.m/m]}$$

Moment at the center of span M_3 :

Solved Examples

Average contact pressure q_{av} :

$$q_{av} = (q_1 + q_2)/2 = (0.186 + 0.09)/2 = 0.138 \text{ [MN/m}^2\text{]}$$

Contact force R :

$$R = q_{av} \frac{\pi d^2}{4} = 0.138 \frac{\pi 5.6^2}{4} = 3.392 \text{ [MN]}$$

Total moment M at mid span:

$$M = \frac{3 + \nu_c}{16 \pi} (1 - \rho^2) R$$

where $\rho = r/a$

For $\rho = 0$:

$$M_c = \frac{3 + \nu_c}{16 \pi} R$$

$$M_c = \frac{3 + 0.2}{16 \pi} 3.392$$

$$M_c = 0.216 \text{ [MN.m/m]}$$

$$M_3 = (M_1 + M_2)/2 - M_c$$

$$M_3 = (0.107 + 0.203)/2 - 0.216 = -0.061 \text{ [MN.m/m]}$$

Computing the maximum negative bending moment M_4 :

$$M^{+ve} = 0.017 x + 0.107$$

$$M = \frac{3 + \nu_c}{16 \pi} (1 - \rho^2) R$$

where $\rho = r/a = r/2.8$ and $x = 2.8 - r$

$$M^{+ve} = 0.155 - 0.17 r$$

$$M = \frac{3 + 0.2}{16 \pi} \left(1 - \frac{r^2}{7.84}\right) 3.392$$

$$M = 0.216 - 0.028 r^2$$

$$M^{-ve} = M^{+ve} - M$$

$$M^{-ve} = 0.028 r^2 - 0.017 r - 0.061$$

Solved Examples

The shear force at mid span Q :

$$Q = d M^{ve} / dr = 0.055 r - 0.017$$

The maximum bending moment occurs at the position of zero shear. Point of zero shear locates at distance r from the center of the raft.

$$0.055r - 0.017 = 0.0$$

$$r = 0.311$$

The maximum negative bending moment M_4 :

$$M_4 = 0.028 \times 0.311^2 - 0.017 \times 0.311 - 0.061$$

$$M_4 = - 0.0635 \text{ [MN.m/m]}$$

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5 \text{ [MN/m}^2\text{]}$ and $f_s = 200 \text{ [MN/m}^2\text{]}$, the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient $k_2 \text{ [MN/m}^2\text{]}$ to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172 \text{ [MN/m}^2\text{]}$.

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_2}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.203}{1.0}} = 0.345 \text{ [m]}$$

Take $d = 0.45 \text{ [m]} > d_m = 0.345 \text{ [m]}$ and $t = 0.5 \text{ [m]}$

5. Computing the area of steel reinforcement A_s

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 50 \times 100 = 7.5 \text{ [cm}^2/\text{m]}$

Take $A_{smin} = 5\Phi 16 = 10.1 \text{ [cm}^2/\text{m]}$.

The bottom reinforcement is chosen to be in radial and tangential directions while the top reinforcement is chosen to be in x -directions and y -directions.

Top reinforcement in x -directions and y -directions $A_{sxt} = A_{syt}$:

$$A_{sxt} = \frac{M_{\max}}{k_2 d_r}$$

$$A_{sxt} = \frac{0.0635}{172 \times 0.45} = 0.0008.22 \text{ [m}^2/\text{m]}$$

Solved Examples

$$A_{st} = 8.22 \text{ [cm}^2\text{/m]}$$

Chosen steel $5\Phi 16 = 10.1 \text{ [cm}^2\text{/m]}$ in both directions.

Bottom reinforcement in radial direction A_{srb} :

$$A_{srb} = \frac{M_2}{k_2 d_r}$$

$$A_{srb} = \frac{0.203}{172 \times 0.45} = 0.002619 \text{ [m}^2\text{/m]}$$

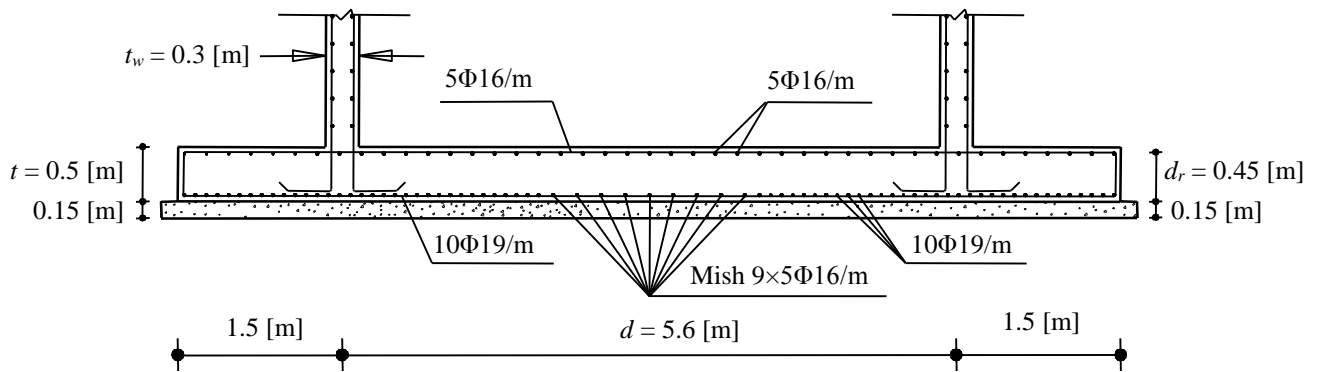
$$A_{srb} = 26.19 \text{ [cm}^2\text{/m]}$$

Chosen steel $10\Phi 19 = 28.353 \text{ [cm}^2\text{/m]}$.

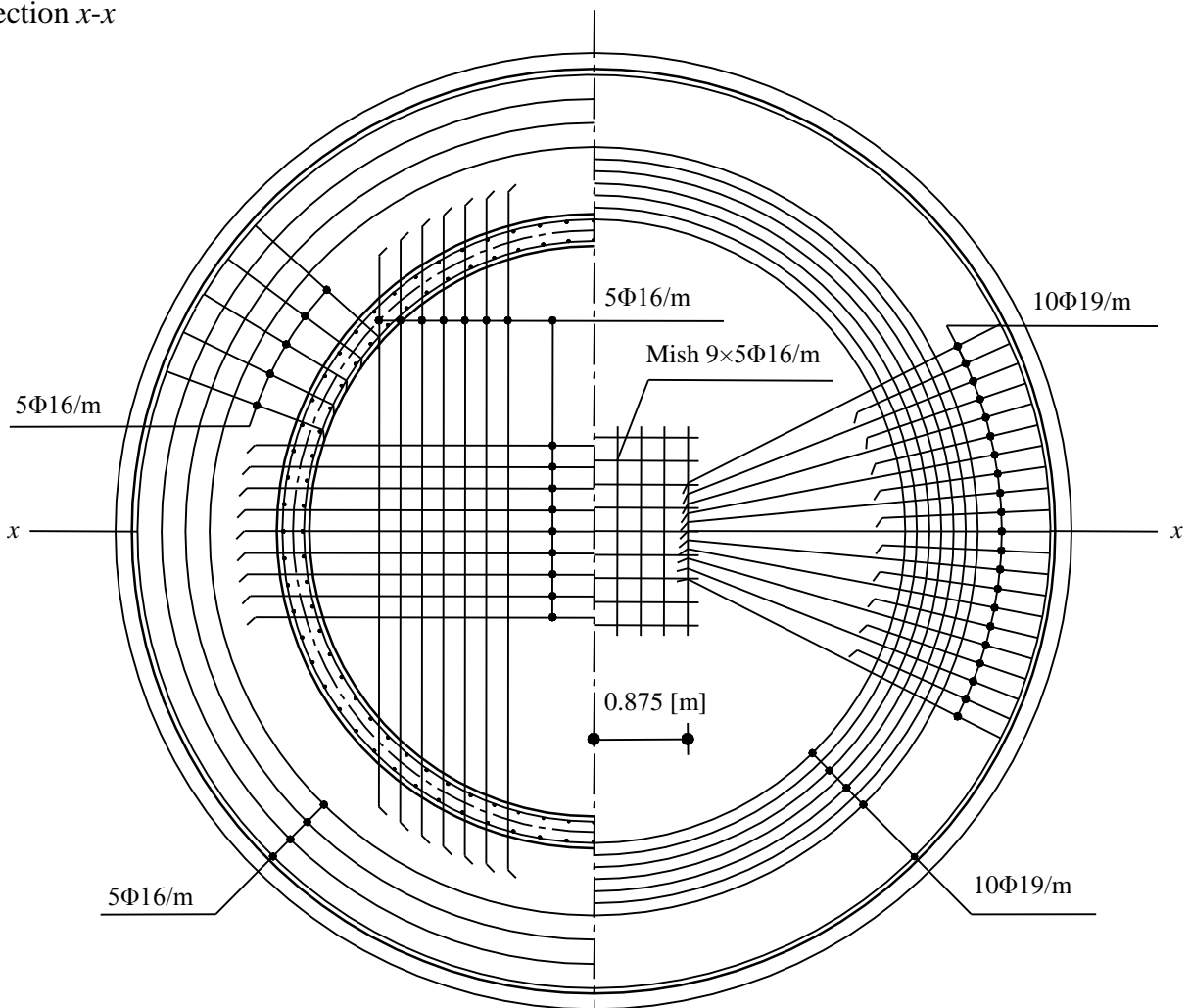
However, the raft is designed for linear contact pressure, but a uniform raft thickness and distribution of steel reinforcement are considered. The raft is reinforced by a square mesh $5\Phi 16$ in the upper surface, while the lower surface is reinforced by $10\Phi 19/\text{m}$ in radial and tangential directions. In addition, an upper radial and tangential reinforcement $5\Phi 16/\text{m}$ are used at the cantilever ring. A small square mesh $5\Phi 16$, each side is 1.75 [m] is used at the center of the raft are shown in Figure 118.

The details of reinforcement of the raft are shown in Figure 118.

Solved Examples



a) Section x-x



b) Plan

Top reinforcement

Bottom reinforcement

Figure 118: Raft dimensions and reinforcement

Solved Examples

Example 37: Design of a square raft for a box core

It is required to design a foundation for a square box core of a building, Figure 119. The side of the core wall is $l_w = 8.0$ [m], while the width of the wall is $t_w = 0.3$ [m]. The wall is reinforced by $5\Phi 13/\text{m}$ in both directions. The core wall carries a vertical load of $P = 300$ [kN/m]. The base of the core is chosen to be a square raft of side L as shown in Figure 119. The allowable soil pressure is $q_{all} = 136$ [kN/m²] at a depth of $D_f = 2.0$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

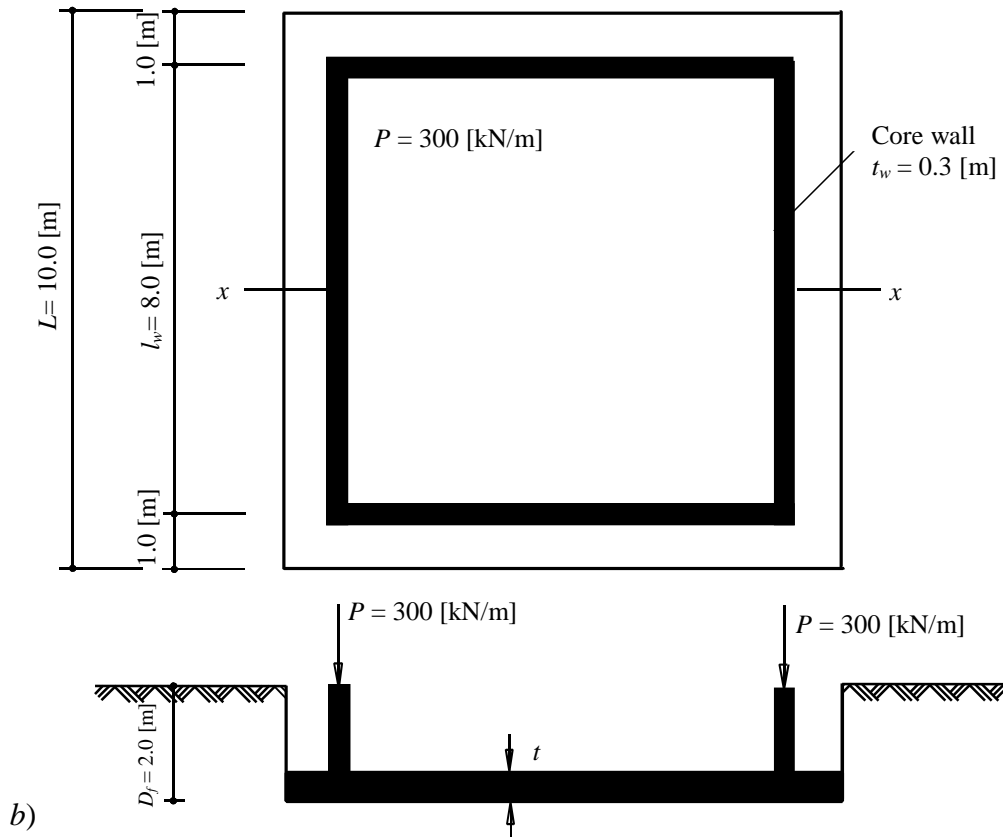


Figure 119: a) Plan of the raft with wall load and dimensions
b) Section x-x through the raft

1. Determining the raft side L

Resultant of loads R at the ground surface level is given by:

$$R = 4 l_w P = 4 \times 8.0 \times 300 = 9600 \text{ [kN]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 136 - 20 \times 2.0 = 96 \text{ [kN/m}^2\text{]}$$

$$A_f = \frac{R}{q_{net}} = \frac{9600}{96} = 100 \text{ [m}^2\text{]}$$

Take $A_f = 10.0 \times 10.0$ [m²] square footing

Solved Examples

2. Computing the contact pressure

Now, take one meter strip, $B = 1.0$ [m] & $P = 300$ [kN], and treat the strip as a beam footing. The contact pressure per meter is given by:

$$q = \frac{2P}{L} = \frac{2 \times 300}{10} = 60 \text{ [kN/m]} = 0.06 \text{ [MN/m]}$$

Figure 120 shows the load diagram and moment diagram for the strip.

3. Computing moments at critical sections

Critical sections of moments are shown in Figure 120.

Moment M_c :

$$M_c = \frac{q l_w^2}{8} = \frac{0.06 \times 8.0^2}{8} = 0.48 \text{ [MN.m]}$$

Moment under the wall $M_1 = M_2$:

$$M_1 = M_2 = q c a = 0.06 \times 0.5 \times 1.0 = 0.03 \text{ [MN.m]}$$

Moment at the center of the raft M_3 :

$$M_3 = M_c - M_1 = 0.48 - 0.03 = 0.45 \text{ [MN.m]}$$

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5$ [MN/m²] and $f_s = 200$ [MN/m²], the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient k_2 [MN/m²] to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172$ [MN/m²]. Take the strap breadth $b = 0.5$ [m].

The maximum depth d_m for the strap as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_3}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.45}{1.0}} = 0.51 \text{ [m]}$$

Take $d = 0.55$ [m] $> d_m = 0.51$ [m] then the section is designed as singly reinforced section.

