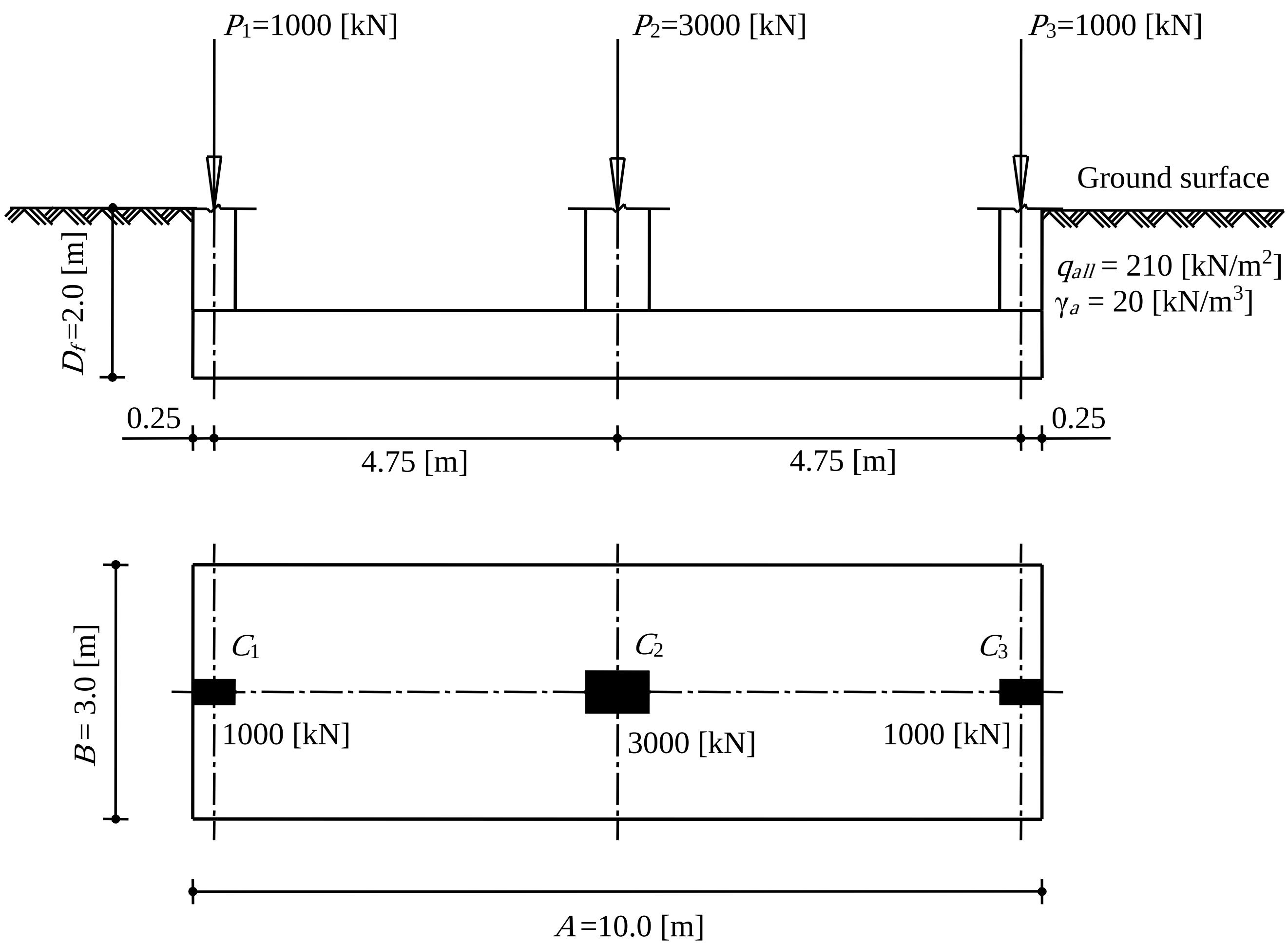
Beam Foundations after *Kany* and *El Gendy* by *GEO Tools* (Analysis and Design)

Part II: Calculation Examples



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2022

Content **Page**

Preface

Various problems in Geotechnical Engineering can be investigated by *GEO Tools*. *M. Kany* and (*M. @ A.*) *El Gendy* developed the original version of *GEO Tools* in *ELPLA* package for analyzing elastic foundation. After the death of *Kany* and (*M. & A.*) *El Gendy* further developed the program to meet the needs of the practice.

This book describes the essential methods used in *GEO Tools* for analyzing beam foundations with verification examples. *GEO Tools* is a simple user interface program and needs little information to define a problem.

There are three soil models with five methods available in *GEO Tools* for analyzing beam foundations. Many test examples are presented to verify and illustrate the soil models and methods for analyzing beam foundations available in *GEO Tools*.

10 Beam Foundations after *Kany and El Gendy* (Calculation Examples)

10.1 Introduction

GEO Tools is a user-friendly computer program. It can be used for analyzing structures with different types of subsoil models. To verify the validity of this computer program, results of numerical examples solved by hand calculation are compared with those obtained by *GEO Tools*. The mathematical solution of the beam on elastic foundation is based on methods of *Kany and El Gendy*.

10.2 Calculation methods

10.2.1 General

It is possible by *GEO Tools* to use the same data for analyzing beam foundations by five different conventional and refined calculation methods based on the three standard subsoil models. The subsoil models for analyzing beam foundations (standard models) available in *GEO Tools* are:

A Simple assumption model
B *Winkler*'s model
C Continuum model

Simple assumption model does not consider the interaction between the beam foundation and the soil. The model assumes a linear distribution of contact pressures beneath the foundation. *Winkler*'s model is the oldest and simplest one that considers the interaction between the beam foundation and the soil. The model represents the soil as elastic springs. Continuum model is the complicated one. The model considers also the interaction between the beam foundation and soil. It represents the soil as a layered continuum medium.

The three standard soil models are described through five different numerical calculation methods. The methods graduate from the simplest one to more complicated one covering the analysis of most common beam foundation problems that may be found in the practice.

According to the three standard soil models (simple assumption model - *Winkler*'s model - Continuum model), five numerical calculation methods are considered to analyze the beam foundation as follows:

1 Linear Contact Pressure
(Simple assumption model)

2 Elastic Beam Foundation using Modulus of Subgrade Reaction by *Kany/ El Gendy* (1995)
(*Winkler*'s model)

3 Elastic Beam Foundation using Modulus of Compressibility by *Kany* (1974)
(Continuum model)

4 Rigid Beam Foundation using Modulus of Compressibility by *Kany* (1972)
(Continuum model)

5 Flexible Beam Foundation using Modulus of Compressibility
(Continuum model)

It is also possible to consider irregular soil layers and the thickness of the base beam that varies in each element. Furthermore, the influence of temperature changes and additional settlement on the beam foundation can be taken into account.

10.2.2 Definition

In the analysis, the beam foundation is divided into equal elements according to **Error! Reference source not found.**. Using the available five calculation methods, the settlement and the contact pressure can be determined in each element.

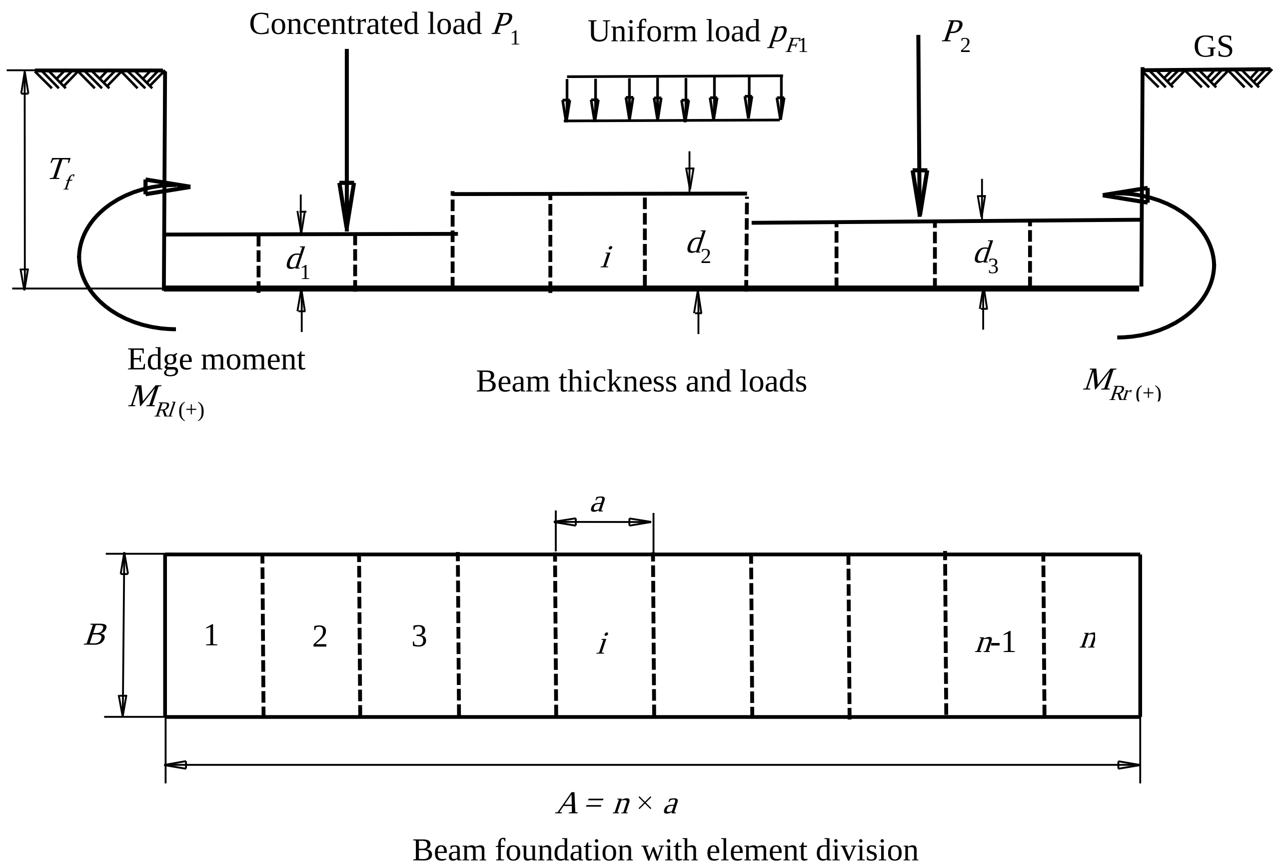


Figure 10.1 Loads, beam thickness und beam foundation with element division

10.3 Example 1: Analysis of a raft for four equal walls

10.3.1 Description of the problem

Figure 10.1 shows plan and section with dimensions and loads for a raft of four equal walls. It is required to find the contact pressure distribution, settlements, moment and shear force diagrams for the raft. The loading and the raft are symmetrical.