The corresponding k_1 for $d = 0.55$ [m] is given by:

Solved Examples

$$k_1 = 0.55 \sqrt{\frac{1.0}{0.45}} = 0.82$$

From Table 2 at $k_1 = 0.82$, $f_c = 8.5$ [MN/m²] and $k_2 = 174$

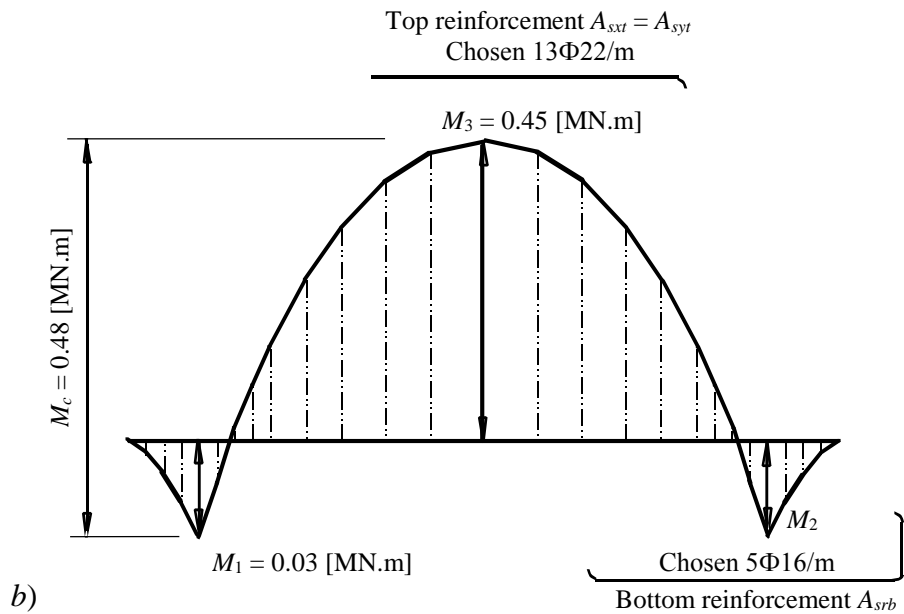
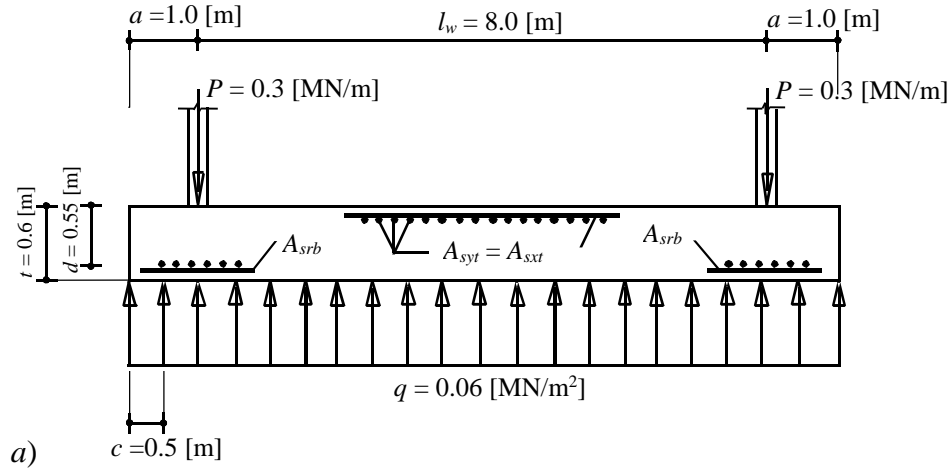


Figure 120: a) Section $x-x$ through the raft

b) Moments at section $x-x$ with main reinforcement A_s at critical sections

5. Computing the area of steel reinforcement

Top reinforcement in x -directions and y -directions $A_{sxt} = A_{syt}$:

$$A_{sxt} = \frac{M_3}{k_2 d}$$

Solved Examples

$$A_{sxt} = \frac{0.45}{174 \times 0.55} = 0.004702 \text{ [m}^2\text{/m]}$$

$$A_{sxt} = 47.02 \text{ [cm}^2\text{/m]}$$

Chosen steel $13\Phi 22/\text{m} = 49.4 \text{ [cm}^2\text{/m]}$ in both directions.

Bottom reinforcement in x -directions and y -directions $A_{sxb} = A_{syb}$:

$$A_{sxb} = \frac{M_1}{k_2 d}$$

$$A_{sxb} = \frac{0.03}{174 \times 0.55} = 0.000314 \text{ [m}^2\text{/m]}$$

$$A_{sxb} = 3.14 \text{ [cm}^2\text{/m]}$$

Chosen steel $5\Phi 16 = 10.1 \text{ [cm}^2\text{/m]}$.

The raft is reinforced by a square mesh $13\Phi 22 \text{ [mm/m]}$ in the upper surface, while the lower surface is reinforced by $5\Phi 16 \text{ [mm/m]}$ in x -directions and y -directions. In addition, an upper reinforcement $6.5\Phi 16 \text{ [mm/m]}$ are used at the cantilever in x -directions and y -directions.

The details of reinforcement of the raft are shown in Figure 121.

Solved Examples

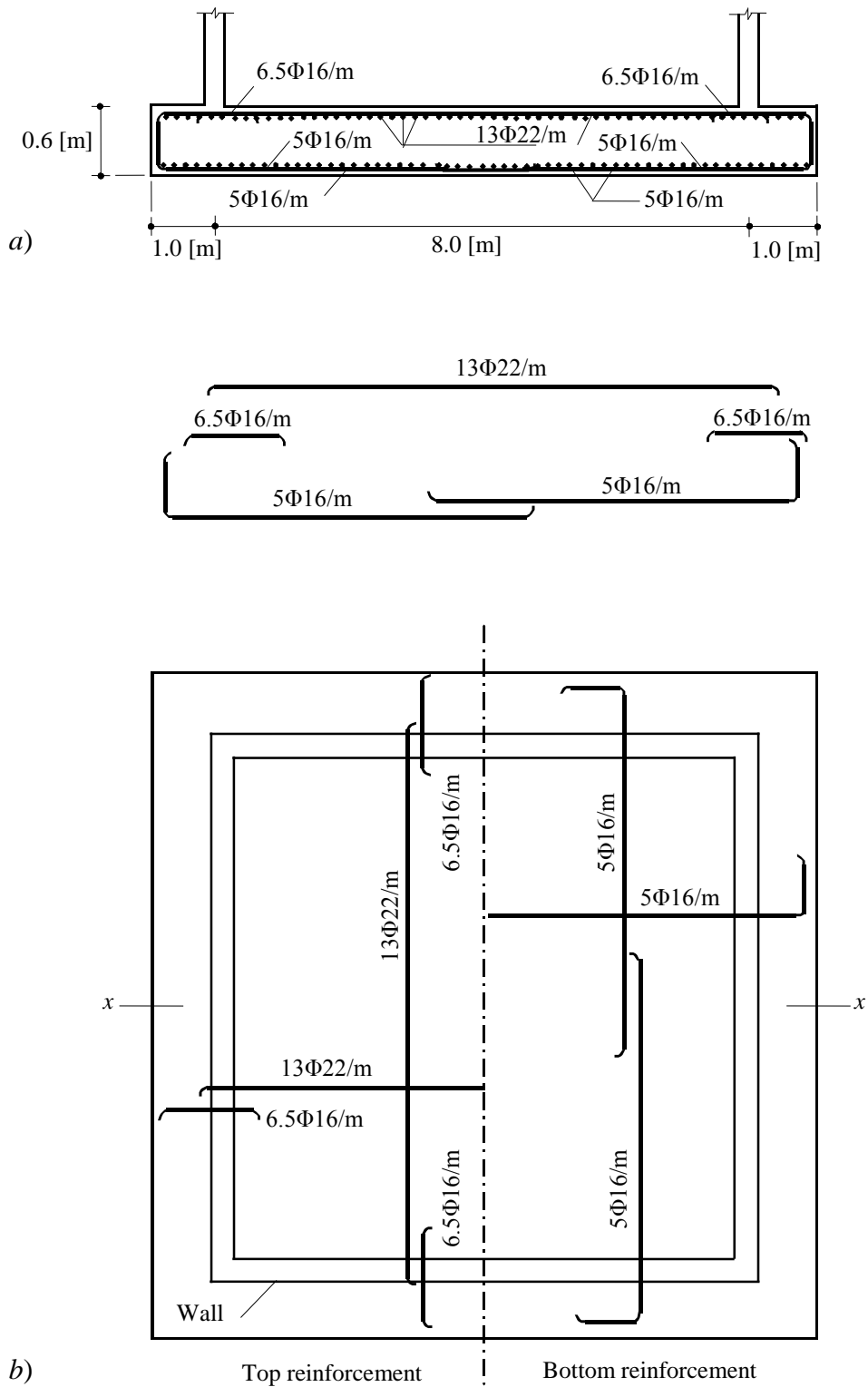


Figure 121: a) Section $x-x$ through the raft with reinforcement
 b) Reinforcement of the raft in plan

Solved Examples

Example 38: Design of a square raft for four equal interior columns

Design a square raft for four equal interior columns, each of 0.50×0.50 [m²], reinforced by $4\Phi 19$, carrying a load of $P = 1500$ [kN] as shown in Figure 122. The distance center to center of columns is 3.0 [m]. The allowable soil pressure is $q_{all} = 210$ [kN/m²] at a depth of $D_f = 2.0$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

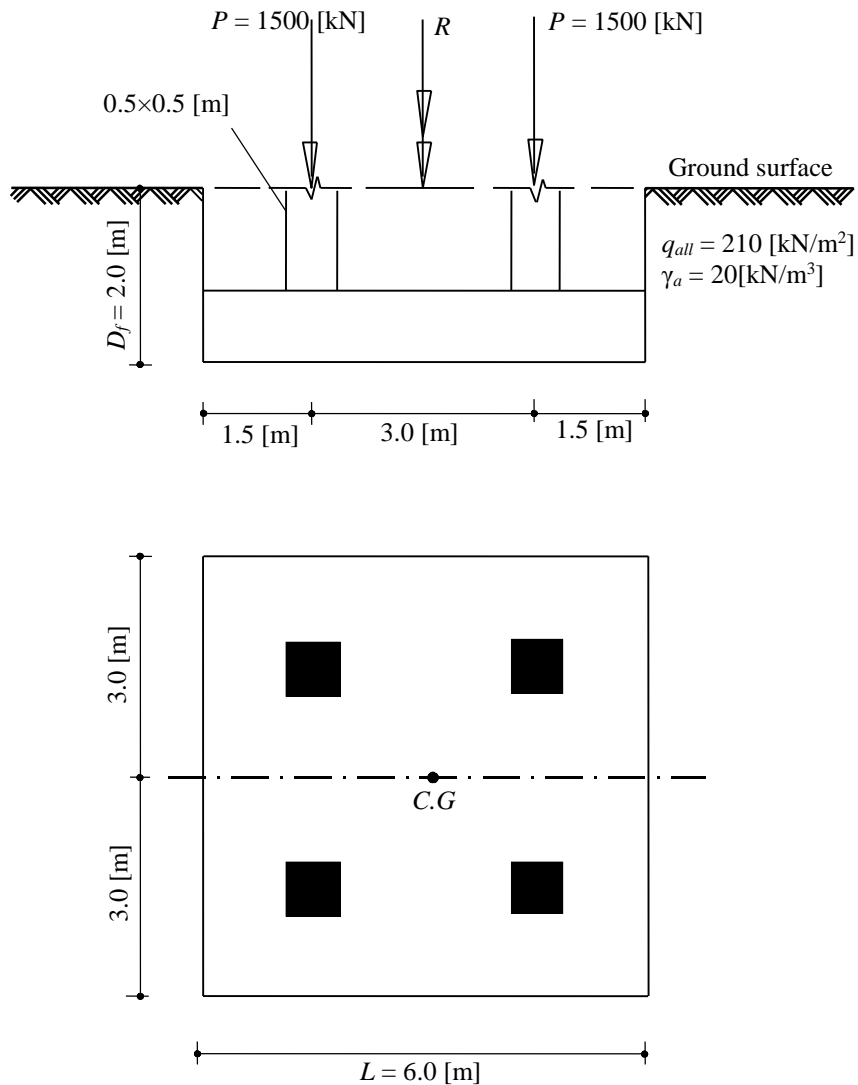


Figure 122: Square raft for four interior edge columns

1. Determining raft side L

Resultant of loads R at the ground surface level is given by:

$$R = 4 P = 4 \times 1500 = 6000 \text{ [kN]}$$

Resultant of total loads R_T at the foundation level is given by:

Solved Examples

$$R_r = \frac{R}{\left(1 - \frac{\gamma_a D_f}{q_{all}}\right)}$$

$$R_r = \frac{6000}{\left(1 - \frac{20 \times 2}{210}\right)} = 7411.76 \text{ [kN]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R_r}{q_{all}} = \frac{7411.76}{210} = 35.29 \text{ [m}^2\text{]}$$

take $A_f = 6.0 \times 6.0 = 36.0 \text{ [m}^2\text{]}$ square raft

2. Computing the contact pressure

The contact pressure per meter square under the base of the raft will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{6000}{6.0 \times 6.0} = 166.67 \text{ [kN/m}^2\text{]} = 0.167 \text{ [MN/m}^2\text{]}$$

Now, take 3.0 [m] strip from centerline to centerline and treat the strip as a beam footing. The contact pressure per meter is given by:

$$q = q_o B = 0.167 \times 3.0 = 0.5 \text{ [MN/m]}$$

Figure 123 shows the load diagram and moment diagram for the strip.

3. Determining the maximum moment M_{max}

From Figure 123, the maximum bending moment in the longitudinal direction occurs at the face of the column which is:

$$M_{max} = q \frac{c^2}{2} = 0.5 \frac{1.25^2}{2} = 0.391 \text{ [MN.m]}$$

Solved Examples

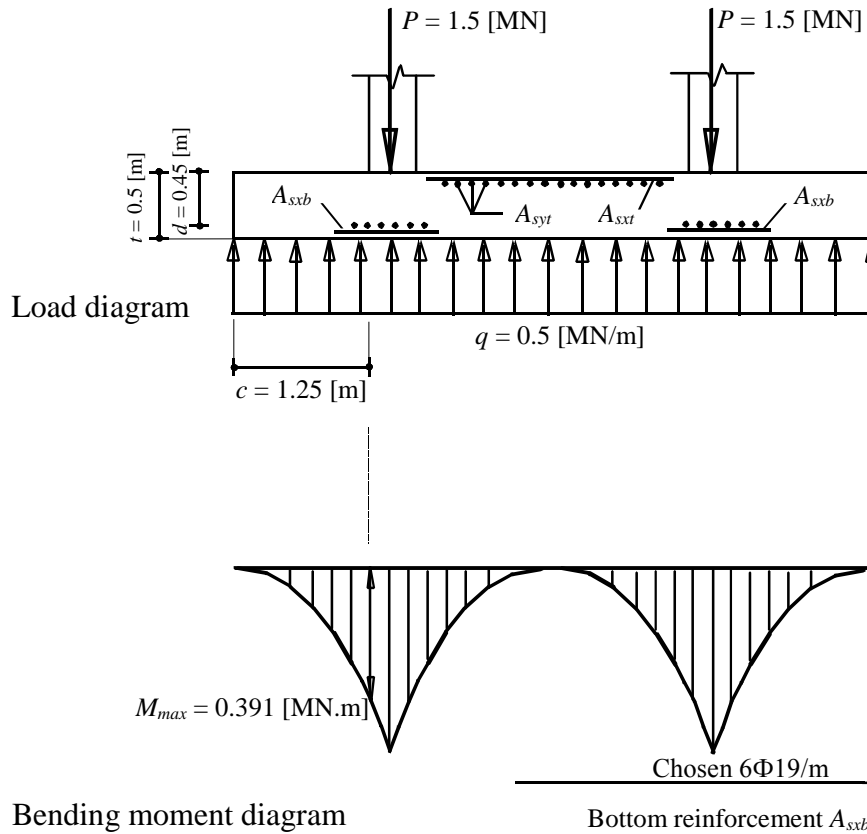


Figure 123: Contact pressure, bending moment and critical section

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5$ [MN/m²] and $f_s = 200$ [MN/m²], the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient k_2 [MN/m²] to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172$ [MN/m²].

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.391}{3.0}} = 0.28 \text{ [m]}$$

Take $d = 0.45$ [m] $>$ $d_m = 0.28$ [m], then the section is designed as singly reinforced section.

The corresponding k_1 for $d = 0.45$ [m] is given by:

$$k_1 = 0.45 \sqrt{\frac{3.0}{0.391}} = 1.25$$

Solved Examples

From Table 2 at $k_1 = 1.27$, $f_c = 5.0$ [MN/m²] and $k_2 = 182$

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.225$ [m] from the face of the column as shown in Figure 124.