Geometry:

Thickness of the raft	d	=0.6	[m]
Dimensions of the raft	A_f	=8×10	[m^2]
Groundwater depth under the ground surface	Tw	=1	[m]
Foundation depth under the ground surface	Tf	=2	[m]

Material properties of the concrete and unit weight of the water

Modulus of elasticity of the concrete	E_b =2×10^7	[kN/m^2]
Unit weight of the concrete	γ_b=25	[kN/ m^3]
Unit weight of the water	γ_w=10	[kN/ m^3]

Soil properties

Modulus of subgrade reaction of the soil k_s = 20000 [kN/m^3].

10.3.2 Preparing the calculation

The raft can be regarded as a beam on elastic foundation subjected to:

- A uniformly distributed loading p_f equal to the weight of the raft itself minus the uplift pressure from the ground water.
- Four concentrated forces from four walls $P_1 = P_4$ = 200 [kN/m] and $P_2 = P_3$ = 300 [kN/m].

Computing the uniform load on the raft

Own weight of the raft	$w_o = \gamma_b \times d$ = 25×0.6	=15	[kN/m^2]
Up lift pressure	$w_w = \gamma_w \times (T_f - Tw)$ = 10(2-1)	=-10	[kN/m^2]
Total	p_f =	=5	[kN/m^2]

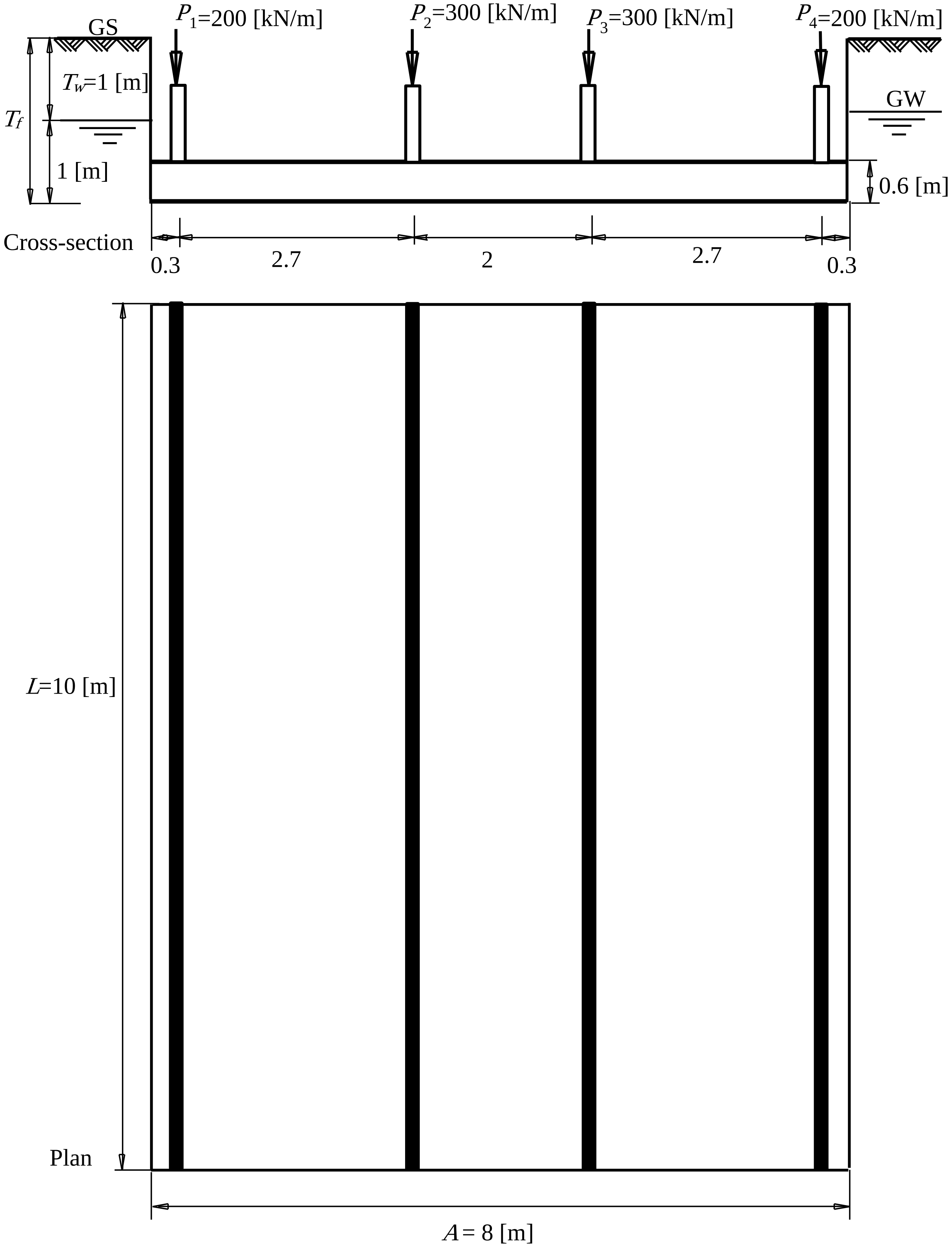


Figure 10.1 Raft of four equal walls

10.3.3 Hand calculation

Assume one-meter strip width from the raft and consider it as a beam on elastic foundation. The beam is divided into eight equal elements, each 1 [m] long (Figure 10.2). Because of the symmetry of the system, the analysis can be carried out by considering only half of the beam. Hence, the total number of equations is reduced to four.

According to *Kany/ El Gendy* (1995), the analysis of a beam on elastic foundation is carried out in the following steps:

10.3.3.1 Calculation of u_i, v_i and w_i:

$$u_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i-1}}\right)$$

$$v_i = \frac{1}{4}\left(\frac{I_i}{I_{i-1}} + 14 + \frac{I_i}{I_{i+1}}\right)$$

$$w_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i+1}}\right)$$

For a constant beam, moment of inertia $I_i = I$, then

$$u_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

$$v_i = \frac{1}{4}\left(\frac{I}{I} + 14 + \frac{I}{I}\right) = \frac{1}{4} \times 16 = 4$$

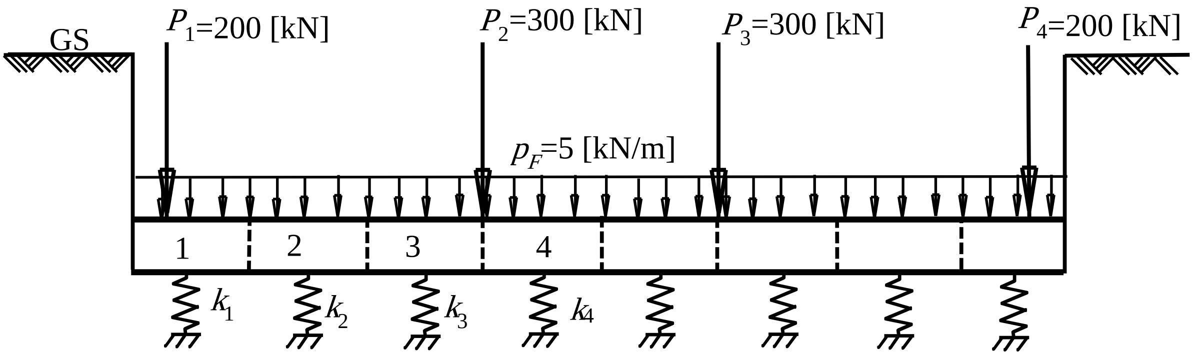
$$w_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

10.3.3.2 Moment of inertia I_i and beam stiffness α_i:

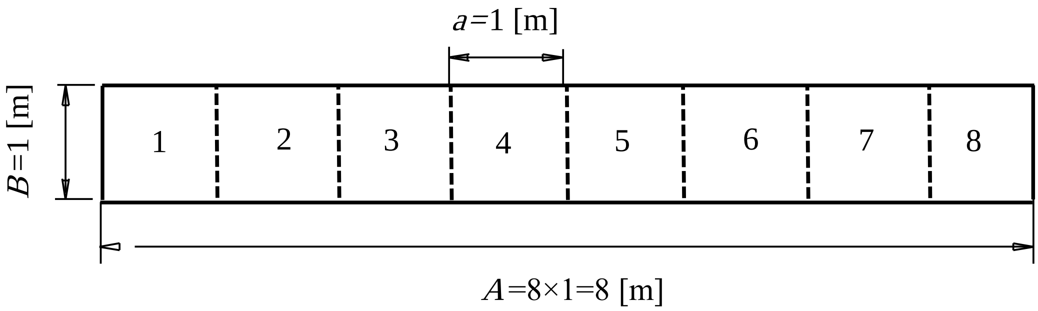
$$I_i = I = \frac{B d_i^3}{12} = \frac{1 \times 0.6^3}{12} = 0.018 [\mathrm{m}^4]$$

and

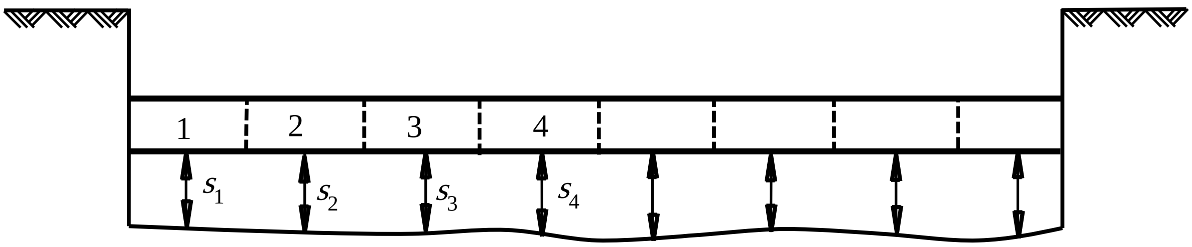
$$\alpha_i = \alpha = \frac{a^4 B}{E_b I} = \frac{1^4 \times 1}{(2 \times 10^7)(0.018)} = \frac{1}{360000} [\mathrm{m}^3/\mathrm{kN}]$$