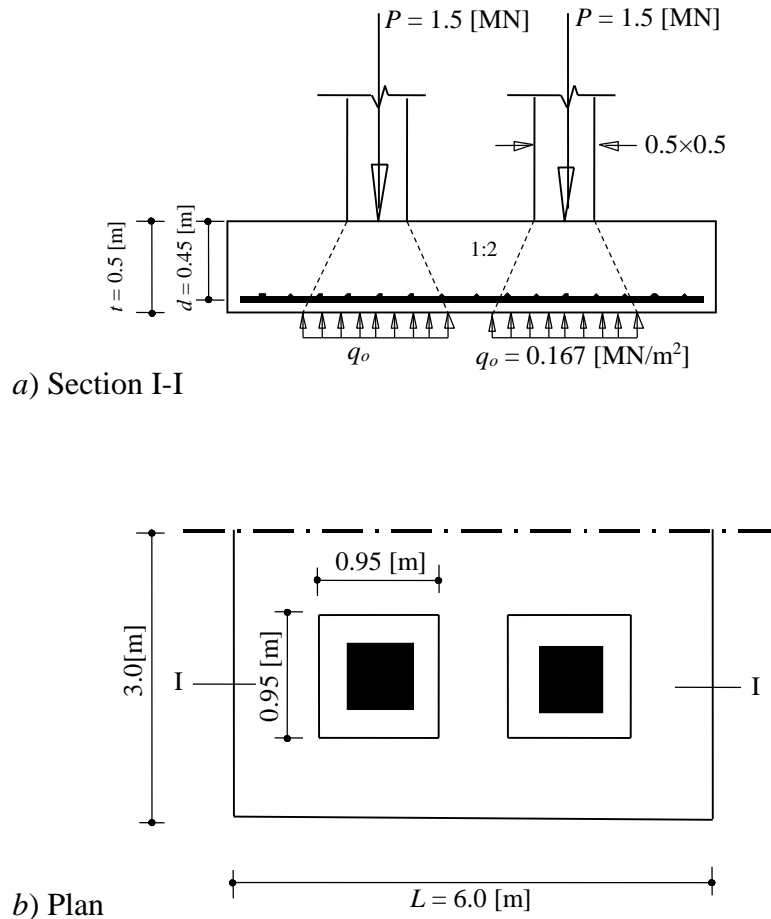


Figure 124: Critical section for punching shear

Geometry (Figure 124)

Effective depth of the section	$d = 0.45$ [m]
Column side	$a = b = 0.5$ [m]
Area of critical punching shear section	$A_p = (a + d)^2 = 0.9025$ [m ²]
Perimeter of critical punching shear section	$b_o = 4(a + d) = 3.8$ [m]

Loads and stresses

Column load	$P = 1.5$ [MN]
Soil pressure under the column	$q_o = 0.167$ [MN/m ²]
Main value of shear strength for concrete C 250	$q_{cp} = 0.9$ [MN/m ²]

Solved Examples

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.5 - 0.167 \times 0.9025 = 1.35 \text{ [MN]}$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{1.35}{3.8 \times 0.45} = 0.79 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9 \text{ [MN/m}^2\text{]} > q_p = 0.79 \text{ [MN/m}^2\text{]}$, the section is safe for punching shear.

6. Computing the area of steel reinforcement

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 50 \times 100 = 7.5 \text{ [cm}^2\text{/m]}$

Take $A_{smin} = 5\Phi 16 = 10.1 \text{ [cm}^2\text{/m]}$.

The required area of steel reinforcement A_{sxb} is:

$$A_{sxb} = \frac{M_{max}}{k_2 d}$$

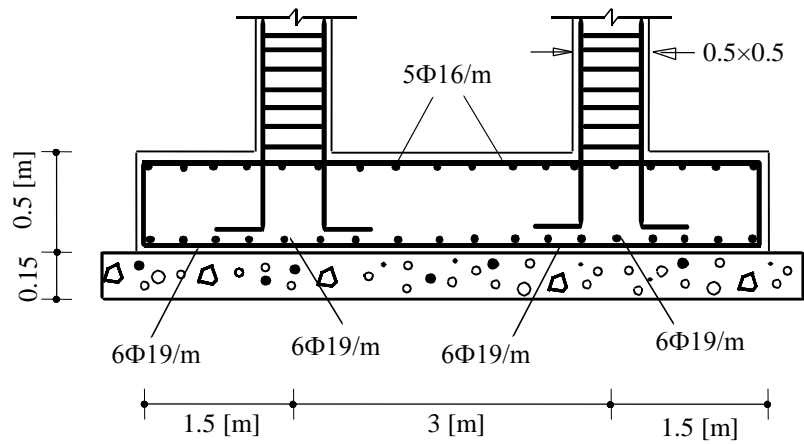
$$A_{sxb} = \frac{0.391}{182 \times 0.45} = 0.004774 \text{ [m}^2\text{/3.0 m]}$$

$$A_{sxb} = 47.74 \text{ [cm}^2\text{/3.0 m]} = 15.91 \text{ [cm}^2\text{/m]}$$

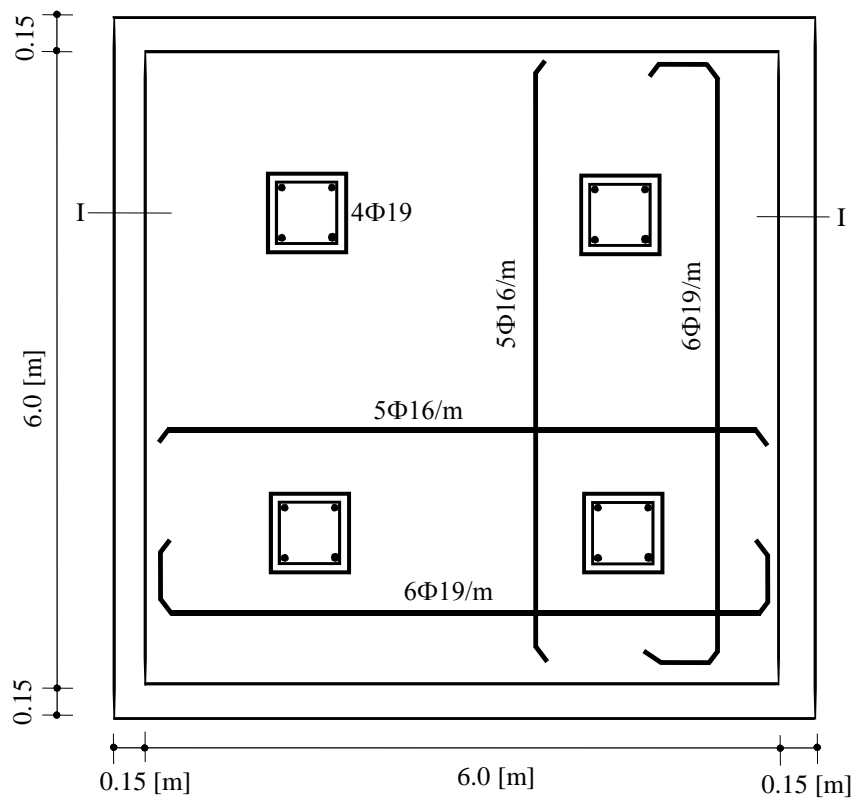
Chosen steel in both x -directions and y -directions at the bottom is $6\Phi 19/\text{m} = 17.0 \text{ [cm}^2\text{/m]}$

The details of reinforcement in plan and section $a-a$ through the footing are shown in Figure 125.

Solved Examples



a) Section I-I



b) Plan

Figure 125: Details of reinforcement in plan and section *a-a* through the raft

Solved Examples

Example 39: Design of a square raft for four equal edge columns

Design a square raft for four equal edge columns, each carrying a load of $P = 1125$ [kN] as shown in Figure 126. Column sides are 0.50×0.50 [m²] while column reinforcement is $4\Phi 19$. The allowable soil pressure is $q_{all} = 210$ [kN/m²] at a depth of $D_f = 1.5$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

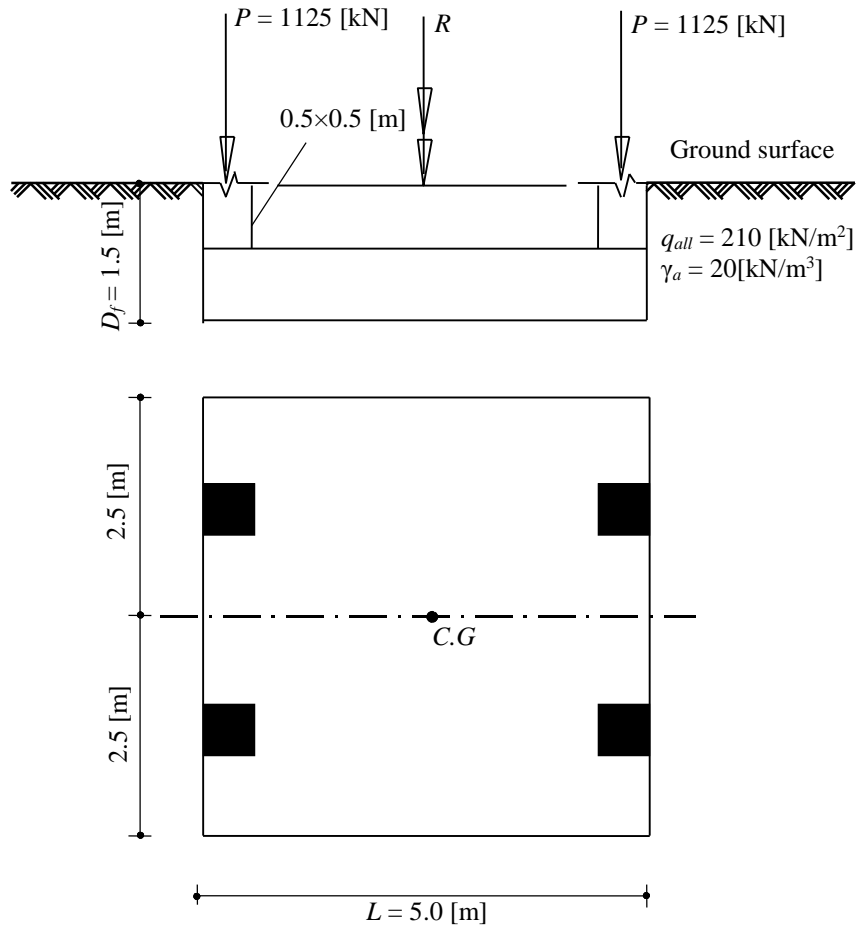


Figure 126: Square raft for four equal edge columns

1. Determining raft side L

Resultant of loads R at the ground surface level is given by:

$$R = 4P = 4 \times 1125 = 4500 \text{ [kN]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 210 - 20 \times 1.5 = 180 \text{ [kN/m}^2\text{]}$$

Area of footing A_f is obtained from:

Solved Examples

$$A_f = \frac{R}{q_{net}} = \frac{4500}{180} = 25 \text{ [m}^2\text{]}$$

take $A_f = 5.0 \times 5.0 = 25.0 \text{ [m}^2\text{]}$ square raft

2. Computing the contact pressure

The contact pressure per meter square under the base of the raft will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{4500}{25.0} = 180 \text{ [kN/m}^2\text{]} = 0.18 \text{ [MN/m}^2\text{]}$$

Now, take 2.5 [m] strip from centerline to centerline and treat the strip as a beam footing. The contact pressure per meter is given by:

$$q = q_o B = 0.18 \times 2.5 = 0.45 \text{ [MN/m]}$$

Figure 127 shows the load diagram and moment diagram for the strip.

3. Determining the maximum moment M_{max}

Critical sections of moments are shown in Figure 127:

$$M_{max} = P \left(\frac{L}{2} - c \right) - q \frac{L^2}{8} = 1.125 \left(\frac{5.0}{2} - 0.25 \right) - 0.45 \frac{5.0^2}{8} = 1.125 \text{ [MN.m]}$$

Solved Examples

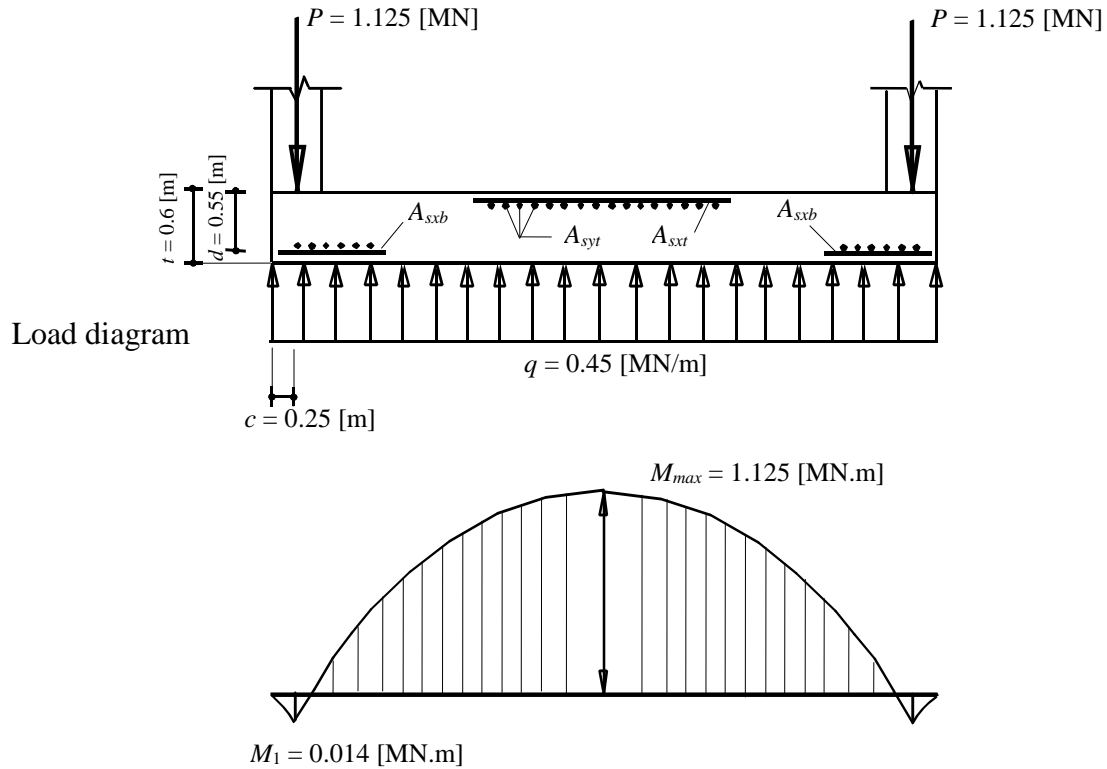


Figure 127: Contact pressure, bending moment and critical section

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5$ [MN/m²] and $f_s = 200$ [MN/m²], the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient k_2 [MN/m²] to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172$ [MN/m²].

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{1.125}{2.5}} = 0.51 \text{ [m]}$$

Take $d = 0.55$ [m] $> d_m = 0.51$ [m], then the section is designed as singly reinforced section.

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.275$ [m] from the face of the column as shown in Figure 128.

Solved Examples

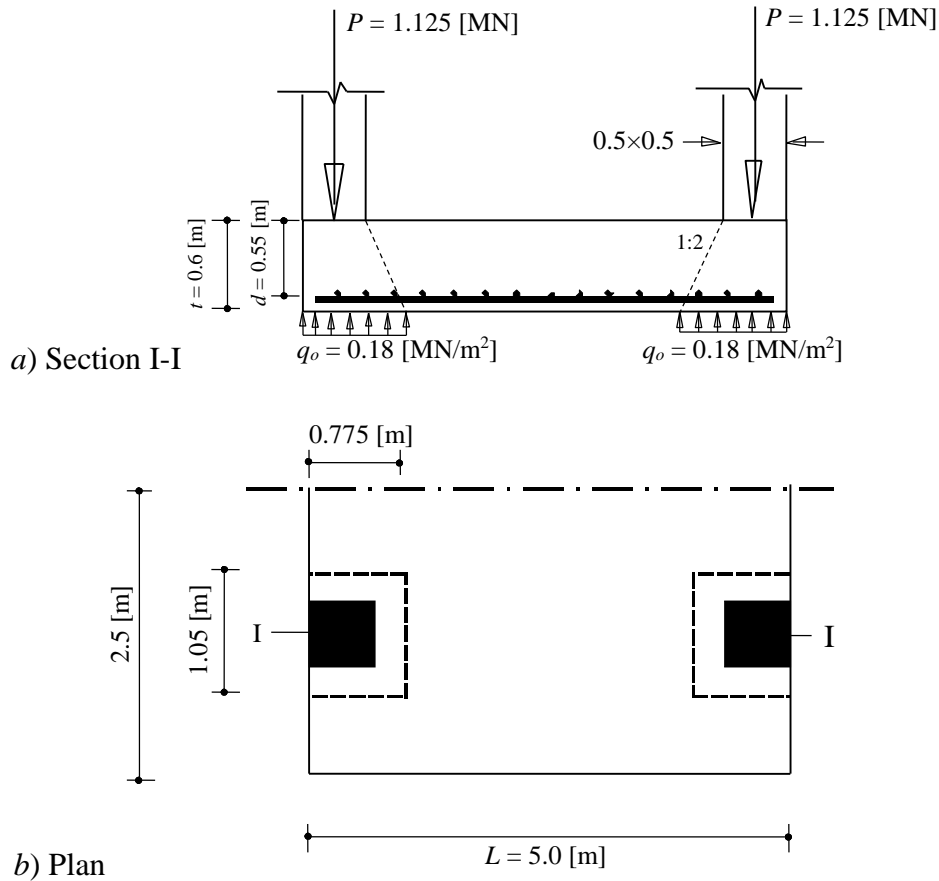


Figure 128: Critical section for punching shear

Geometry (Figure 128)

Effective depth of the section	$d = 0.55 \quad [\text{m}]$
Column side	$a = b = 0.5 \quad [\text{m}]$
Area of critical punching shear section	$A_p = (a + d) (a + d/2) = 0.81 \quad [\text{m}^2]$
Perimeter of critical punching shear section	$b_o = 2 (a + d/2) + (a + d) = 2.6 \quad [\text{m}]$

Loads and stresses

Column load	$P = 1.125 \quad [\text{MN}]$
Soil pressure under the column	$q_o = 0.18 \quad [\text{MN}/\text{m}^2]$
Main value of shear strength for concrete C 250	$q_{cp} = 0.9 \quad [\text{MN}/\text{m}^2]$

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.125 - 0.18 \times 0.81 = 0.98 \quad [\text{MN}]$$

The punching shear stress q_p is given by:

Solved Examples

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{0.98}{2.6 \times 0.55} = 0.69 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9 \text{ [MN/m}^2\text{]} > q_p = 0.69 \text{ [MN/m}^2\text{]}$, the section is safe for punching shear.