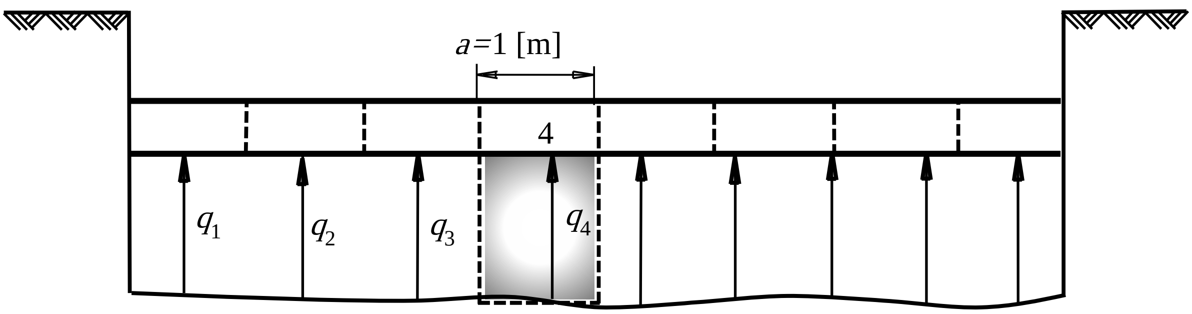
Beam foundation with loads and springs



Plan of beam foundation with elements



Settlement



Contact pressure

Figure 10.2 One meter strip width of the raft

10.3.3.3 Determining external moments $M_i^{(l)}$

The external moments $M_i^{(l)}$ at points 2, 3, 4 and 5 are:

$$M_1^{(l)} = 0$$
$$M_2^{(l)} = 200(1.5 - 0.3) + 5\frac{1.5^2}{2} = 245.625 \text{ [kN.m]}$$
$$M_3^{(l)} = 200(2.5 - 0.3) + 5\frac{2.5^2}{2} = 455.625 \text{ [kN.m]}$$
$$M_4^{(l)} = 200(3.5 - 0.3) + 5\frac{3.5^2}{2} + 300 \times 0.5 = 820.625 \text{ [kN.m]}$$
$$M_5^{(l)} = 200(4.5 - 0.3) + 5\frac{4.5^2}{2} + 300 \times 1.5 = 1340.625 \text{ [kN.m]}$$

10.3.3.4 Determining the right hand side R_i

The right hand side R_i of the contact pressure equation is:

$$R_i = \left(M^{(l)}_{i-1} + 4M^{(l)}_i + M^{(l)}_{i+1}\right)\frac{a^2}{6E\,I_i}$$
$$R_i = \left(M^{(l)}_{i-1} + 4M^{(l)}_i + M^{(l)}_{i+1}\right)\frac{1^2}{6 \times 2 \times 10^7 . \times 0.018}$$
$$R_i = \frac{1}{2160000}\left(M^{(l)}_{i-1} + 4M^{(l)}_i + M^{(l)}_{i+1}\right)$$

Apply the above equation at points 2, 3 and 4:

$$R_2 = \frac{1}{2160000}(0 + 4 \times 245.625 + 455.625) = \frac{1438.125}{216000}$$
$$R_3 = \frac{1}{2160000}(245.625 + 4 \times 455.625 + 820.625) = \frac{2888.75}{216000}$$
$$R_4 = \frac{1}{2160000}(455.625 + 4 \times 820.625 + 1340.625) = \frac{5078.75}{216000}$$

10.3.3.5 Determining contact pressures

The contact pressure equation is:

$$\left(\frac{1}{k_{i+1}}\right)q_{i+1} - \left(\frac{2}{k_i} - \frac{\alpha_i}{6}w_i\right)q_i + \left(\frac{1}{k_{i-1}} + \frac{\alpha_i}{6}(v_i + 2w_i)\right)q_{i-1} + \frac{\alpha_i}{6}\left(\sum_{j=1}^{i-2}[(i-j-1)u_i + (i-j)v_i + (i-j+1)w_i]\,q_j\right) = R_i$$

$$\left(\frac{1}{k}\right) q_{i+1} - \left(\frac{2}{k} - \frac{\alpha}{6}\right) q_i + \left(\frac{1}{k} + \alpha\right) q_{i-1} + \alpha \left(\sum_{j=1}^{i-2} (i-j)\, q_j\right) = R_i$$

$$\left(\frac{1}{20000}\right) q_{i+1} - \left(\frac{2}{20000} - \frac{1}{360000 \times 6}\right) q_i + \left(\frac{1}{20000} + \frac{1}{360000}\right) q_{i-1} + \frac{1}{360000} \left(\sum_{j=1}^{i-2} (i-j) q_j\right) = R_i$$

or

$$108\ q_{i+1} - 215\ q_i + 114\ q_{i-1} + 6 \left(\sum_{j=1}^{i-2} (i-j) q_j\right) = 2160000\ R_i$$

Apply the above equation at points 2, 3 and 4:

$$108\ q_3 - 215\ q_2 + 114\ q_1 = 1438.125$$
$$108\ q_4 - 215\ q_3 + 114\ q_2 + 12\ q_1 = 2888.75$$
$$-107\ q_4 - 114\ q_3 + 12\ q_2 + 18\ q_1 = 5078.75$$

There are four unknown q_1, q_2, q_3, and q_4, so a farther equation is required. This can be obtained by considering the overall equilibrium of vertical forces.

$$a \times B(q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8) = P_1 + P_2 + P_3 + P_4 + A \times B \times P_f$$

or

$$q_1 + q_2 + q_3 + q_4 = 520$$

Contact pressure equations in matrix form:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 114 & -215 & 108 & 0 \\ 12 & 114 & -215 & 108 \\ 18 & 12 & 114 & -107 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 520 \\ 1438.125 \\ 2888.75 \\ 5078.75 \end{Bmatrix}$$

Solving the above system of linear equations using *Gaussian's* elimination to obtain the contact pressures q_1, q_2, q_3, and q_4.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2.886 & -0.053 & -1 \\ 0 & 8.5 & -18.917 & 8 \\ 0 & -0.333 & 5.333 & -6.944 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 520 \\ -507.385 \\ -279.271 \\ -237.845 \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0.982 & 0.653 \\ 0 & 1 & 0.018 & 0.347 \\ 0 & 0 & -2.244 & 0.594 \\ 0 & 0 & -16.018 & 20.486 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 344.191 \\ 175.809 \\ -208.664 \\ 537.739 \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0.913 \\ 0 & 1 & 0 & 0.352 \\ 0 & 0 & 1 & -0.265 \\ 0 & 0 & 0 & -1.014 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 252.877 \\ 174.135 \\ 92.988 \\ -126.559 \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 138.925 \\ 130.202 \\ 126.063 \\ 124.811 \end{Bmatrix}$$

$q_1 =$	139.925	[kN/m^2]
$q_2 =$	130.202	[kN/m^2]
$q_3 =$	126.063	[kN/m^2]
$q_4 =$	124.811	[kN/m^2]

10.3.3.6 Determining settlements s_i

The settlement s_i can be given by:

$$s_i = \frac{q_i}{k_i} = \frac{q_i}{20000} \text{ [m]}$$

$s_1 = 0.70$ [cm]
$s_2 = 0.65$ [cm]
$s_3 = 0.63$ [cm]
$s_4 = 0.62$ [cm]

The contact pressure distribution, settlement, moment and shear force diagrams for the raft are shown in Figure 10.3 to Figure 10.6. Once the internal forces are obtained at various sections, the design of the raft can be completed in the normal manner.

10.3.3.7 Computer calculation

The input data and results of *GEO Tools* are presented on the pages 10.15 to 10.26. By comparison, one can see an agreement with the hand calculation.

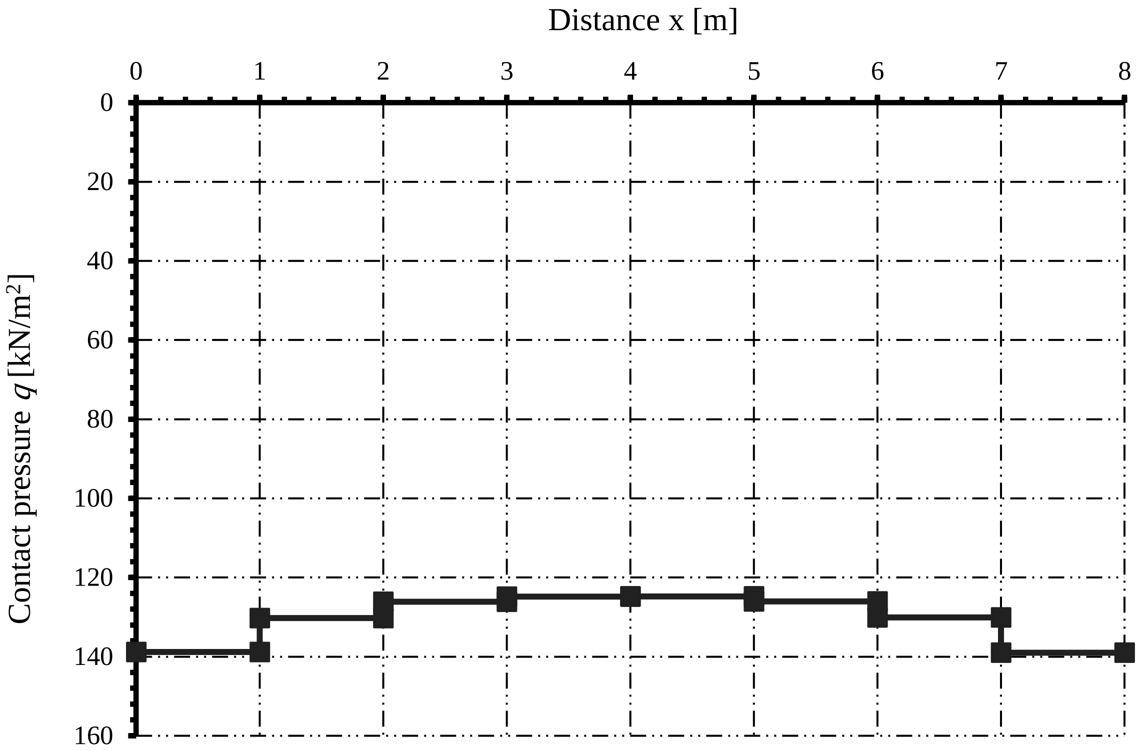


Figure 10.3 Contact pressures

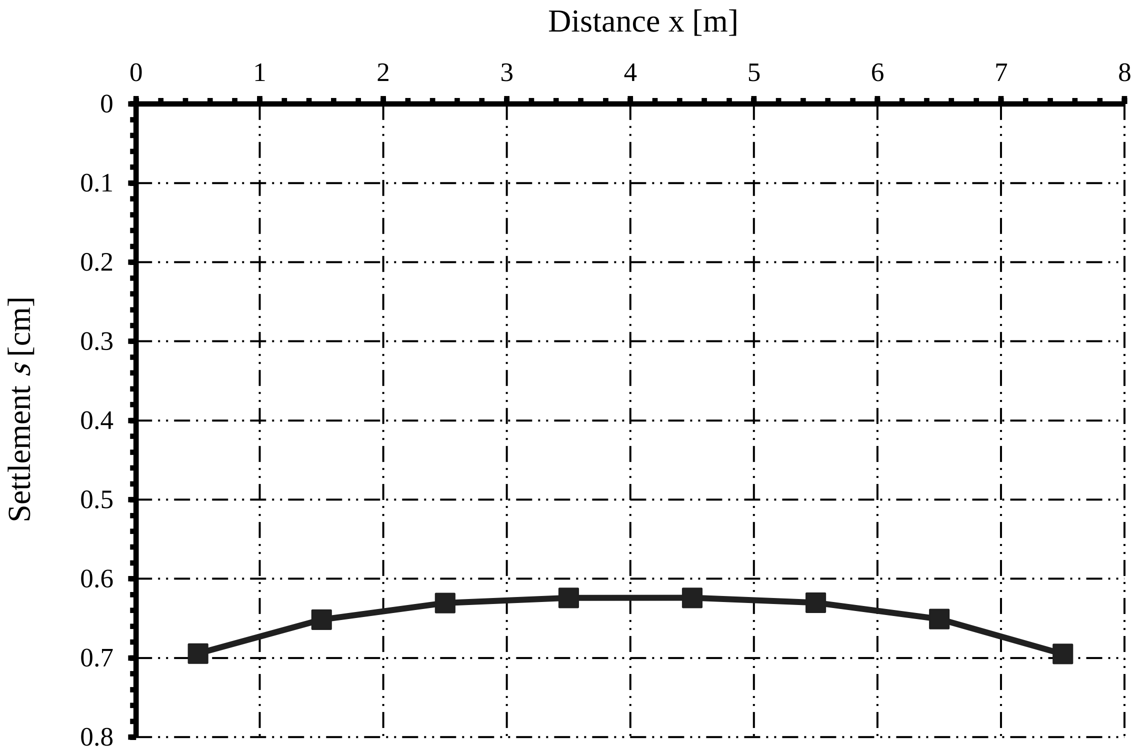


Figure 10.4 Settlements

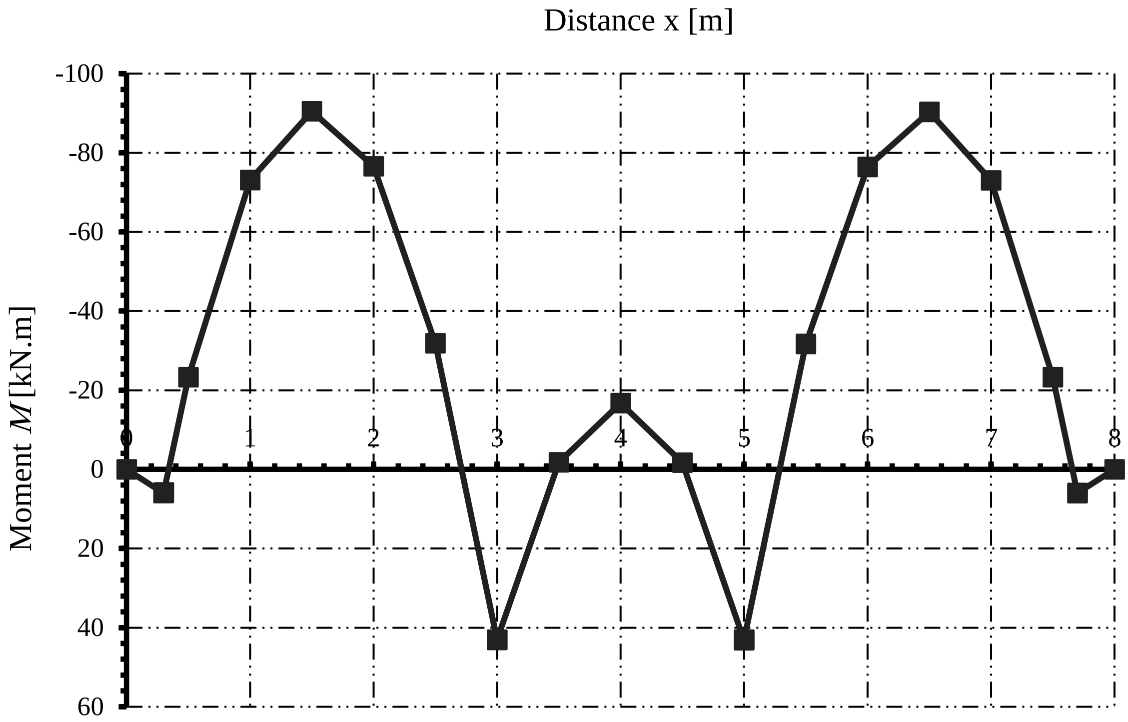


Figure 10.5 Moments

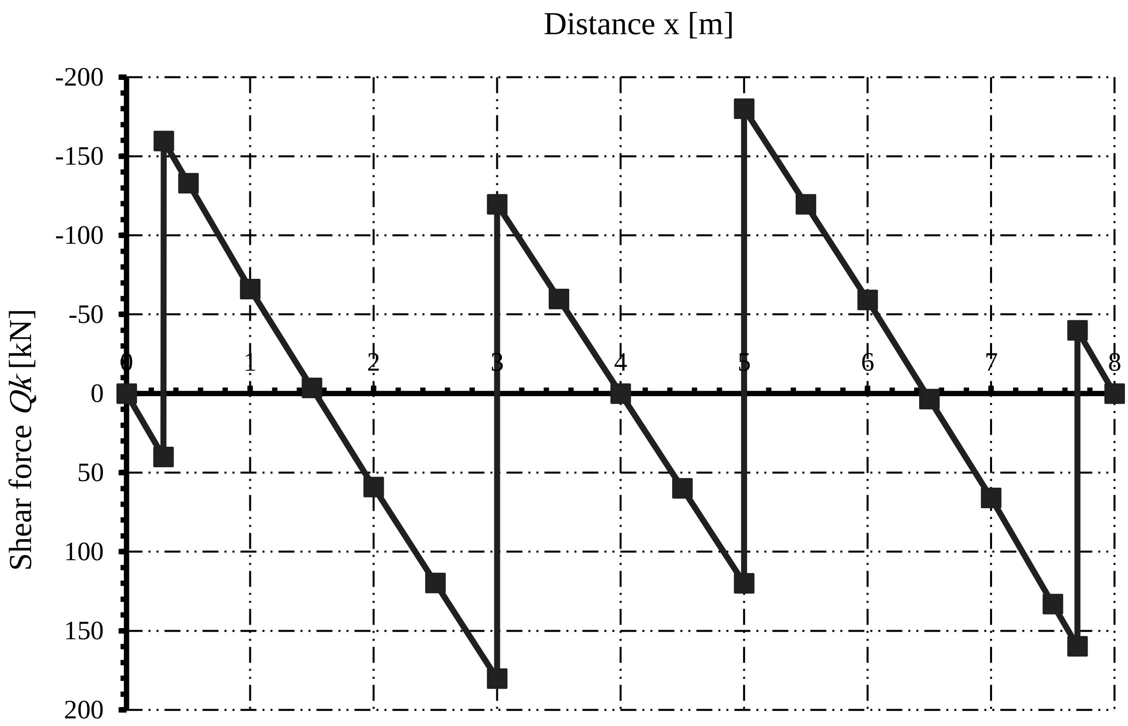


Figure 10.6 Shear forces

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*************************************************************
                        GEO Tools
                        Version 12.3
          Program authors M. El Gendy/ A. El Gendy
*************************************************************
Title: Beam Foundations after Kany and El Gendy
Date: 05-05-2022
Project: Analysis of a raft for four equal walls
File: Raft 4Walls

-------------------------------------------------------------
Analysis of beam a foundation
Calculation method: Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy
-------------------------------------------------------------

Data:

Main Soil Data:

Groundwater depth under the ground surface   Tw           [m]       = 1.00
Foundation depth under ground surface        Tf           [m]       = 2.00

Summary of loading:

Self weight                                  Pe           [kN]      = 120
Load on Footing                              Pa           [kN]      = 920
Groundwater force                            Pw           [kN]      = 80
Total load                                   Po=Pe+Pa-Pw  [kN]      = 1040

Groundwater pressure                         Qw           [kN/m2]   = 10.0
Average soil pressure                        Qo           [kN/m2]   = 130.0

Beam Material:

Modulus of elasticity of the concrete        Eb           [kN/m2]   = 20000000.00
Unit weight of footing concrete              γb           [kN/m3]   = 25.0

Dimensions:

Depth of the foundation surface under ground Tk           [m]       = 1.40
Beam thickness                               d            [m]       = 0.60
Moment of inertia of the beam                I            [m4]      = 0.018
Beam stiffness                               αB           1/[kN/m3] = 2.78E-06
Beam length (longitudinal)                   A            [m]       = 8.00
Beam width (transversal)                     B            [m]       = 1.00
Length/width ratio                           A/B          [-]       = 8.00
Element size                                 a            [m]       = 1.00
Number of elements of the beam               N            [-]       = 8

Loads:

Point Loads:
-----------------------------------------------------------
No.     Load   Load position    Column    Column    Column
       value        from the      side      side     label
                   left edge
  I        P              Xp         a         b        Lb
[-]     [kN]             [m]       [m]       [m]       [-]
-----------------------------------------------------------
  1      200            0.30      0.20      1.00        W1
  2      300            3.00      0.20      1.00        W2
  3      300            5.00      0.20      1.00        W3
  4      200            7.70      0.20      1.00        W4
-----------------------------------------------------------