6. Computing the area of steel reinforcement

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 60 \times 100 = 9.0 \text{ [cm}^2\text{/m]}$

Take $A_{smin} = 5\Phi 16 = 10.1 \text{ [cm}^2\text{/m]}$.

The required area of steel reinforcement A_{sxt} is:

$$A_{sxt} = \frac{M_{\max}}{k_2 d}$$

$$A_{sxt} = \frac{1.125}{172 \times 0.55} = 0.011892 \text{ [m}^2\text{/2.5 m]}$$

$$A_{sxt} = 118.92 \text{ [cm}^2\text{/2.5 m]} = 47.57 \text{ [cm}^2\text{/m]}$$

Chosen steel in x -directions at the top is $13\Phi 22\text{/m} = 49.4 \text{ [cm}^2\text{/m]}$

From Figure 129, the maximum bending moment in y -direction occurs at the face of the column which is:

$$M_y = q \frac{c^2}{2} = 0.45 \frac{1.0^2}{2} = 0.225 \text{ [MN.m]}$$

Solved Examples

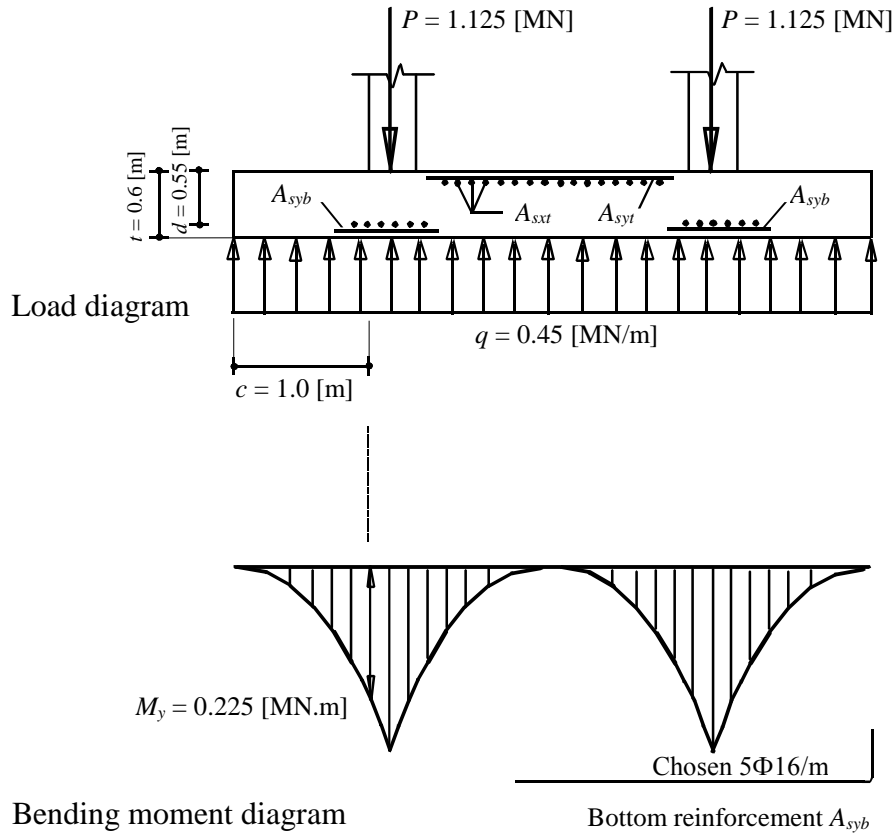


Figure 129: Contact pressure, bending moment and critical section

The required area of steel reinforcement A_{syb} is:

$$A_{syb} = \frac{M_{\max}}{k_2 d}$$

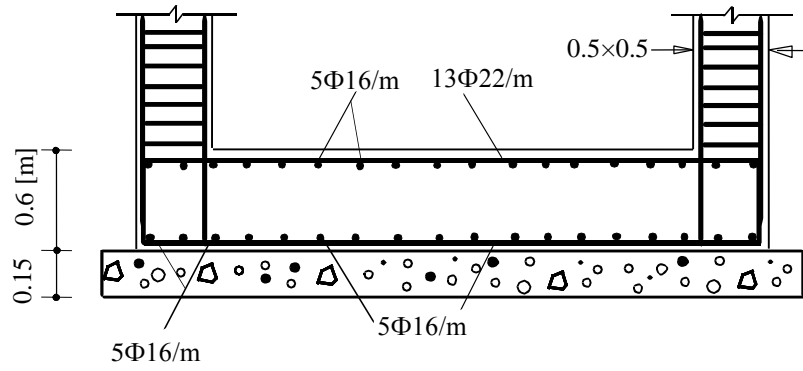
$$A_{syb} = \frac{0.225}{172 \times 0.55} = 0.002378 \text{ [m}^2/2.5 \text{ m]}$$

$$A_{syb} = 23.78 \text{ [cm}^2/2.5 \text{ m]} = 9.54 \text{ [cm}^2/m]$$

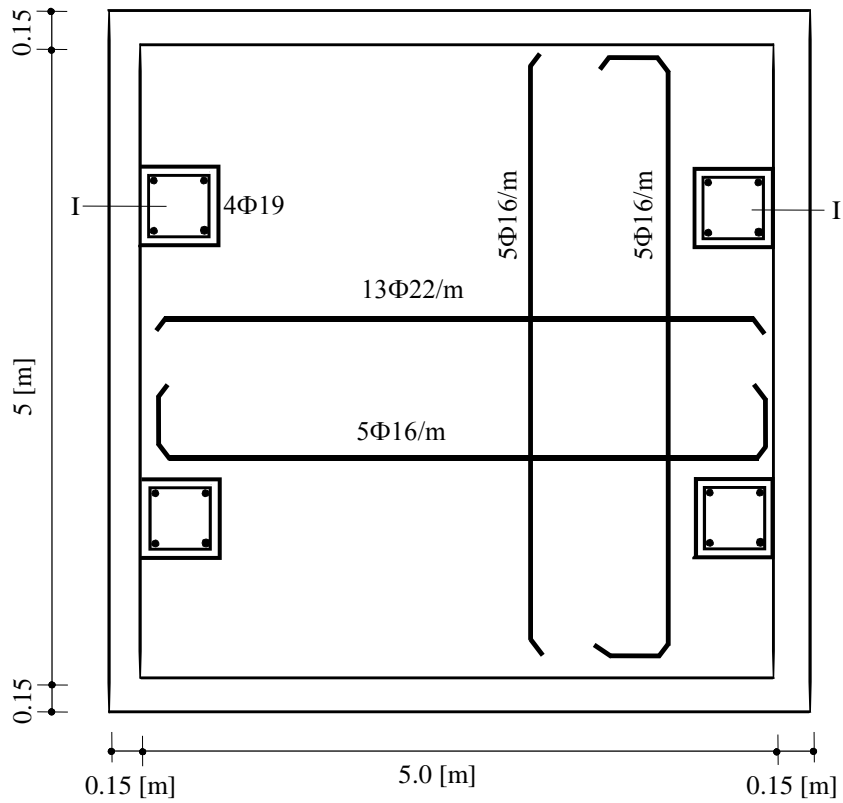
Chosen steel in y -directions at the bottom is $5\Phi 16/m = 10.1 \text{ [cm}^2/m]$

The details of reinforcement in plan and section $a-a$ through the footing are shown in Figure 130.

Solved Examples



a) Section I-I



b) Plan

Figure 130: Details of reinforcement in plan and section *a-a* through the raft

Solved Examples

Example 40: Design of a square raft for 9 equal columns

Design a square raft for 9 equal columns, each of 0.50×0.50 [m], reinforced by $4\Phi 19$, carrying a load of $P = 1500$ [kN] as shown in Figure 131. The distance center to center of columns is 3.0 [m]. The allowable soil pressure is $q_{all} = 210$ [kN/m²] at a depth of $D_f = 2.0$ [m] and average unit weight of the soil and concrete is $\gamma_a = 20$ [kN/m³].

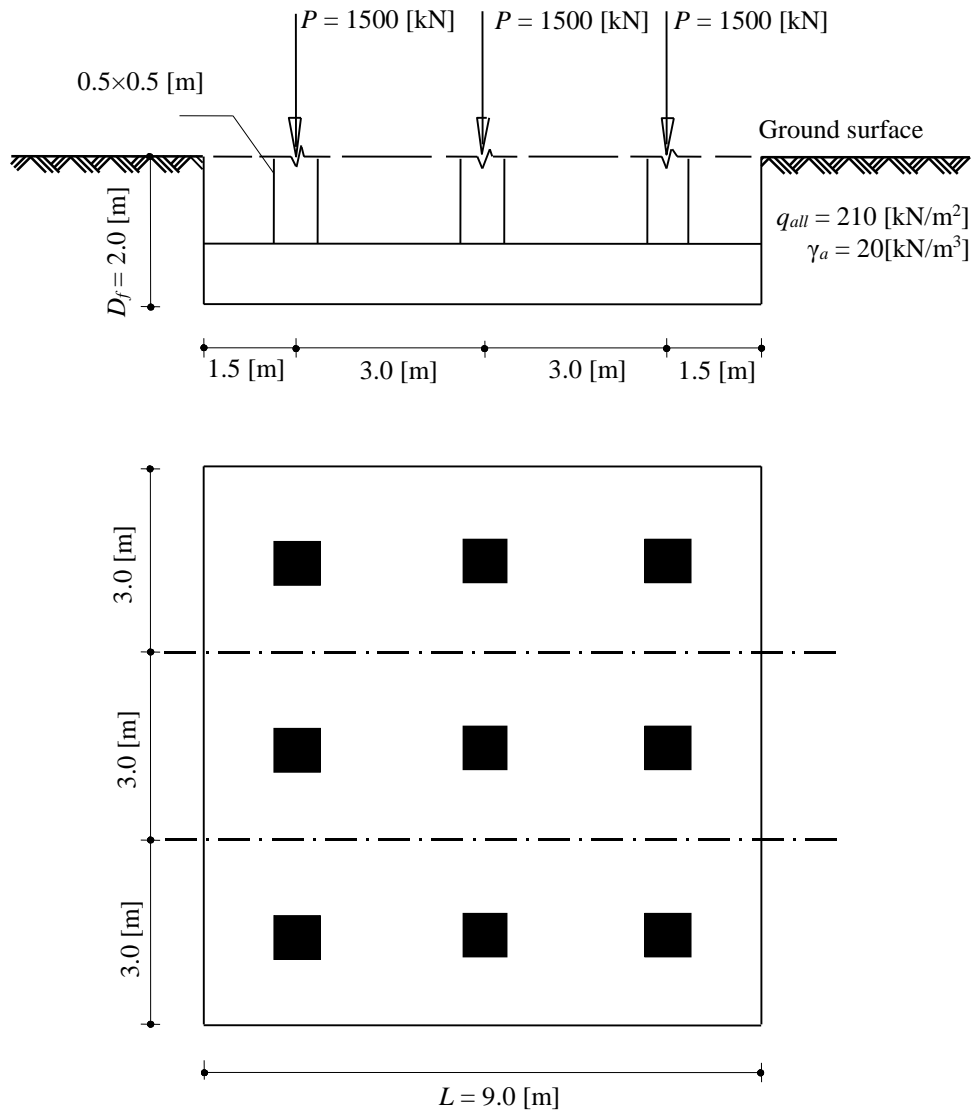


Figure 131: Square raft for 9 equal columns

1. Determining the raft side L

Resultant of loads R at the ground surface level is given by:

$$R = 9 P = 9 \times 1500 = 13500 \text{ [kN]}$$

Allowable net soil pressure q_{net} is given by:

$$q_{net} = q_{all} - \gamma_a D_f = 210 - 20 \times 2.0 = 170 \text{ [kN/m}^2\text{]}$$

Solved Examples

Area of footing A_f is obtained from:

$$A_f = \frac{R}{q_{net}} = \frac{13500}{170} = 79.41 \text{ [m}^2\text{]}$$

take $A_f = 9.0 \times 9.0 = 81.0 \text{ [m}^2\text{]}$ square raft

2. Computing the contact pressure

The contact pressure per meter square under the base of the footing will be uniform. It is given by:

$$q_o = \frac{R}{A_f} = \frac{13500}{81} = 166.67 \text{ [kN/m}^2\text{]} = 0.167 \text{ [MN/m}^2\text{]}$$

Now, take 3.0 [m] strip from centerline to centerline and treat the strip as a beam footing. The contact pressure per meter is given by:

$$q = q_o B = 0.167 \times 3.0 = 0.5 \text{ [MN/m]}$$

Figure 132 shows the load diagram and moment diagram for the beam footing.

3. Determining the maximum moment M_{max}

From Figure 132, the maximum bending moment occurs at the face of the column which is:

$$M_{max} = q \frac{c^2}{2} = 0.5 \frac{1.25^2}{2} = 0.391 \text{ [MN.m]}$$

Solved Examples

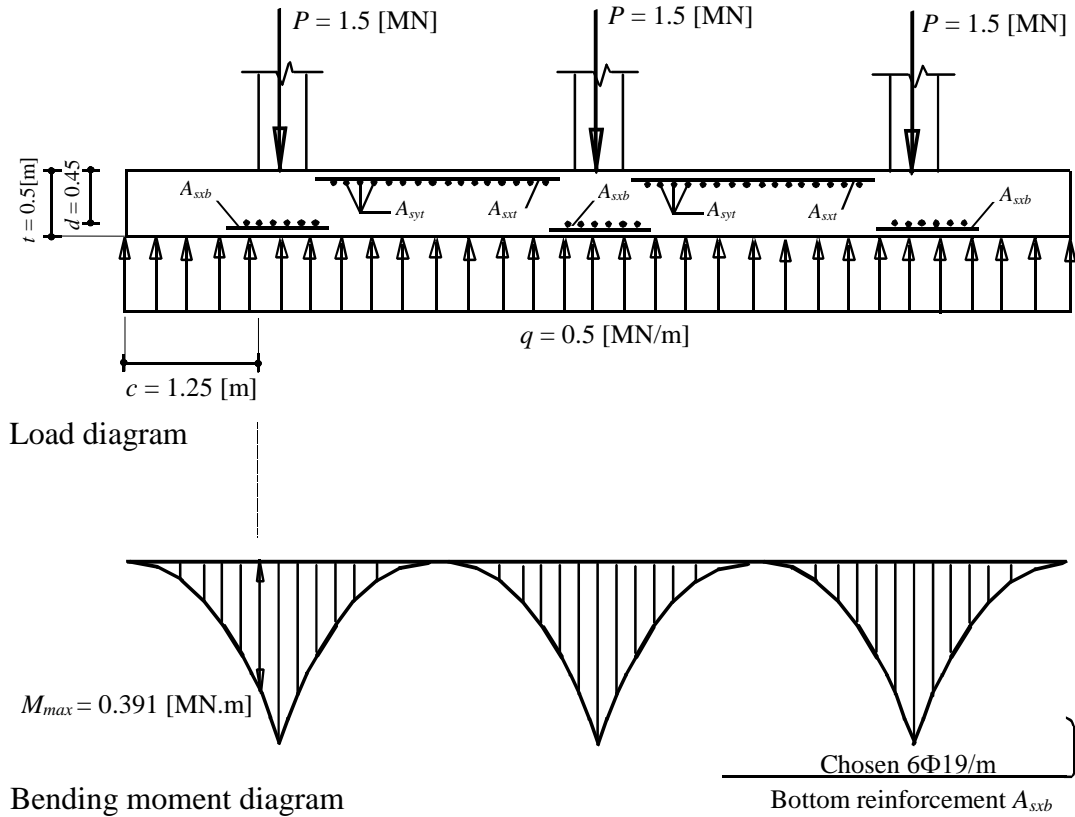


Figure 132: Contact pressure, bending moment and critical sections

4. Determining the depth required to resist the moment d_m

From Table 2 for $f_c = 9.5$ [MN/m²] and $f_s = 200$ [MN/m²], the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient k_2 [MN/m²] to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172$ [MN/m²].