Distributed Loads:
-----------------------------------------------------------
No.      Load   Load start from   Load end from   Load type
        value     the left edge   the left edge
  I        Pf               Xpl             Xpr
[-]   [kN/m2]               [m]             [m]         [-]
-----------------------------------------------------------
  1       -10              0.00            8.00  (Groundwater pressure)
  2        15              0.00            8.00  (Self weight)
-----------------------------------------------------------
```

Right sides of the system of equations:

Element No. I [-]	Right sides of the system of equations Rv [m]
1	4.16E+03
2	1.04E+03
3	6.658E-04
4	1.3374E-03
5	2.3513E-03
6	3.7957E-03
7	5.6707E-03
8	7.9068E-03

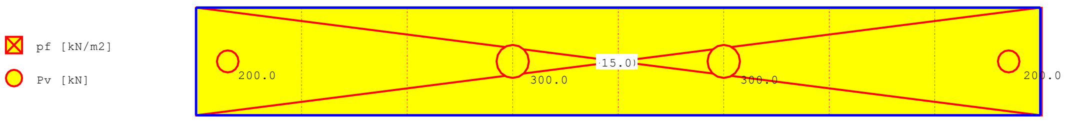
Settlements/ Contact pressures/ Moduli of subgrade reactions:

Element No. I [-]	Contact pressure q [kN/m2]	Settlement s [cm]	Modulus of subgrade reaction ks [kN/m3]
1	138.9	0.69	20000
2	130.3	0.65	20000
3	126.1	0.63	20000
4	124.8	0.62	20000
5	124.8	0.62	20000
6	126.0	0.63	20000
7	130.1	0.65	20000
8	139.0	0.70	20000

Moments/ Shear Forces:

No. I [-]	Distance x [m]	Distance [-]	Moment Mm [kN.m]	Shear force Qk [kN/m]
1	0.00		0.0	0.0
2	0.20	CL	2.7	26.8
3	0.30	CC	6.0	40.2
4	0.30	CC	5.9	-159.7
5	0.40	CR	-9.3	-146.5
6	0.50		-23.3	-133.1
7	1.00		-73.1	-66.1
8	1.50		-90.5	-3.5
9	1.53	MM	-90.5	0.0
10	2.00		-76.6	59.1
11	2.50		-31.9	119.7
12	2.90	CL	25.7	168.1
13	3.00	CC	43.1	180.2
14	3.00	CC	43.0	-119.6
15	3.10	CR	31.7	-107.8
16	3.50		-1.8	-59.9
17	4.00	MM	-16.7	0.0
18	4.00		-16.7	0.1
19	4.50		-1.7	60.0
20	4.90	CL	31.8	107.9
21	5.00	CC	43.2	119.9
22	5.00	CC	43.0	-180.0
23	5.10	CR	25.8	-168.0
24	5.50		-31.7	-119.6
25	6.00		-76.4	-59.1
26	6.47	MM	-90.4	0.0
27	6.50		-90.3	3.4
28	7.00		-73.0	66.0
29	7.50		-23.2	133.0
30	7.60	CL	-9.3	146.4
31	7.70	CC	6.0	159.8
32	7.70	CC	6.0	-40.1
33	7.80	CR	2.7	-26.8
34	8.00		0.0	0.0

Analysis of beam foundation
Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



System of loading

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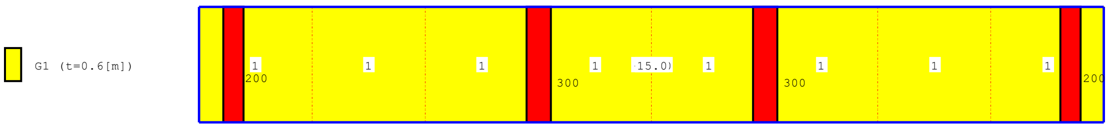
File: Raft 4Walls	Date: 05-05-2022
Page No.:	Project: Analysis of a raft for four equal walls

Analysis of beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Element groups t [m]

No. of element groups = 1

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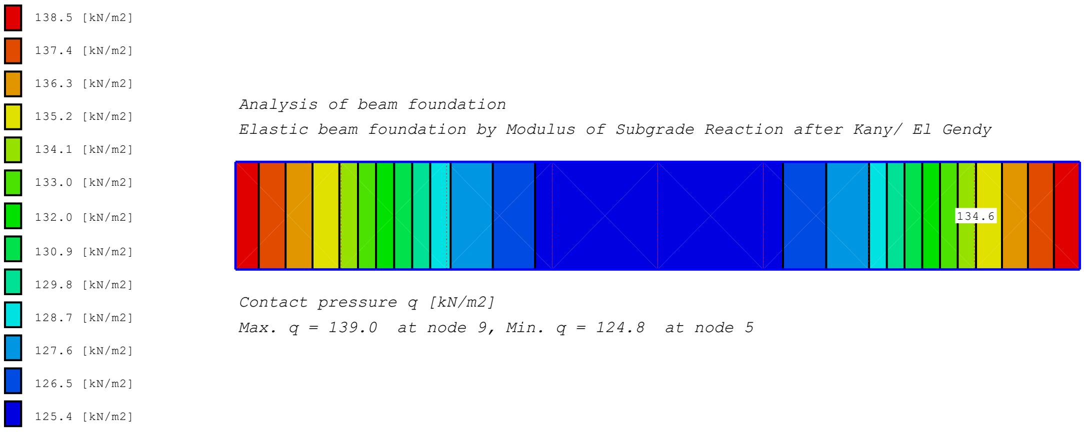

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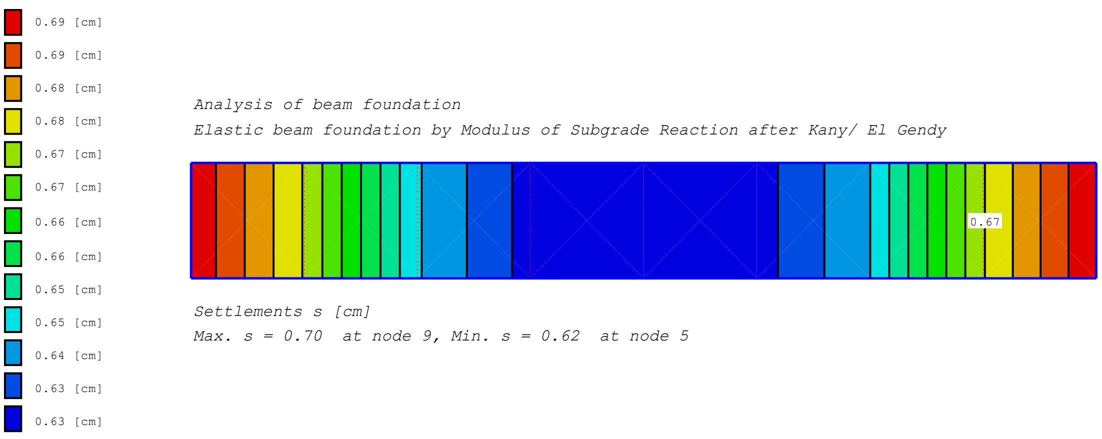

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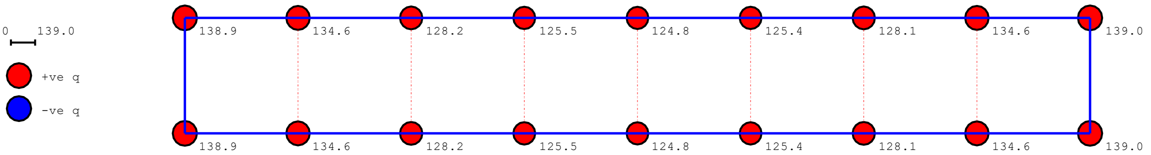
Analysis of beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Contact pressure q [kN/m2]

Max. q = 139.0 at node 9, Min. q = 124.8 at node 5

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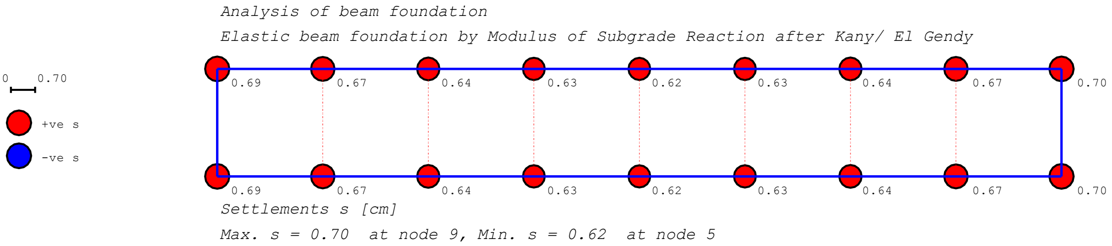
Scale: 36	**Title: Beam Foundations after Kany and El Gendy**
File: Raft 4Walls	**Date: 05-05-2022**
Page No.:	**Project: Analysis of a raft for four equal walls**



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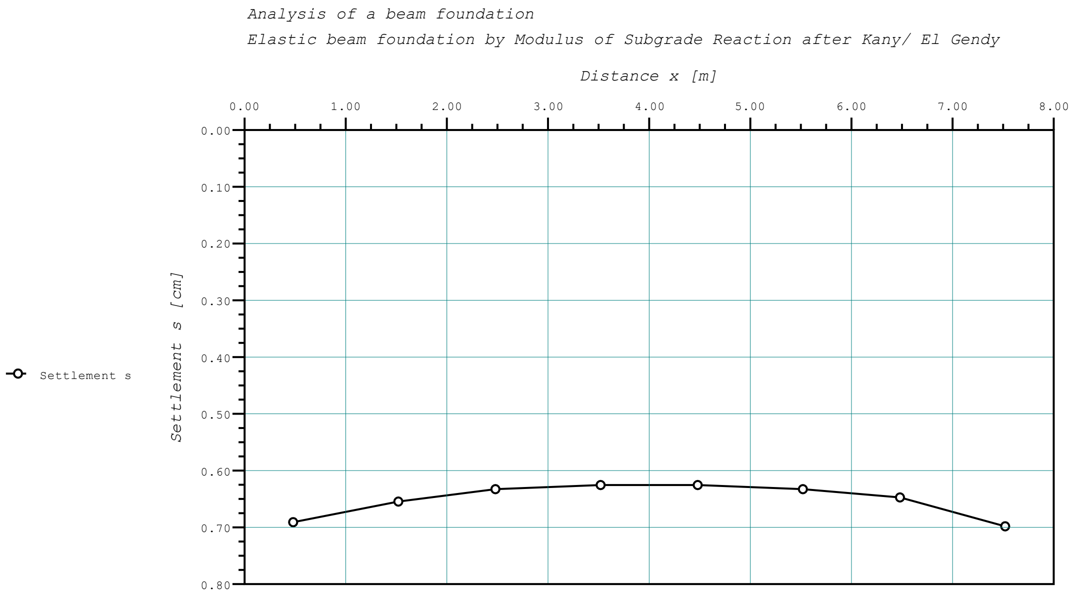
Scale: 38	**Title: Beam Foundations after Kany and El Gendy**
File: Raft 4Walls	**Date: 05-05-2022**
Page No.:	**Project: Analysis of a raft for four equal walls**



Settlements

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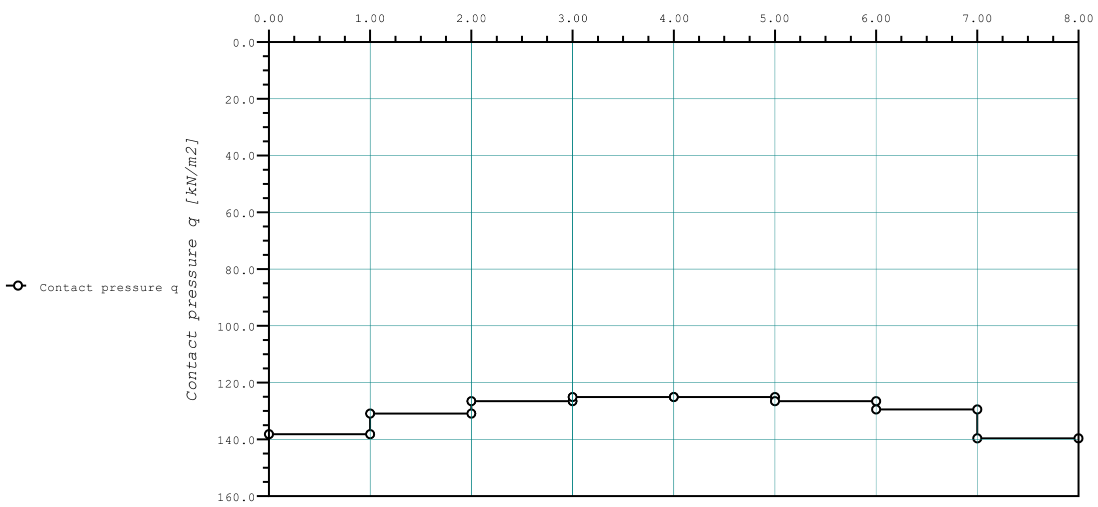
Scale: 40	Title: Beam Foundations after Kany and El Gendy
File: Raft 4Walls	Date: 05-05-2022
Page No.:	Project: Analysis of a raft for four equal walls

Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Contact pressure