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M_{\max}}{B}}$$

$$d_m = 0.766 \sqrt{\frac{0.391}{3.0}} = 0.28 \text{ [m]}$$

Take $d = 0.45$ [m] $> d_m = 0.28$ [m], then the section is designed as singly reinforced section.

The corresponding k_1 for $d = 0.45$ [m] is given by:

$$k_1 = 0.45 \sqrt{\frac{3.0}{0.391}} = 1.25$$

From Table 2 at $k_1 = 1.27$, $f_c = 5.0$ [MN/m²] and $k_2 = 182$

Solved Examples

5. Check for punching shear

The critical punching shear section lies on a perimeter at a distance $d/2 = 0.225$ [m] from the face of the column as shown in Figure 133.

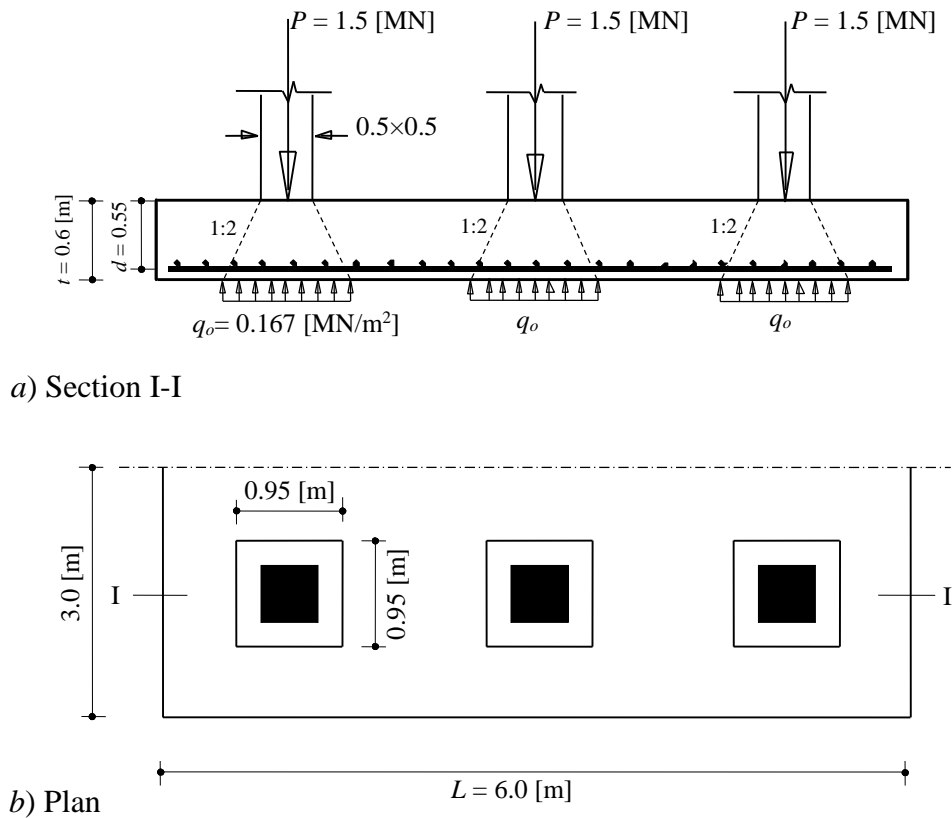


Figure 133: Critical section for punching shear

Geometry (Figure 133)

Effective depth of the section	$d = 0.45$ [m]
Column side	$a = b = 0.5$ [m]
Area of critical punching shear section	$A_p = (a + d)^2 = 0.9025$ [m ²]
Perimeter of critical punching shear section	$b_o = 4(a + d) = 3.8$ [m]

Loads and stresses

Column load	$P = 1.5$ [MN]
Soil pressure under the column	$q_o = 0.167$ [MN/m ²]
Main value of shear strength for concrete C 250	$q_{cp} = 0.9$ [MN/m ²]

Check for section capacity

The punching shear force Q_p is:

$$Q_p = P - q_o A_p$$

$$Q_p = 1.5 - 0.167 \times 0.9025 = 1.35 \text{ [MN]}$$

Solved Examples

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o d}$$

$$q_p = \frac{1.35}{3.8 \times 0.45} = 0.79 \text{ [MN/m}^2\text{]}$$

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.5}{0.5}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \text{ [MN/m}^2\text{]}$$

$q_{pall} = 0.9 \text{ [MN/m}^2\text{]} > q_p = 0.79 \text{ [MN/m}^2\text{]}$, the section is safe for punching shear.

6. Computing the area of steel reinforcement

Minimum area of steel reinforcement $A_{smin} = 0.15\% A_c = 0.0015 \times 50 \times 100 = 7.5 \text{ [cm}^2/\text{m]}$

Take $A_{smin} = 5\Phi 16 = 10.1 \text{ [cm}^2/\text{m]}$.

The required area of steel reinforcement A_s is:

$$A_{sxb} = \frac{M_{\max}}{k_2 d}$$

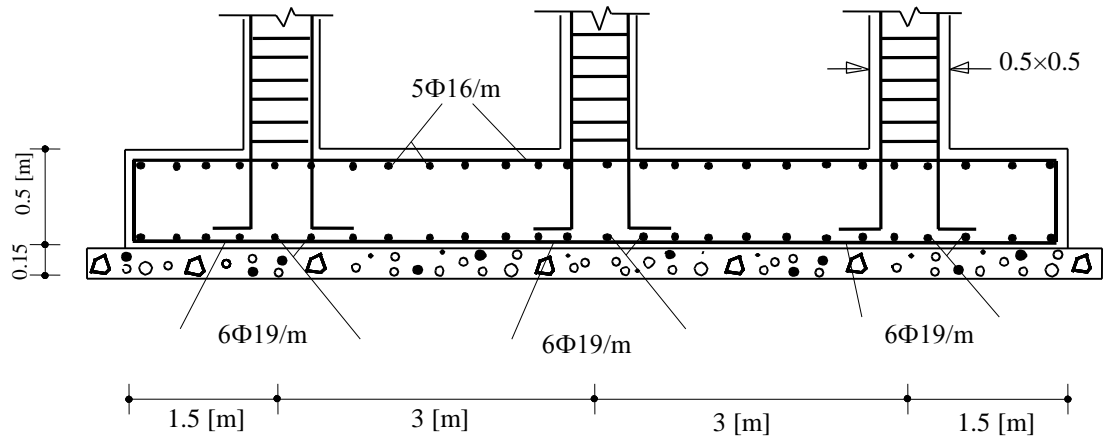
$$A_{sxb} = \frac{0.391}{182 \times 0.45} = 0.004774 \text{ [m}^2/3.0 \text{ m]}$$

$$A_{sxb} = 47.74 \text{ [cm}^2/3.0 \text{ m]} = 15.91 \text{ [cm}^2/\text{m]}$$

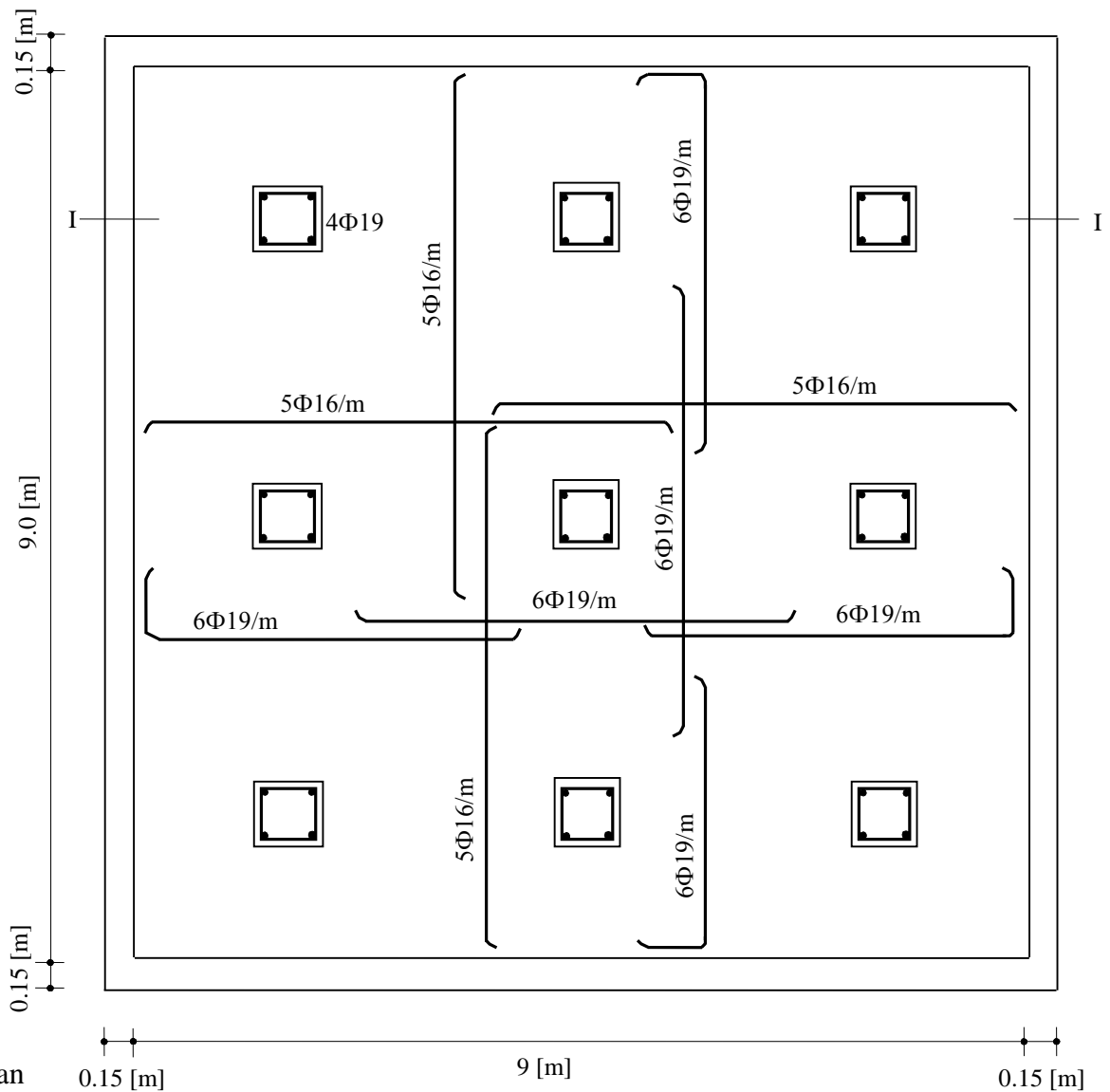
Chosen steel in both x -directions and y -directions at the bottom is $6\Phi 19/\text{m} = 17.0 \text{ [cm}^2/\text{m]}$

The details of reinforcement in plan and section $a-a$ through the footing are shown in Figure 134.

Solved Examples



a) Section I-I



b) Plan

Figure 134: Details of reinforcement in plan and section *a-a* through the raft

Part B
Theoretical Basis

1- Determine the bearing capacity of footing

Determine the bearing capacity of footing may be carried out according to the Egyptian Code of Practice [1], *ECP* as follows:

1.1- Vertical concentrated load

Ultimate bearing capacity q_{ult} is given by:

$$q_{ult} = C N_c \lambda_c + \gamma_1 D_f N_q \lambda_q + \gamma_2 B N_\gamma \lambda_\gamma \quad (1)$$

where:

- q_{ult} Ultimate bearing capacity, [kN/m²].
- N_c, N_q, N_γ Bearing capacity factors depended on the angle of internal friction ϕ , [-].
- $\lambda_c, \lambda_q, \lambda_\gamma$ Shape factors depended on the shape of the footing in plan, [-].
- C Cohesion of the soil, [kN/m²].
- ϕ Angle of internal friction of the soil, [°].
- γ_1 Unit weight of the soil above the foundation level, [kN/m³].
- γ_2 Unit weight of the soil under the foundation level, [kN/m³].
- D_f Foundation depth, [m].
- B Width of the footing (the smallest side), [m].
- L Length of the footing, [m].

Bearing capacity factors N_q, N_c and N_γ are determinate from:

a) Case of soil having $\phi > 0$

$$N_q = e^{\pi \tan \phi} \tan^2 \left(45 + \frac{\phi}{2} \right) \quad (2)$$

$$N_c = (N_q - 1) \cot \phi \quad (3)$$

$$N_\gamma = (N_q - 1) \tan \phi \quad (4)$$

b) Case of soil having $\phi = 0$

$$N_c = 5, \quad N_q = 1 \quad \text{and} \quad N_\gamma = 0$$

Shape factors λ_c , λ_q and λ_γ are obtained from Table 14

Table 14 Shape factors λ_c , λ_q and λ_γ

Footing shape in plan	λ_c, λ_q	λ_γ
Strip	1	1
Rectangular	$1+0.3B/L$	$1-0.3B/L$
Squar or circular	1.3	0.7

1.2- Vertical eccentric load

When determining ultimate bearing capacity for a footing having an eccentric load (Figure 135), the total footing area of $(B \times L)$ is replaced by an effective footing area of $(B' \times L')$ that load applies in the center of it. Ultimate bearing capacity q_{ult} is determined by the effective footing area as:

$$q_{ult} = C N_c \lambda_c + \gamma_1 D_f N_q \lambda_q + \gamma_2 B' N_\gamma \lambda_\gamma \quad (5)$$

$$B' = B - 2e_x \quad (6)$$

$$L' = L - 2e_y \quad (7)$$

where:

- e_x Eccentricity in x -direction, [m].
- e_y Eccentricity in y -direction, [m].
- B' Width of the effective footing (the smallest side), [m].
- L' Length of the effective footing, [m].

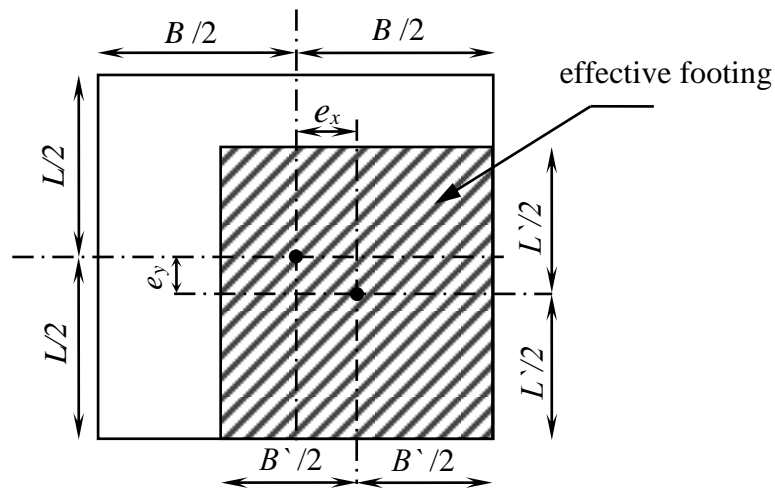


Figure 135: Vertical eccentric load and the effective footing

1.3- Inclined load

Ultimate bearing capacity reduces when the angle δ between the applied load and the vertical direction exceeds as shown in Figure 136. In this case ultimate bearing capacity q_{ult} is given by:

$$q_{ult} = C N_c \lambda_c i_c + \gamma_1 D_f N_q \lambda_q i_q + \gamma_2 B N_\gamma \lambda_\gamma i_\gamma \quad (8)$$

where:

i_c, i_q, i_γ Factors depended on the angle δ between the load and the vertical direction, [-].
 δ Angle between the load and the vertical direction, [°].

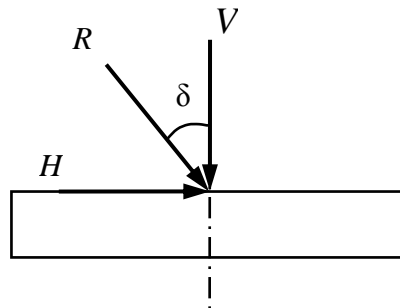


Figure 136: Inclined load

a) Case of soil having $\phi = 0$ and $C \neq 0$

$$i_q = 1 \quad (9)$$

$$i_c = 0.5 + 0.5 \sqrt{1 - \frac{H_b}{AC}} \quad (10)$$

where:

H_b The horizontal component for the ultimate bearing capacity, [kN].

A Area of the footing, [m²].

First it should be chosen an area to achieve the condition ($A \times C > H_b$)

b) Case of soil having $\phi \neq 0$ and $C = 0$

$$i_q = (1 - 0.7 \tan \delta)^3 \quad (11)$$

$$i_\gamma = (1 - \tan \delta)^3 \quad (12)$$

c) Case of soil having $\varphi \neq 0$ and $C \neq 0$

$$i_q = \left[1 - 0.7 \frac{H_b}{V_b + A C \cot \varphi} \right]^3 \quad (13)$$

$$i_\gamma = \left[1 - \frac{H_b}{V_b + A C \cot \varphi} \right]^3 \quad (14)$$

$$i_c = i_q - \frac{1 - i_q}{N_q - 1} \quad (15)$$

where:

V_b Vertical component of the ultimate bearing capacity, [kN].

H_b Horizontal component of the ultimate bearing capacity, [kN].

$$H_b = F_b \times H \quad (16)$$

$$V_b = F_b \times V \quad (17)$$

where:

F_b Factor of safety, [-].

H Horizontal component of the footing load, [kN].

V Vertical component of the footing load, [kN].

1.4- Layered soil

Ultimate bearing capacity for a footing resting on layered soil may be carried simply as indicated in the following steps:

- 1) Determine the ultimate bearing capacity for the first layer.
- 2) Determine the ultimate bearing capacity for the second layer with assuming an imaginary footing lies at the top of the layer with dimension $(B+h_1) \times (L+h_1)$ as shown in Figure 137.
- 3) Repeat the previous step for the other layers.
- 4) The ultimate bearing capacity is the smallest value.

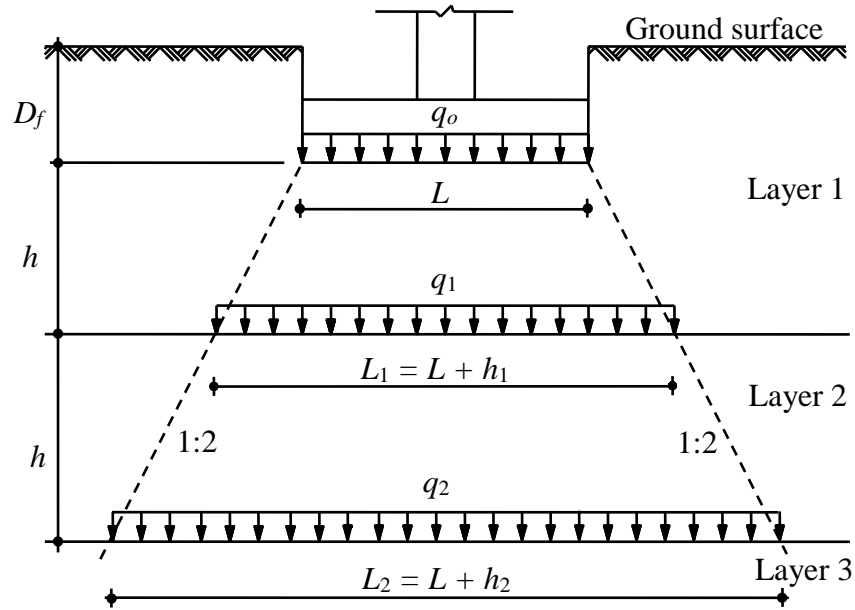


Figure 137: Layered soil

1.5- Effect of water table on bearing capacity

Water table has an effect on the third term of the bearing capacity equation. Three cases for the water table effect may be considered as follows:

a) The foundation level is under the ground water table as shown in Figure 138.

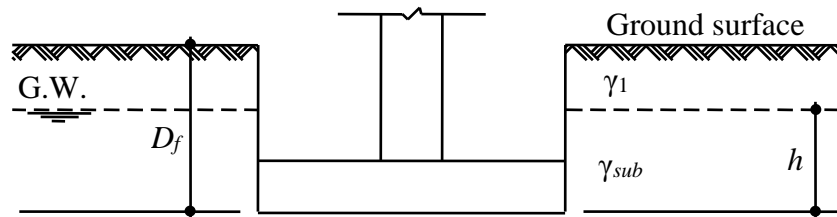


Figure 138: Foundation level is under the ground water table

Ultimate bearing capacity q_{ult} is given by:

$$q_{ult} = C N_c \lambda_c i_c + q' N_q \lambda_q + \gamma_{sub} B N_\gamma \lambda_\gamma \quad (18)$$

$$q' = \gamma_1 \times (D_f - h) + \gamma_{sub} h \quad (19)$$

where:

q' Effective pressure at foundation level, [kN/m²].
 γ_{sub} Submerged unit weight of the soil, [kN/m³].

- b) The foundation level is under the bed level of a canal or a river as for a pair as shown in Figure 139.

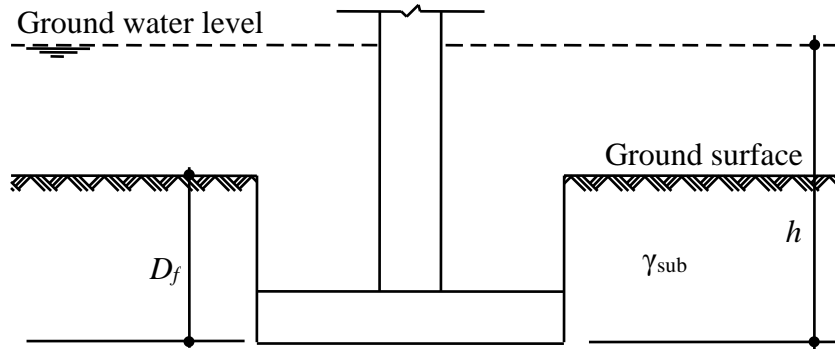


Figure 139: Foundation level is under the bed level of a canal

Ultimate bearing capacity q_{ult} is given by:

$$q_{ult} = C N_c \lambda_c i_c + q' N_q \lambda_q + \gamma_{sub} B N_\gamma \lambda_\gamma \quad (20)$$

$$q' = \gamma_{sub} D_f \quad (21)$$

- c) The ground water table under the foundation level as shown in Figure 140.

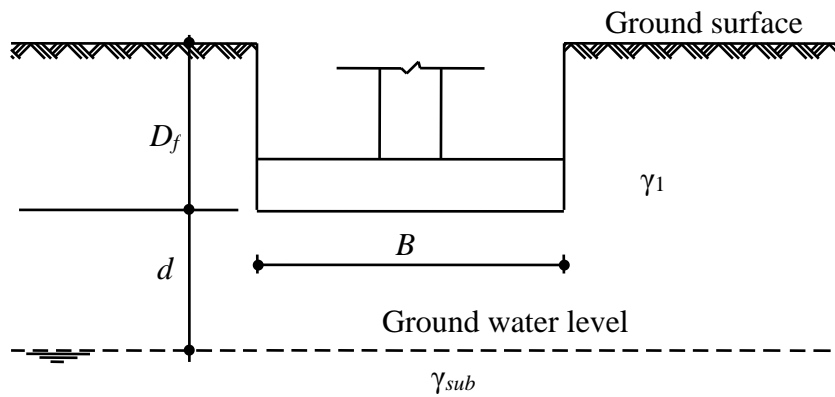


Figure 140: Ground water table under the foundation level

Ultimate bearing capacity q_{ult} is given by:

$$q_{ult} = C N_c \lambda_c + \gamma_1 D_f N_q \lambda_q + \gamma_2' B N_\gamma \lambda_\gamma \quad (22)$$

$$\gamma_2' = \gamma_{sub} + \frac{d}{B} (\gamma - \gamma_{sub}) \quad (23)$$

1.6- Allowable and net bearing capacities

It will be important to define some terms relating to bearing capacities.

a) Ultimate bearing capacity q_{ult}

The ultimate bearing capacity q_{ult} is defined as the least pressure on the soil which would cause shear failure of the supporting soil immediately below the footing.

b) Allowable bearing capacity q_{all}

The allowable bearing capacity is allowable pressure on the soil due to the footing load that can be carried safely by the soil. The allowable bearing capacity is equal to the ultimate bearing capacity divided by a specified factor of safety FS , as example 2.5.

$$q_{all} = \frac{q_{ult}}{FS} \quad (24)$$

c) Net allowable bearing capacity q_{net}

It is noticed that the allowable bearing capacity equations are based on gross pressure under the footing (Eq. 25 and Figure 141), while those of designing the footing or determining the settlement are based on the net pressure under the footing (Eq. 26 and Figure 141).

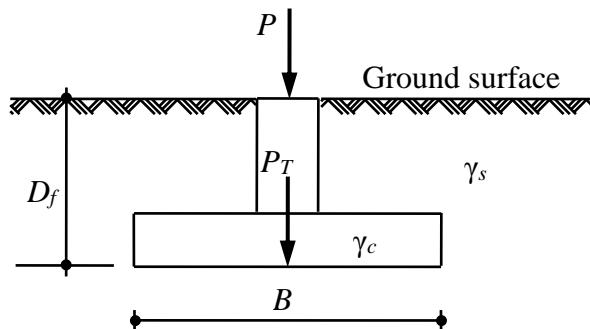


Figure 141: Gross and net load

$$q_{all} = \frac{P_T}{A} \quad (25)$$

$$q_{net} = \frac{P}{A} \quad (26)$$

where:

- P_T Total load at the foundation level including applied load at the surface, the self weight of the footing and the weight of the soil, [kN].
- A Footing area in plan, [m].
- P Applied load at the surface, [kN].

Theoretical Basis

A relation between the allowable bearing capacity of the soil and the net allowable bearing capacity is given by:

$$q_{net} = q_{all} - \gamma_a D_f \quad (27)$$

where:

γ_a Average unit weight of the soil and the footing material, [kN/m³].
 γ_a may be taken as 20 [kN/m³].

A relation between the total load at the foundation level and the applied load at the surface can be obtained from Eq. 25, 26 and 27 as:

$$P_T = \frac{P}{\left(1 - \frac{\gamma_a D_f}{q_{all}}\right)} \quad (28)$$

2- Determine Reinforced Concrete Design of Shallow Foundations

Design of shallow foundations in this book is carried out according to the Egyptian Code of Practice [1], *ECP*, using the method of working stress. Design of section for footings and rafts is carried out only for the moment, first. Then a check for shearing force for strip footing or check for punching shear for other types of foundations is carried out. Other checks such as bond stress and dowels are not required. The section is chosen to resist moment and shear without additional reinforcement.

2.1- Critical sections for moment

Sections for computing moment are taken at (Figure 142):

- The face of the column or the reinforced concrete wall.
- The mid-distance between the face of the steel column and the edge of the base plate.
- The mid-distance between the face of the brick wall and the axis of the wall.

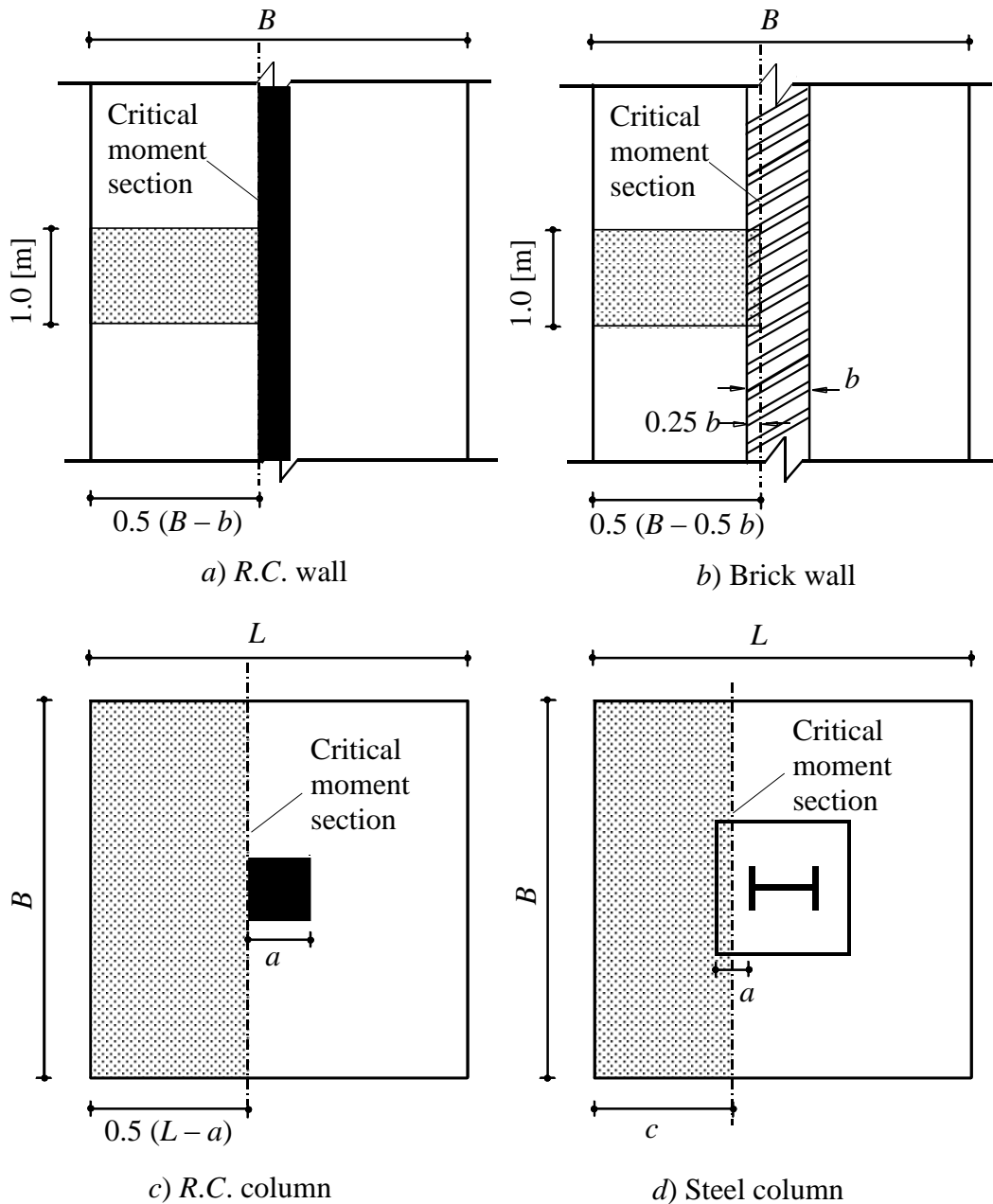


Figure 142: Critical sections for moments

2.2- Design for flexure moment

The design procedure is based on the stress diagram shown in Figure 143. In this method, a linearly elastic relationship between stresses and strains is assumed for both the concrete and the reinforcing steel. The codes assume that the maximum stress produced by the worst combinations of working loads does not exceed a specified allowable working stress value. When the applied moment exceeds the moment capacity at the designed balanced section, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

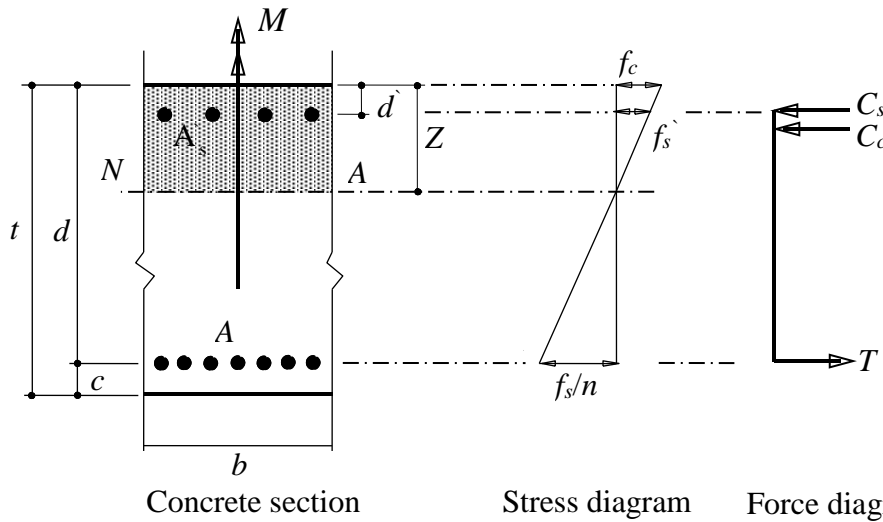


Figure 143: Contact Distribution of stresses and forces according to *ECP* (working stress method)

In designing procedure, a suitable depth d [m] of the section is assumed first. Then, the maximum depth d_m [m] required to resist the applied moment as a singly reinforced section is obtained. The reinforcing steel area is determined based on whether the assumed depth d greater than, less than, or equal to d_m .