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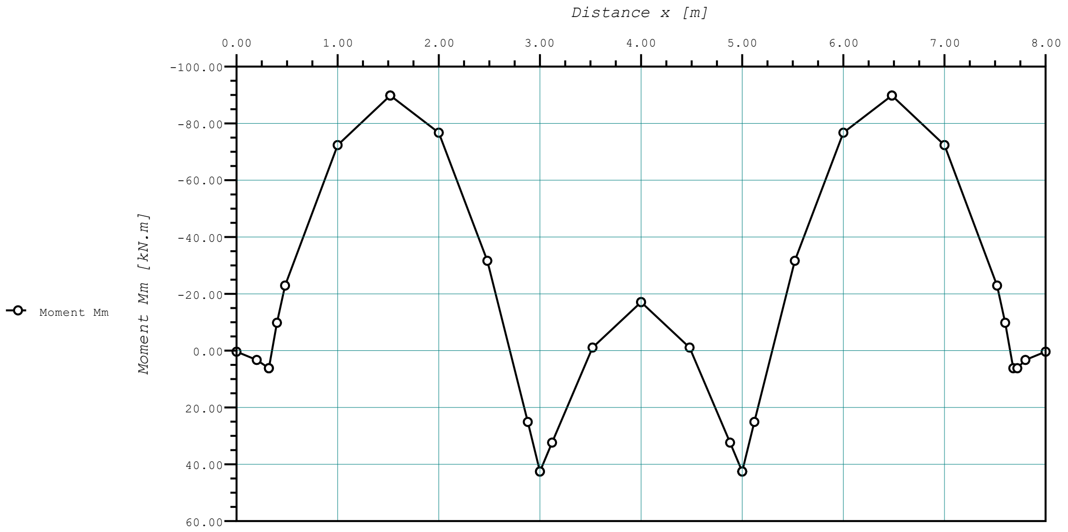
Scale: 40	Title: Beam Foundations after Kany and El Gendy
File: Raft 4Walls	Date: 05-05-2022
Page No.:	Project: Analysis of a raft for four equal walls

Analysis of a beam foundation
Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Moments

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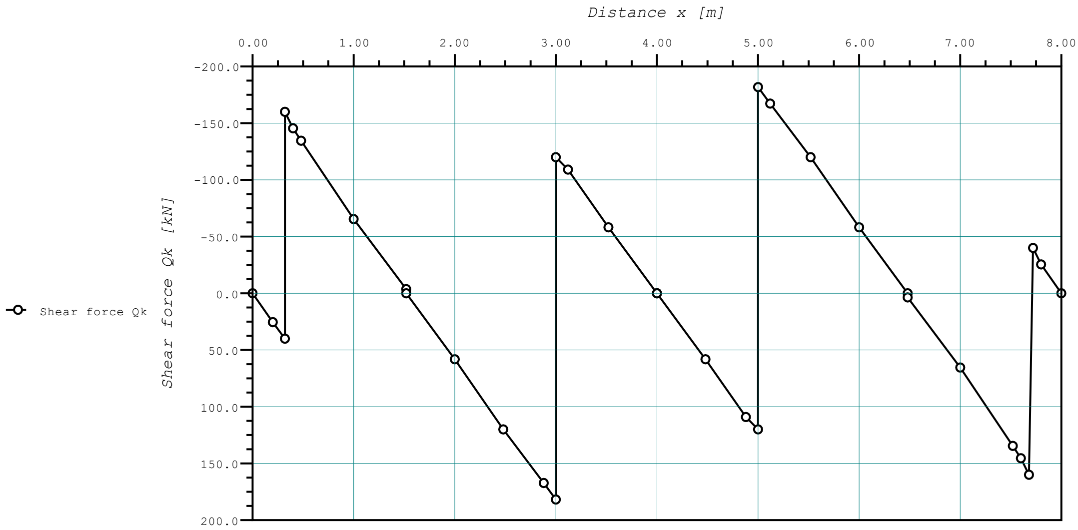
Scale: 40	**Title: Beam Foundations after Kany and El Gendy**
File: Raft 4Walls	**Date: 05-05-2022**
Page No.:	**Project: Analysis of a raft for four equal walls**

Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Shear Forces

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Scale: 40	**Title: Beam Foundations after Kany and El Gendy**
File: Raft 4Walls	**Date: 05-05-2022**
Page No.:	**Project: Analysis of a raft for four equal walls**

10.4 Example 2: Analysis of a bottom slab for an aqueduct

10.4.1 Description of the problem

Figure 10.7 shows a cross-section of a concrete aqueduct filled with water. It is required to find the contact pressure distribution and the settlement under the bottom slab. The loading and the bottom slab are symmetrical.

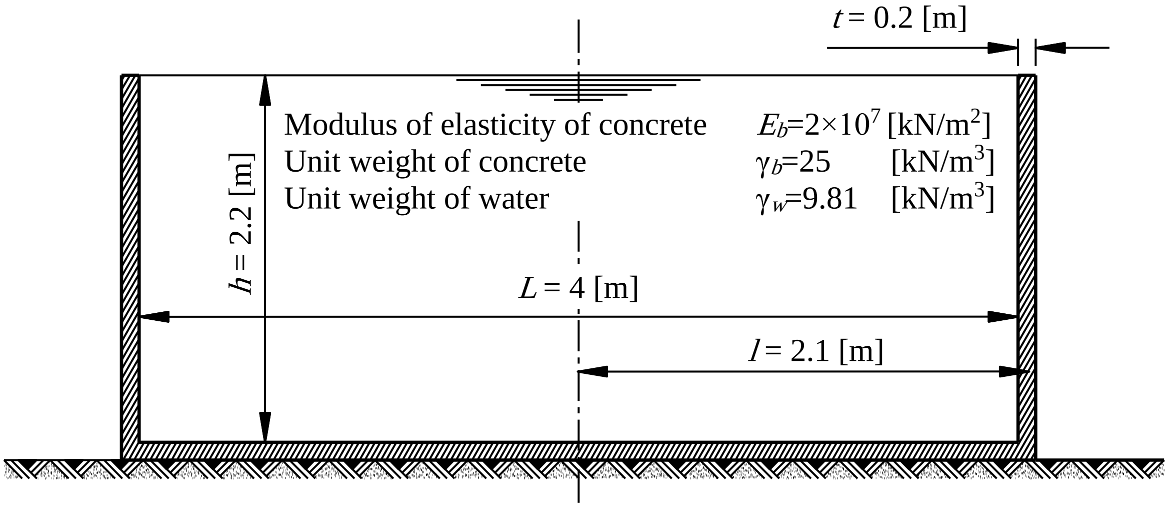


Figure 10.7 Cross-section of the aqueduct with dimensions

Geometry:
The bottom slab and the wall of the aqueduct have a thickness of d = 0.2 [m]. The cross section dimensions of the aqueduct are = 4.2 [m] ×2.2 [m].

Material properties of the concrete and the water

Modulus of elasticity of the concrete	$E_b = 2\times10^7$	[kN/m^2]
Unit weight of the concrete	γ_b=25	[kN/m^3]
Unit weight of the water	γ_w=9.81	[kN/m^3]

Soil properties
Modulus of subgrade reaction of the soil k_s=50000 [kN/m^3].

10.4.2 Preparing the calculation

The bottom slab can be regarded as a beam on elastic foundation subjected to:

- A uniformly distributed loading p_f equal to the weight of the bottom slab itself plus the weight of the water.
- Two concentrated forces P_1 and P_2 due to the weight of the sidewalls.
- Two moments M_l and M_r due to the water pressure on the walls.

Computing the loads on the bottom slab

Own weight of the bottom slab	$w_o = \gamma_b \times d = 25 \times 0.2$	=5	[kN/m^2]
Own weight of the water	$w_w = \gamma_w \times h = 9.81 \times 2.2$	=21.582	[kN/m^2]
Total	$p_f =$	=26.582	[kN/m^2]

Own weight of the wall $P_1 = P_2 = \gamma_b \times d \times h = 25 \times 0.2 \times 2.3 = 11.5$ [kN/m]

Moment due to water pressure $M_{rl} = M_{rr} = -\gamma_w \times h^3/6 = -9.81 \times 2.3^3/6 = -17.41$ [kN.m/m]

Assume one-meter strip width from the bottom slab and consider it as a beam on elastic foundation. The beam is divided into eight equal elements, each 0.525 [m] long (Figure 10.8). Because of the symmetry of the system, the analysis can be carried out by considering only half of the beam. Hence, the total number of equations is reduced to four.

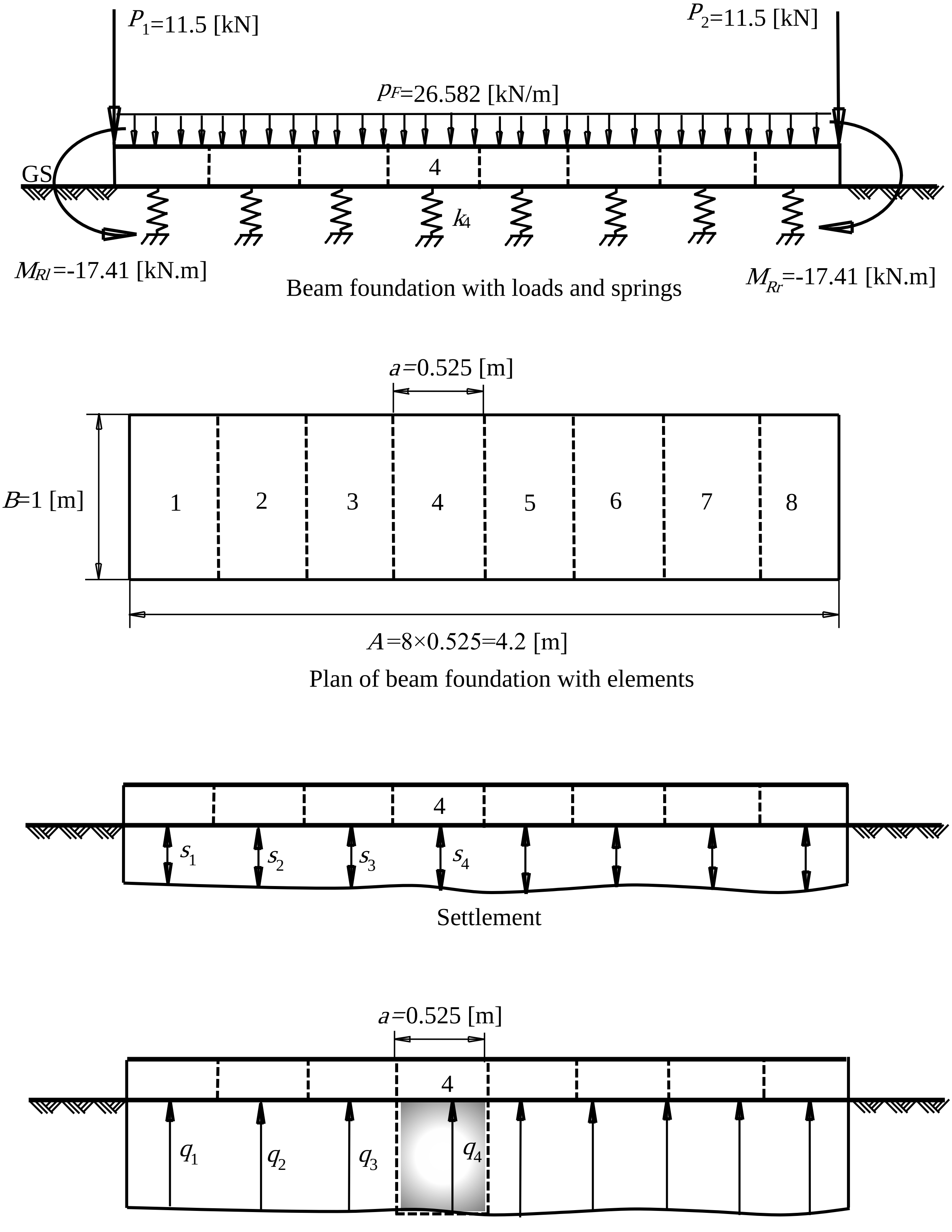


Figure 10.8 One meter strip width of the foundation

10.4.3 Hand calculation

According to *Kany/ El Gendy* (1995), the analysis of a beam on elastic foundation is carried out in the following steps:

10.4.3.1 Moment of inertia I and beam stiffness α:

$$I = \frac{Bd_i^3}{12} = \frac{1 \times 0.2^3}{12} = 0.000667\,[\text{m}^4]$$

and

$$\alpha = \frac{a^4 B}{E_b I} = \frac{0.525^4 \times 1}{(2 \times 10^7)(0.000667)} = 5.7 \times 10^{-6}\ [\text{m}^3/\text{kN}]$$

10.4.3.2 Determining external moments $M_i^{(l)}$

The external moments $M_i^{(l)}$ at points 2, 3, 4 and 5 are:

$$M_1^{(l)} = 17.41\ \ [\text{kN.m}]$$

$$M_2^{(l)} = 17.41 + 11.5 \times 1.50. \times 525 + 26.582 \frac{(1.5 \times 0.525)^2}{2} = 34.71\ [\text{kN.m}]$$