The value of the ratio $\xi = z/d$ of neutral axis to effective depth at balanced condition is given by:

$$\xi = \frac{n}{n + \frac{f_s}{f_c}} \quad (29)$$

where:

- z The neutral axis depth, [m].
- f_s Tensile stress of steel, [MN/m²].
- f_c Compressive stress of concrete, [MN/m²].
- n Modular ratio, $n = E_s/E_c$, is the ratio between moduli of elasticity of steel and concrete. The value of the modular ratio is $n = 15$

The coefficient k_1 to obtain the section depth at balanced condition is given by:

$$k_1 = \sqrt{\frac{2}{f_c \xi (1 - \frac{\xi}{3})}} \quad (30)$$

The maximum depth d_m [m] as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M}{b}} \quad (31)$$

where:

M Moment at a section obtained from analysis, [MN.m].
 b Width of the section to be designed, [m].

Check if the assumed depth d is not exceeded the maximum depth d_m to resist moment as a singly reinforced section.

Singly reinforced section

If $d \geq d_m$, then the section is designed as singly reinforced section. The tensile steel reinforcement is calculated as follows:

Determine the neutral axis z [m] corresponding to the depth d by iteration from:

$$z = \sqrt{\frac{2 n M (d - z)}{b f_s (d - \frac{z}{3})}} \quad (32)$$

The value of the ratio ξ corresponding to the depth d is given by:

$$\xi = \frac{z}{d} \quad (33)$$

The coefficient k_2 [MN/m²] to obtain the tensile reinforcement for singly reinforced section, is given by:

$$k_2 = f_s \left(1 - \frac{\xi}{3}\right) \quad (34)$$

The area of tensile steel reinforcement A_s [m²] is then given by:

$$A_s = \frac{M}{k_2 d} \quad (35)$$

The coefficients k_1 and k_2 may be also obtained from the known charts of reinforced concrete. Tables 1 and 2 show coefficients k_1 and k_2 for singly reinforced rectangular sections. The units used to obtain coefficients k_1 and k_2 in the tables are [MN] and [m].

Doubly reinforced section

If $d_m > d$ then the section is designed as doubly reinforced section. Both top and bottom reinforcement are required. However, it is prefer to design the section of foundation as a singly reinforced section by increasing the depth but in some cases, the designer refers to this solution. The tension and compression reinforcement is calculated as follows:

The limiting moment resisted by concrete compression and tensile steel M_{lim} [MN.m] as a singly reinforced section is given by:

Theoretical Basis

$$M_{lim} = \left(\frac{d}{d_m} \right)^2 M \quad (36)$$

Therefore, the moment resisted by compression steel and additional tensile steel ΔM [MN.m] is:

$$\Delta M = M - M_{lim} \quad (37)$$

The coefficient k_2 [MN/m²] to obtain the tensile steel as singly reinforced section, is given by:

$$k_2 = f_s \left(1 - \frac{\xi}{3} \right) \quad (38)$$

The area of tensile reinforcement A_{s1} [m²] to resist M_{lim} is then given by:

$$A_{s1} = \frac{M_{lim}}{k_2 d} \quad (39)$$

The required additional tensile steel A_{s2} [m²] to resist ΔM is given by:

$$A_{s2} = \frac{\Delta M}{f_s (d - d')} \quad (40)$$

where:

d' Concrete cover to center of compression reinforcing, [m].

The total required tensile reinforcement A_s [m²] to resist $M_{lim} + \Delta M$ is given by:

$$A_s = A_{s1} + A_{s2} \quad (41)$$

The required compression steel A_s' [m²] to resist ΔM is given by:

$$A_s' = \frac{\Delta M}{f_s' (d - d')} \quad (42)$$

where:

f_s' Compressive stress of steel in compression [MN/m²], which is obtained from:

$$f_s' = n f_c \frac{z - d'}{z} \quad (43)$$

2.3- Check for punching shear

The *ECP* code for working stress method assumes the critical punching shear section on a perimeter at a distance $d/2$ from the face of the column as shown in Figure 144.

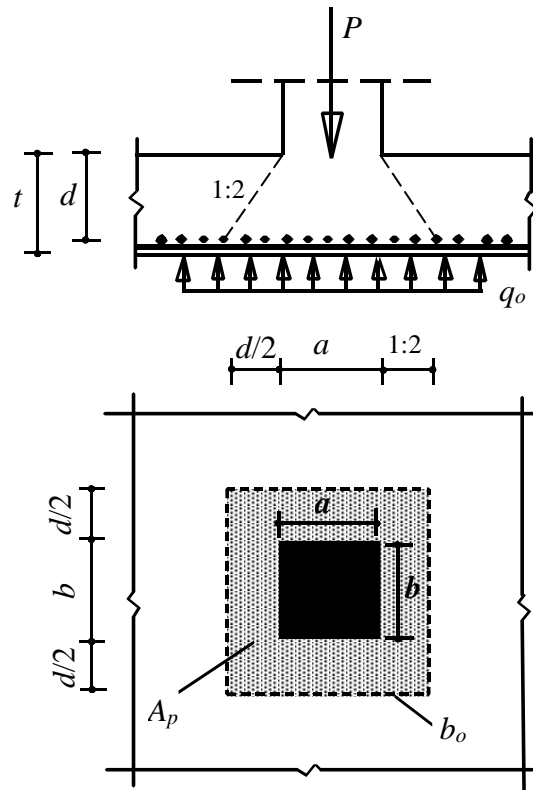


Figure 144: Critical section for punching shear according to *ECP* (working stress method)

The punching shear force at the section Q_p [MN] is given by:

$$Q_p = P - q_o A_p \quad (44)$$

where:

$$q_p = \frac{Q_p}{b_o d} \quad (45)$$

- P Column load, [MN].
 q_o Upward soil pressure under the column, [MN/m²].
 A_p Area of critical punching shear section, [m²].
 = $(a + d)^2$ for square columns
 = $(a + d)(b + d)$ for rectangular columns
 = $\pi(D + d)^2/4$ for circular columns
 a, b Column sides, [m].
 D Column diameter, [m].

The punching shear stress q_p [MN/m²] is given by:

where:

- d Depth to resist punching shear [m]

Theoretical Basis

b_o Perimeter of critical punching shear section [m]
 $= 4 (a + d)$ for square columns
 $= 2 (a + b + 2 d)$ for rectangular columns
 $= \pi (D + d)$ for circular columns

The allowable concrete punching strength q_{pall} [MN/m²] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b} \right) q_{cp} \leq q_{cp} \quad (46)$$

where:

q_{cp} The main value of shear strength [MN/m²] according to Table 3.
 a The smallest column side.

Normally, it is impracticable to provide shear reinforcement in slabs and footings. In such cases, concrete alone should resist the punching shear without contribution of the shear reinforcement. The slab thickness is considered to be safe for punching stress, if the punching shear stress is less than the allowable concrete punching strength where:

$$q_p \geq q_p \quad (47)$$

If the above basic condition is not satisfied, the thickness will have to be increased to resist the punching shear.

Mechanical properties of concrete according to ECP

ECP defines the concrete grade according to the standard cube compressive strength of concrete f_{cu} (characteristic strength). The standard cube has 15 [cm] side length. Thus, grade C250 concrete has $f_{cu} = 250$ [kg/cm²]. According to *ECP* the mechanical properties of concrete, characteristic strength f_{cu} , concrete cylinder strength f_c^{\prime} , compressive stress of concrete for bending or compression with big eccentricity f_c , and main value of punching shear strength q_{cp} can be taken as in Table 15.

Table 15: Determination of tension reinforcement in longitudinal direction

Concrete grade	C150	C175	C200	C225	C250	C275	C300
f_{cu} [kg/cm ²]	150	175	200	225	250	275	300
$f_c^{\prime} = 0.8 f_{cu}$ [kg/cm ²]	120	140	160	180	200	220	250
f_c [kg/cm ²]	65	70	80	90	95	100	105
q_{cp} [kg/cm ²]	7	7	8	8	9	9	10

To convert to [MN/m²], divide by 10

3- Types of shallow foundations

3.1. Isolated footing

An isolated footing is a concrete slab or block under an individual load or column. It may be a square, rectangular or circular footing, Figure 145.

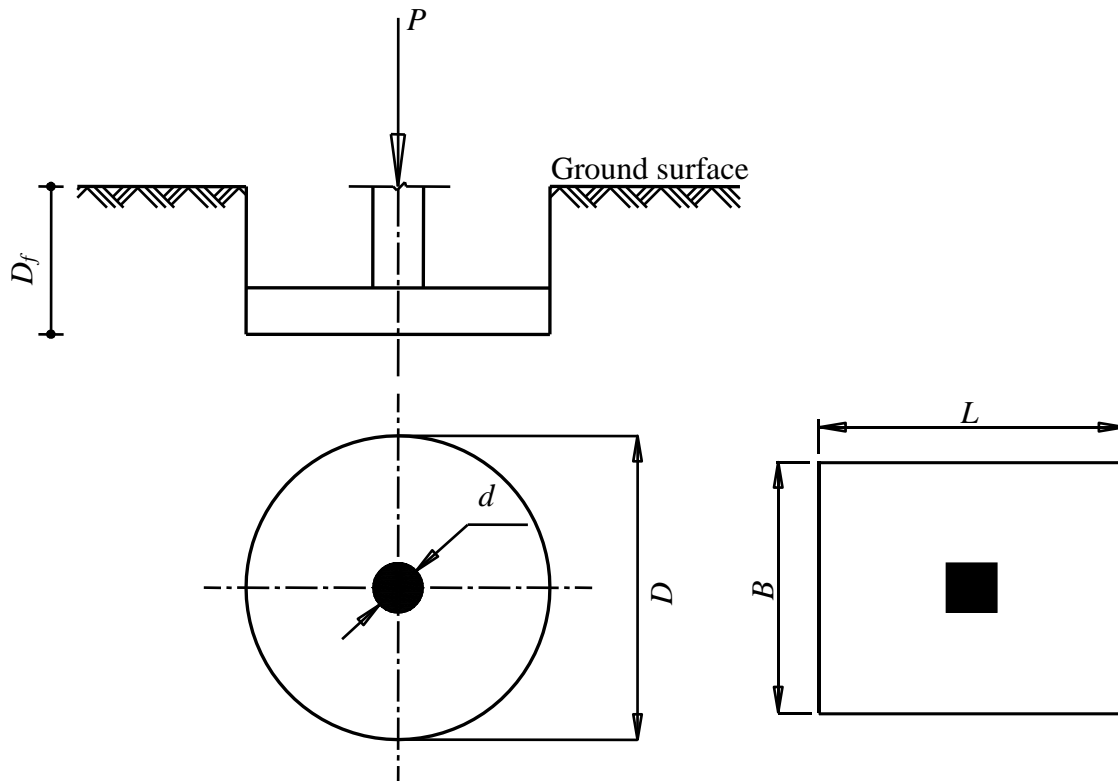


Figure 145: Isolated footing

3.2. Strip footing

When a number of columns in a row are to be supported on a soil of low bearing capacity, isolated footings if provided would overlap. A practical solution would be a footing to support all the columns of the row. Such a footing is called a "Strip footing", Figure 146. It may be used also under a brick or R. C. wall, Figure 146.

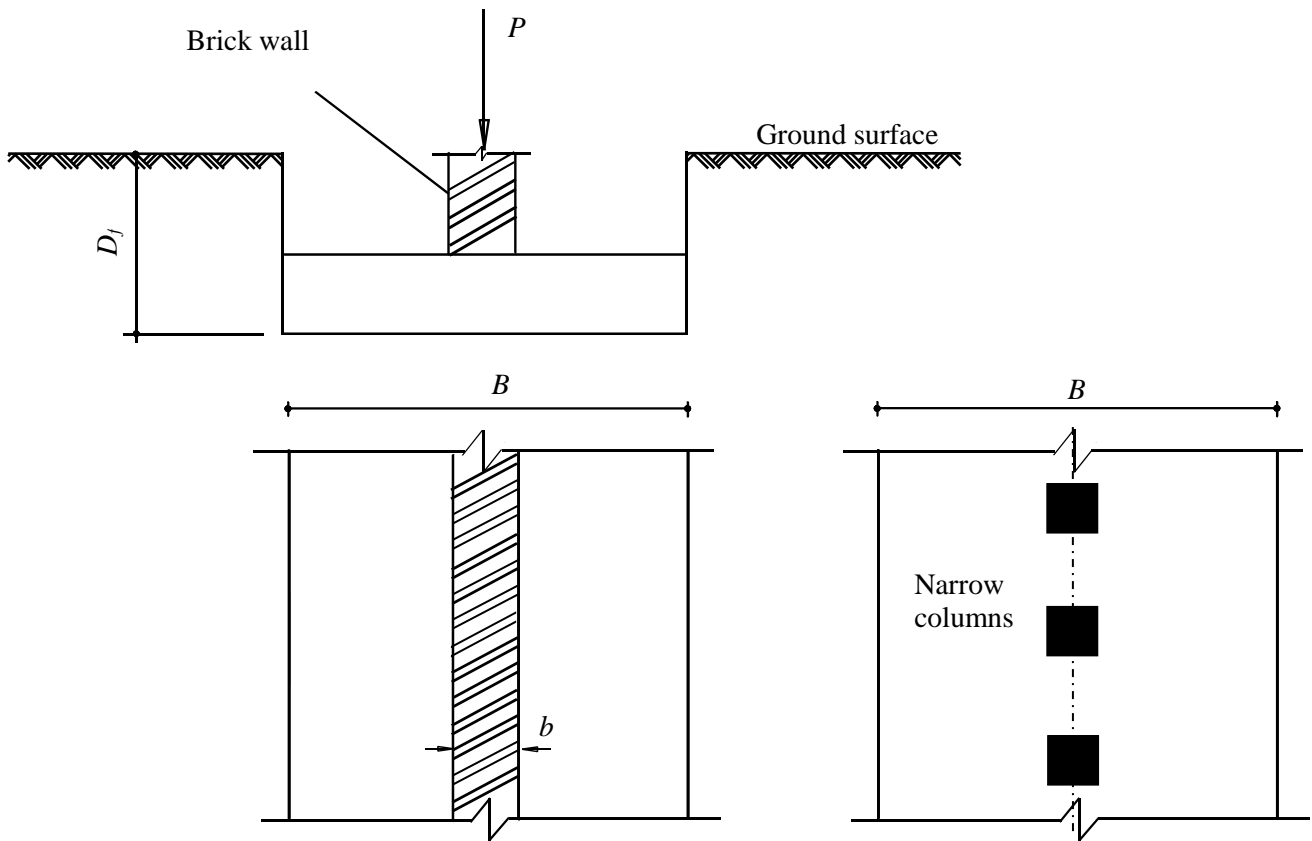


Figure 146: Strip footing

3.3. Combined footing

If a spread footing is provided for an external column, it will lead to unstable structure system. Therefore, the external column must be connected at least with another internal column by a footing. Such a footing is called a "Combined footing". Combined footing is also commonly used when isolated footing for two columns or more if provided would overlap.

a) Combined rectangular footing

In the design of rectangular footing, it is generally assumed that the contact pressure distribution is uniform. In fact, many column loads are unsymmetrical. To achieve a uniform contact pressure distribution under the footing, it is necessary to coincide the center of the footing area with the point of application of the resultant. It can be carried out by extending the footing so that the center of area of the footing coincides with the center of gravity of the external loads. This is easy to be done by rectangular footing, Figure 147.

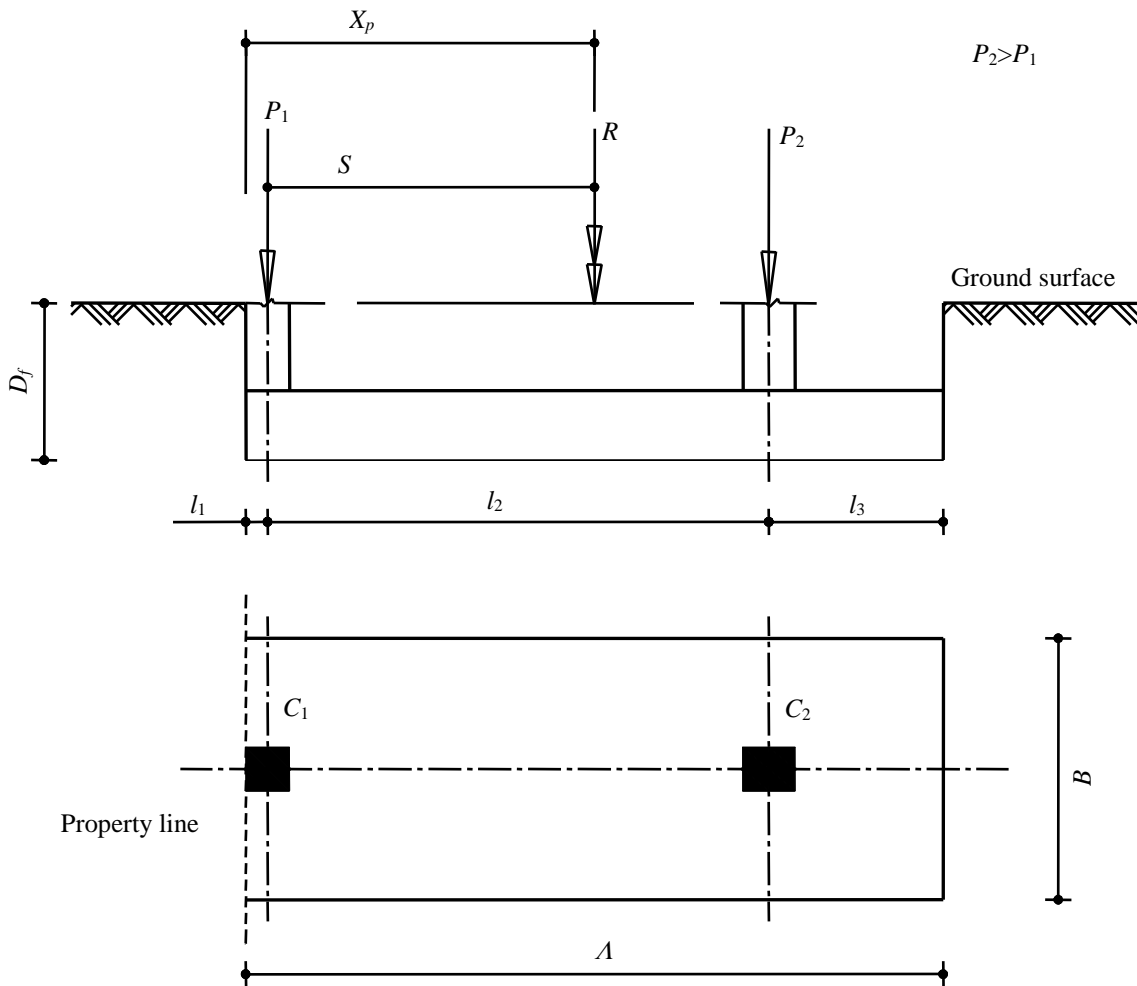


Figure 147: Combined rectangular footing

b) Combined trapezoidal footing

A special case of footings is the trapezoidal footing (Figure 148), which may be used to carry two columns of unequal loads when distance outside the column of the heaviest load is limited. In such case, using a rectangular footing may lead to the resultant of loads does not fall at the middle length of the footing. To overcome this difficulty, a trapezoidal footing is used in such a way that the center of gravity of the footing lies under the resultant of the loads. Consequently, the distribution of contact pressure will be uniform.

Theoretical Basis

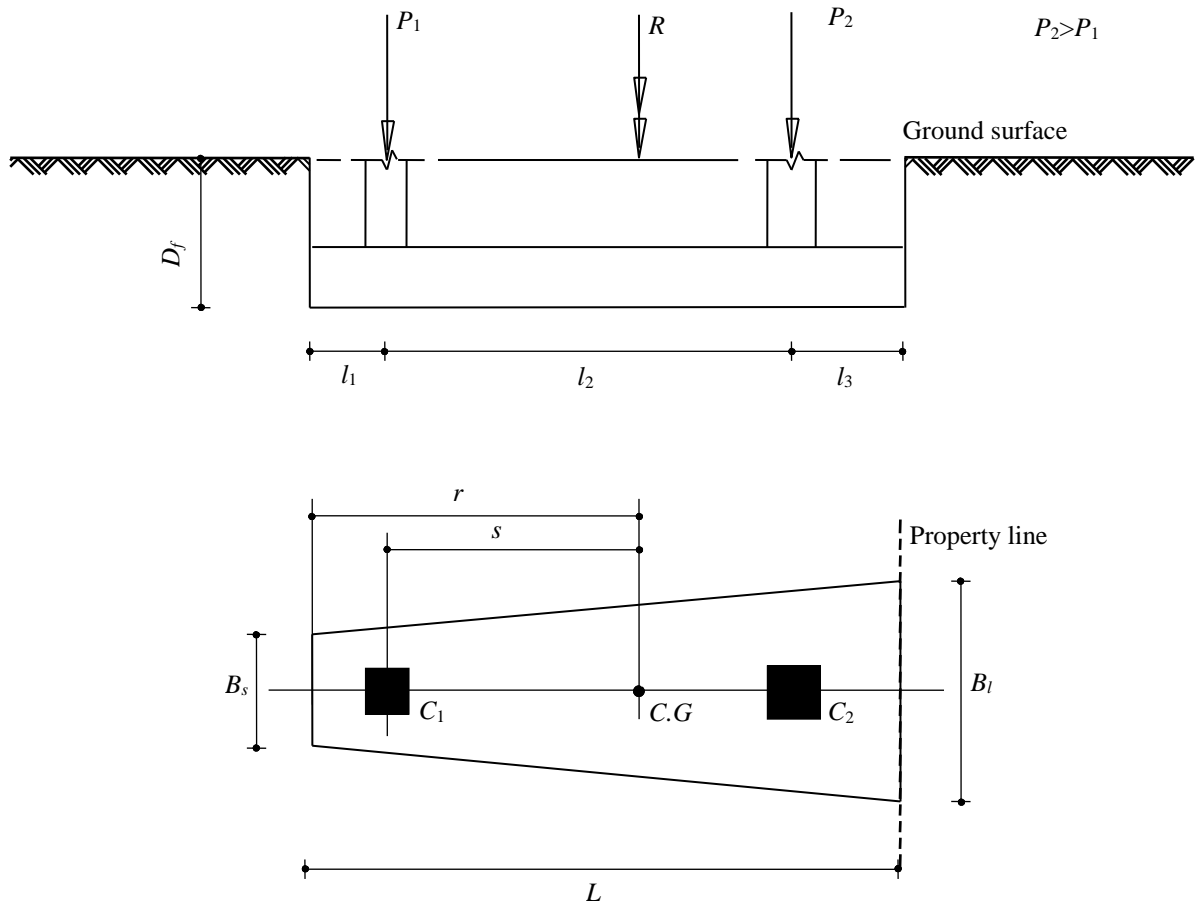


Figure 148: Combined trapezoidal footing

3.4. Strap footing

If the distances between columns are large, a composed of two spread footings connected by a tie girder or a strap may be used as an economical system instead of combined footing (Figure 149). Strap footing may also serve a need of bridging over areas that cannot be loaded.

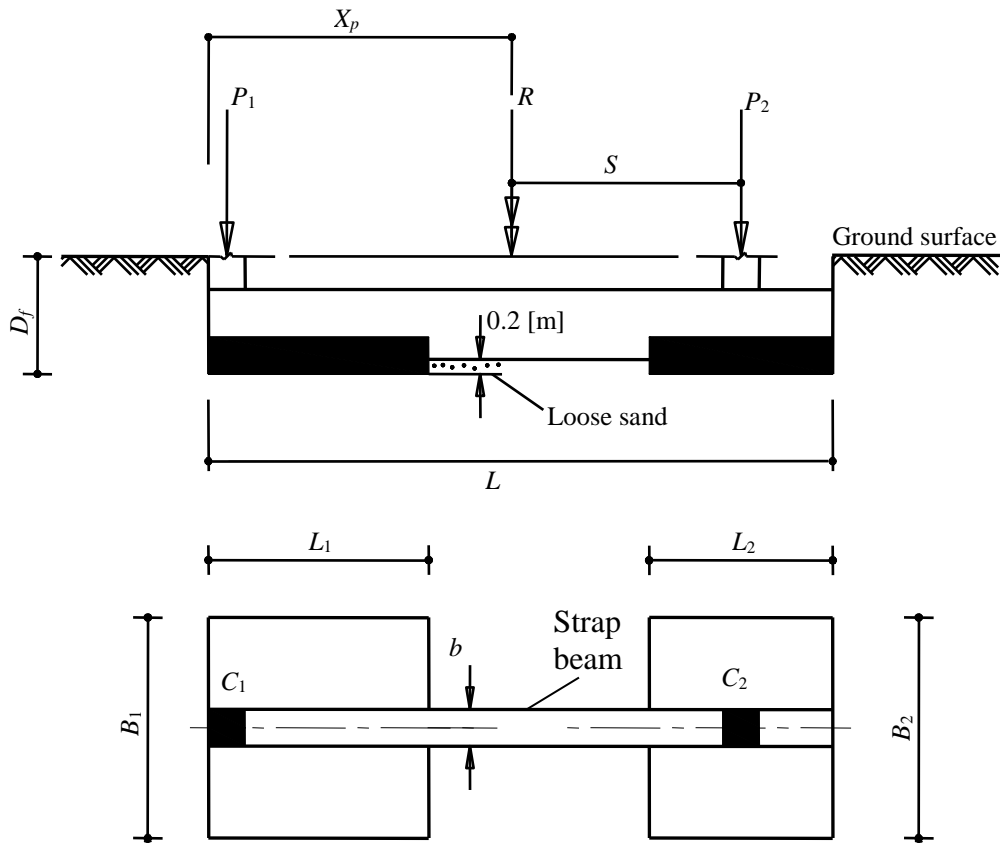


Figure 149: Combined strap footing

3.5. Circular raft

Ring or circular rafts can be used for cylindrical structures such as chimneys, silos, storage tanks, TV-towers and other structures. In this case, ring or circular raft is the best suitable foundation to the natural geometry of such structures (Figure 150). The design of circular rafts is quite similar to that of other rafts.

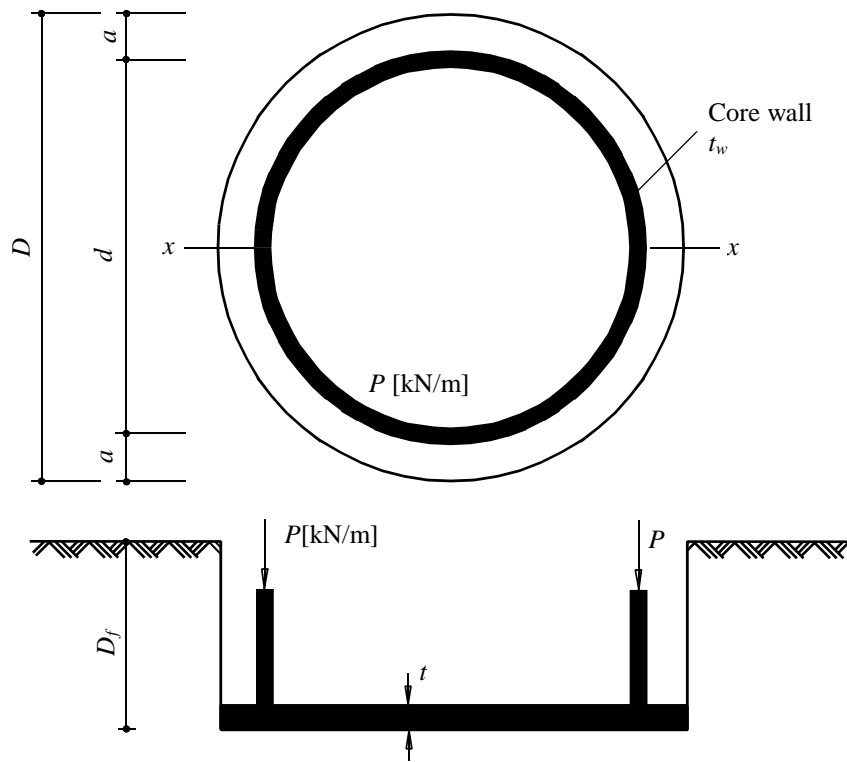


Figure 150: Circular raft

3.6. Raft

If distances between columns are small, a slab contains all columns called a raft may be used instead of spread footings, Figure 151.

Theoretical Basis

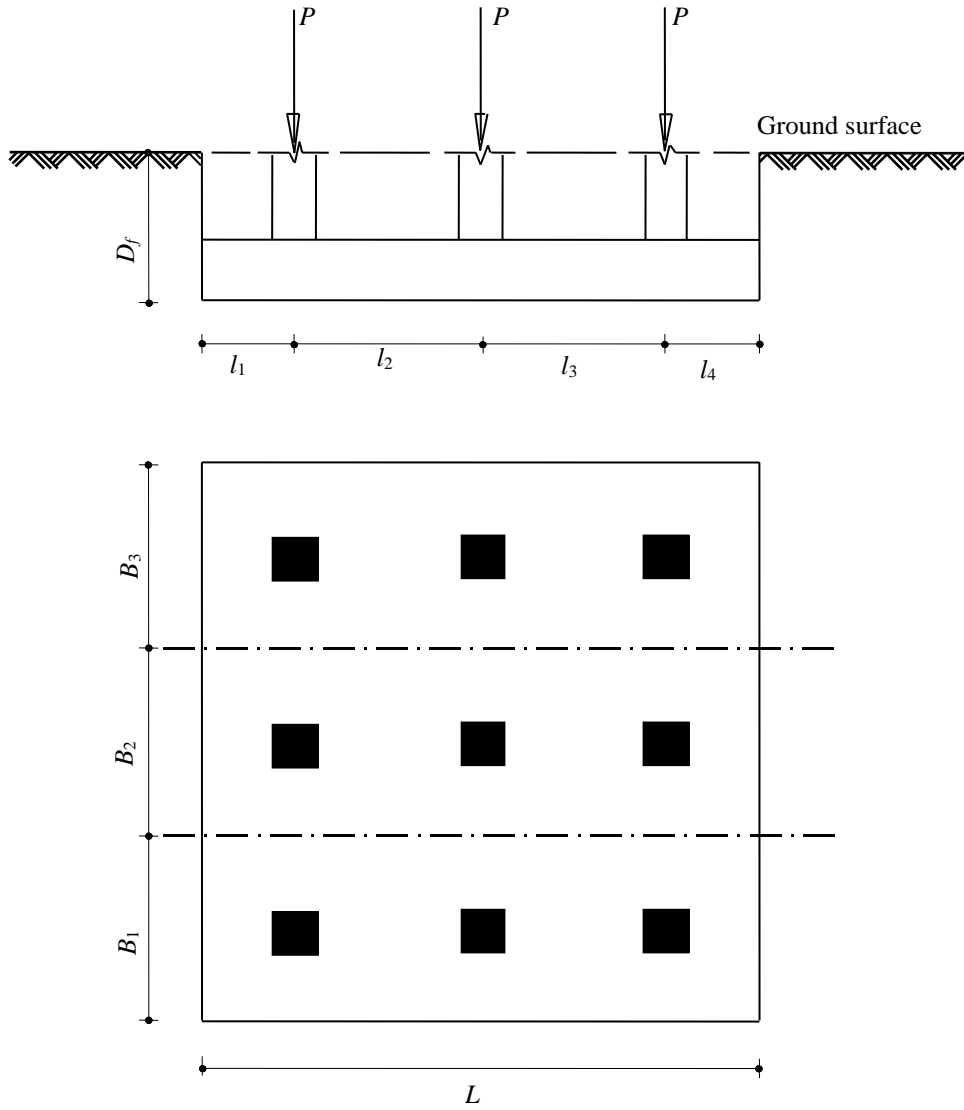


Figure 151: Raft

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