$$M_3^{(l)} = 17.14 + 11.5 \times 2.50. \times 525 + 26.582 \frac{(2.5 \times 0.525)^2}{2} = 55.40\ [\text{kN.m}]$$

$$M_4^{(l)} = 17.41 + 11.5 \times 3.50. \times 525 + 26.582 \frac{(3.5 \times 0.525)^2}{2} = 83.42\ [\text{kN.m}]$$

$$M_5^{(l)} = 17.41 + 11.5 \times 4.50. \times 525 + 26.582 \frac{(4.5 \times 0.525)^2}{2} = 118.76\ [\text{kN.m}]$$

10.4.3.3 Determining the right hand side R_i

The right hand side R_i of the contact pressure equation is:

$$R_i = \left(u_i M^{(l)}{}_{i\text{-}1} + v_i M^{(l)}{}_i + w_i M^{(l)}{}_{i+1}\right)\frac{a^2}{6E\,I_i}$$

$$R_i = \left(M^{(l)}{}_{i\text{-}1} + 4\,M^{(l)}{}_i + M^{(l)}{}_{i+1}\right)\frac{0.525^2}{6 \times 2 \times 10^{70}. \times 0.000667}$$

$$R_i = 3.445 \times 10^{-6}\left(M^{(l)}{}_{i\text{-}1} + 4\,M^{(l)}{}_i + M^{(l)}{}_{i+1}\right)$$

Apply the above equation at points 2, 3 and 4:

$$R_2 = 3.445 \times 10^{-6}(17.1 + 4 \times 34.399 + 55.09) = 7.228 \times 10^{-4}$$

$$R_3 = 3.445 \times 10^{-6}(34.399 + 4 \times 55.09 + 83.107) = 1.164 \times 10^{-3}$$

$$R_4 = 3.445 \times 10^{-6}(55.09 + 4 \times 83.107 + 118.451) = 1.743 \times 10^{-3}$$

10.4.3.4 Determining contact pressures

The contact pressure equation is:

$$\left(\frac{1}{k}\right)q_{i+1} - \left(\frac{2}{k} - \frac{\alpha}{6}\right)q_i + \left(\frac{1}{k} + \alpha\right)q_{i-1} + \alpha\left(\sum_{j=1}^{i-2}(i-j)q_j\right) = R_i$$

$$\left(\frac{1}{50000}\right)q_{i+1} - \left(\frac{2}{50000} - \frac{5.7\times10^{-6}}{6}\right)q_i + \left(\frac{1}{50000} + 5.7\times10^{-6}\right)q_{i-1} + 5.7\times10^{-6}\left(\sum_{j=1}^{i-2}(i-j)q_j\right) = R_i$$

or

$$q_{i+1} - 1.953\ q_i + 1.285\ q_{i-1} + 0.285\left(\sum_{j=1}^{i-2}(i-j)q_j\right) = 50000\ R_i$$

Apply the above equation at points 2, 3 and 4:

$$q_3 - 1.953\ q_2 + 1.285\ q_1 = 36.45$$

$$q_4 - 1.953\ q_3 + 1.285\ q_2 + 0.57\ q_1 = 58.5$$

$$-0.953 q_4 + 1.285\ q_3 + 0.57\ q_2 + 0.855\ q_1 = 87.5$$

There are four unknown q_1, q_2, q_3, and q_4, so a farther equation is required. This can be obtained by considering the overall equilibrium of vertical forces.

$$a\times B(q_1+q_2+q_3+q_4+q_5+q_6+q_7+q_8) = P_1 + P_2 + A\times B\times P_f$$

or

$$q_1+q_2+q_3+q_4 = 128.23$$

Contact pressure equations in matrix form:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1.285 & -1.953 & 1 & 0 \\ 0.57 & 1.285 & -1.953 & 1 \\ 0.855 & 0.57 & 1.285 & -0.953 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 128.23 \\ 36.14 \\ 58.2 \\ 87.15 \end{Bmatrix}$$

Solving the above system of linear equations to obtain the contact pressures q_1, q_2, q_3, and q_4.

q_1 = 66.24 [kN/m^2]
q_2= 33.74 [kN/m^2]
q_3= 17.22 [kN/m^2]
q_4= 11.02 [kN/m^2]

10.4.3.5 Determining settlements s_i

The settlement s_i can be given by:

$$s_i = \frac{q_i}{k_i} = \frac{q_i}{50000} \text{ [m]}$$

$s_1 = 0.13$ [cm]
$s_2 = 0.07$ [cm]
$s_3 = 0.03$ [cm]
$s_4 = 0.02$ [cm]

The contact pressure distribution, settlement, moment and shear force diagrams for the raft are shown in Figure 10.9 to Figure 10.12. Once the internal forces are obtained at various sections, the design of the raft can be completed in the normal manner.

10.4.3.6 Computer calculation

The input data and results of *GEO Tools* are presented on the pages 10.35 to 10.46. By comparison, one can see an agreement with the hand calculation.

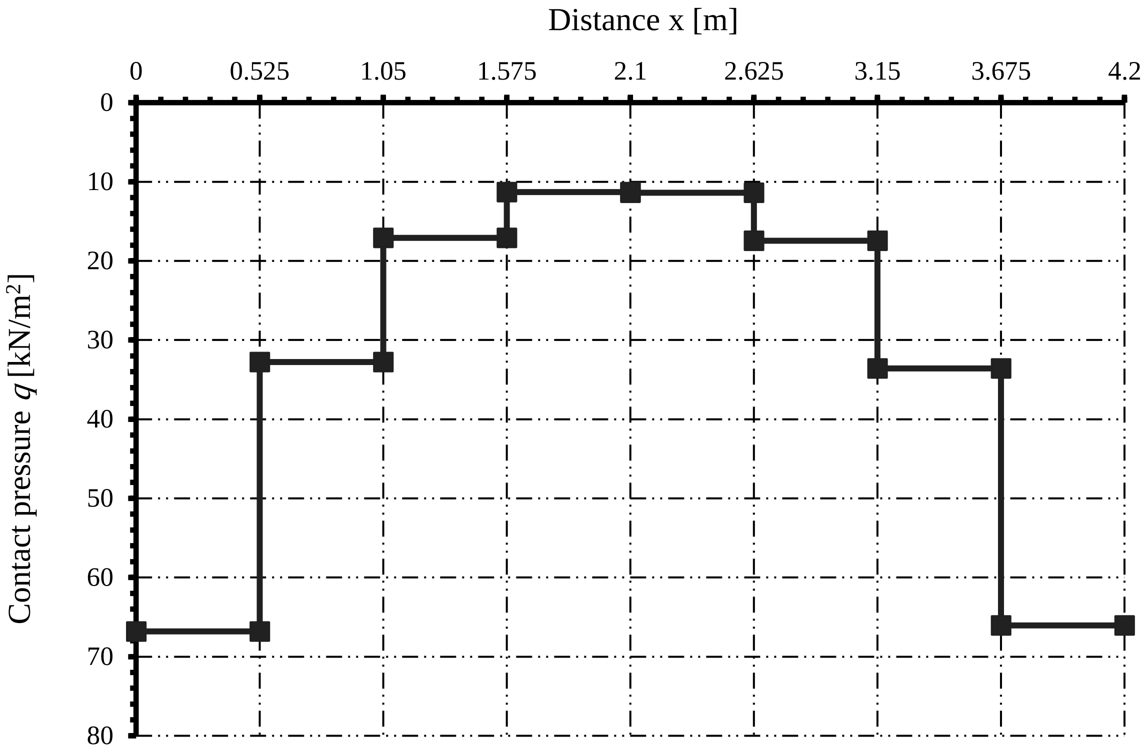


Figure 10.9 Contact pressures

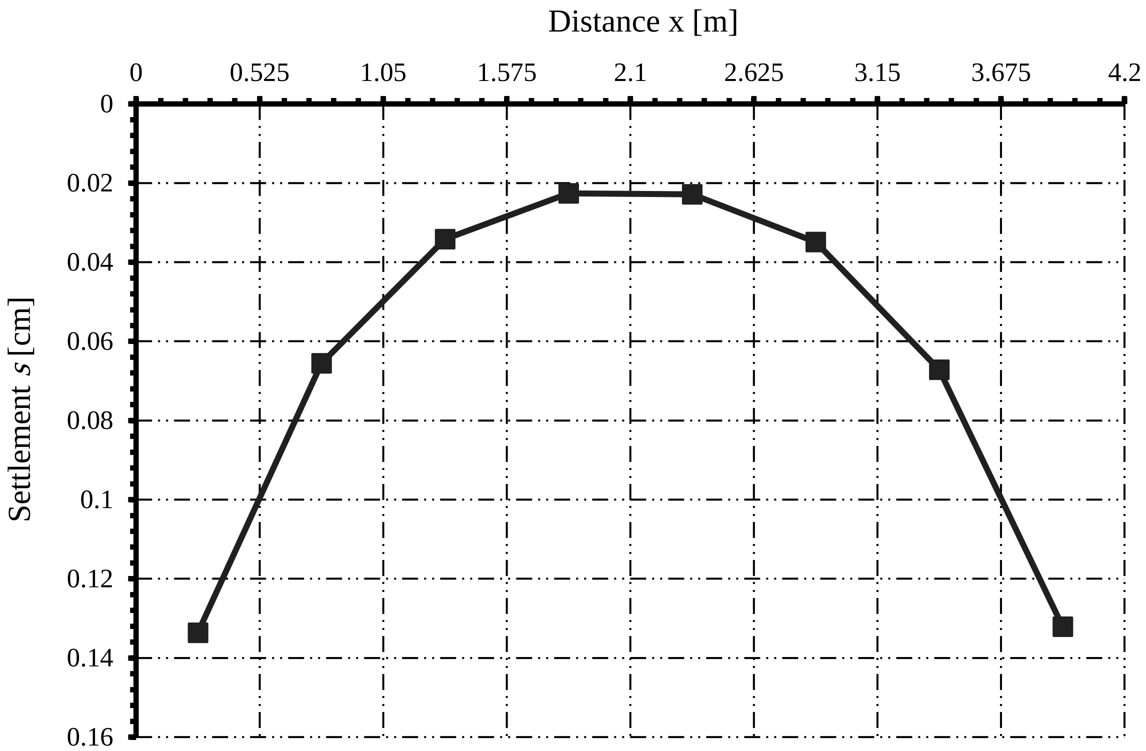


Figure 10.10 Settlements

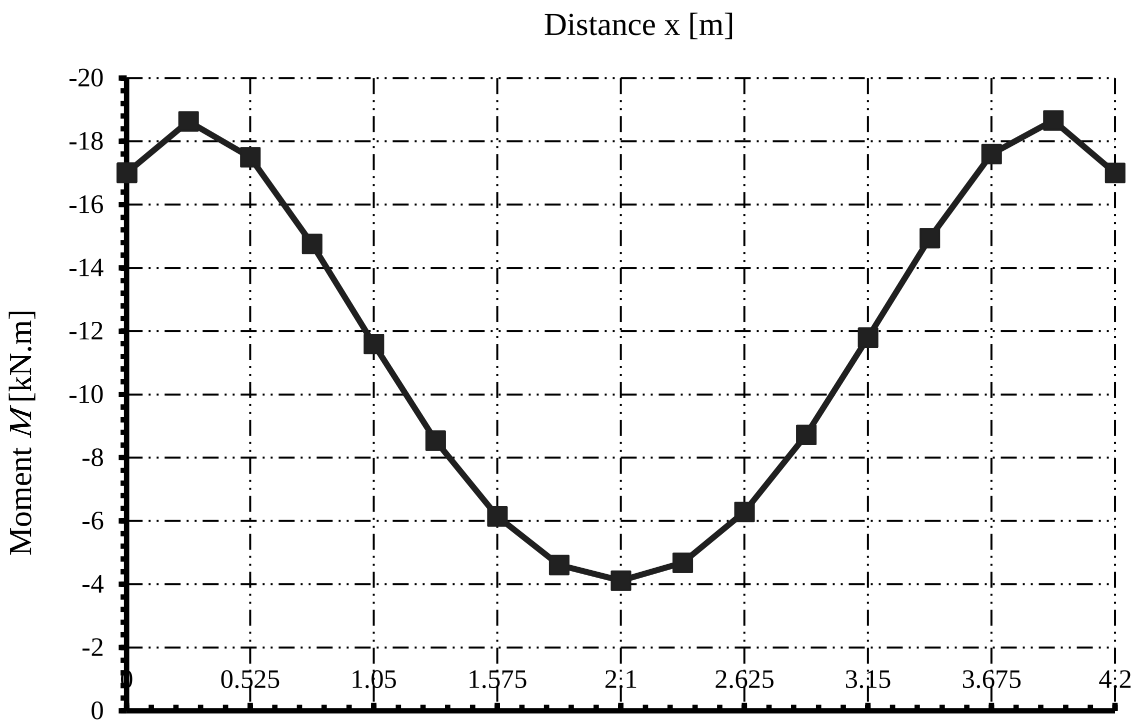


Figure 10.11 Moments

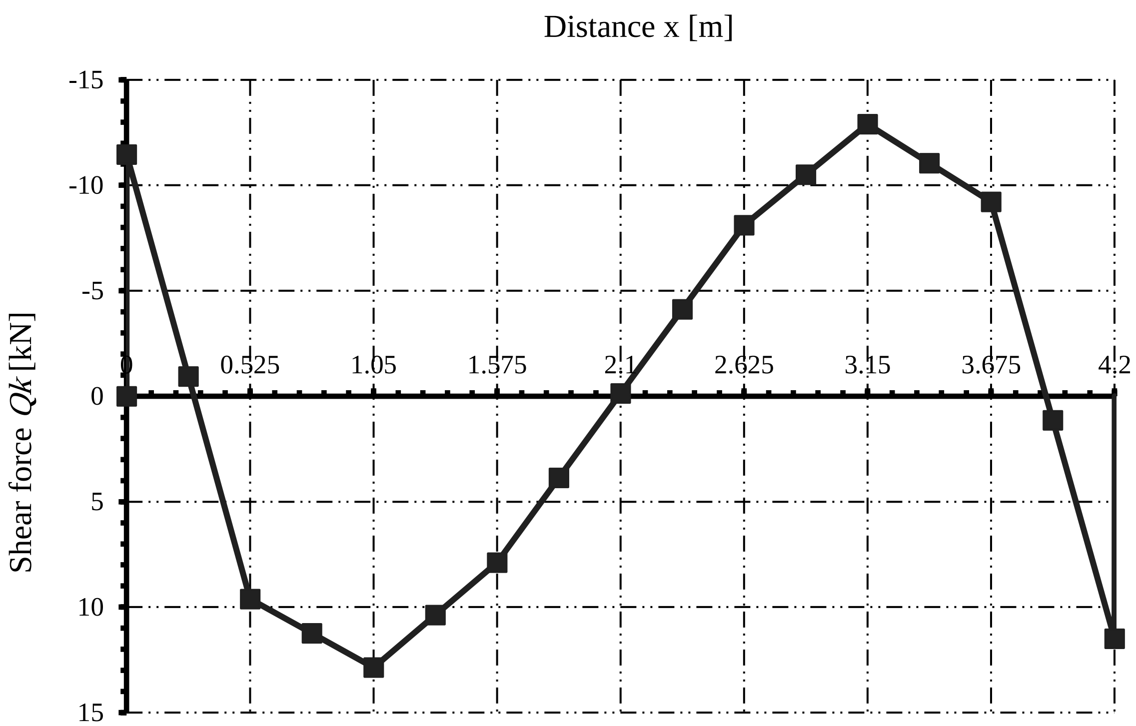


Figure 10.12 Shear forces

```
*************************************************************
                        GEO Tools
                        Version 12.3
          Program authors M. El Gendy/ A. El Gendy
*************************************************************
Title: Beam Foundations after Kany and El Gendy
Date: 05-05-2022
Project: Analysis of a bottom slab for an aqueduct
File: Aqueduct

-------------------------------------------------------------
Analysis of a beam foundation
Calculation method: Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy
-------------------------------------------------------------

Data:

Main Soil Data:

Groundwater depth under the ground surface   Tw           [m]        = 0.00
Foundation depth under ground surface        Tf           [m]        = 0.00

Summary of loading:

Self weight                                  Pe           [kN]       = 21.000
Load on Footing                              Pa           [kN]       = 113.644
Groundwater force                            Pw           [kN]       = 0.000
Total load                                   Po=Pe+Pa-Pw  [kN]       = 134.644

Groundwater pressure                         Qw           [kN/m2]    = 0.0
Average soil pressure                        Qo           [kN/m2]    = 32.1

Beam Material:

Modulus of elasticity of the concrete        Eb           [kN/m2]    = 20000000.00
Unit weight of footing concrete              γb           [kN/m3]    = 25.0

Dimensions:

Depth of the foundation surface under ground Tk           [m]        = -0.20
Beam thickness                               d            [m]        = 0.20
Moment of inertia of the beam                I            [m4]       = 0.000667
Beam stiffness                               αB           1/[kN/m3]  = 5.70E-06
Beam length (longitudinal)                   A            [m]        = 4.20
Beam width (transversal)                     B            [m]        = 1.00
Length/width ratio                           A/B          [-]        = 4.20
Element size                                 a            [m]        = 0.52
Number of elements of the beam               N            [-]        = 8

Loads:

Edge moments:

Edge moment left (clockwise)                 Mrl          [kN.m]     = -17.41
Edge moment right (counterclockwise)         Mrr          [kN.m]     = -17.41

Point Loads:
```

No. I [-]	Load value P [kN]	Load position from the left edge Xp [m]	Column side a [m]	Column side b [m]	Column label Lb [-]
1	11.500	0.00	0.20	1.00	W1
2	11.500	4.20	0.20	1.00	W2

Distributed Loads:

No. I [-]	Load value Pf [kN/m2]	Load start from the left edge Xpl [m]	Load end from the left edge Xpr [m]	Load type [-]
1	21.582	0.00	4.20	
2	5.000	0.00	4.20	(Self weight)

Right sides of the system of equations:

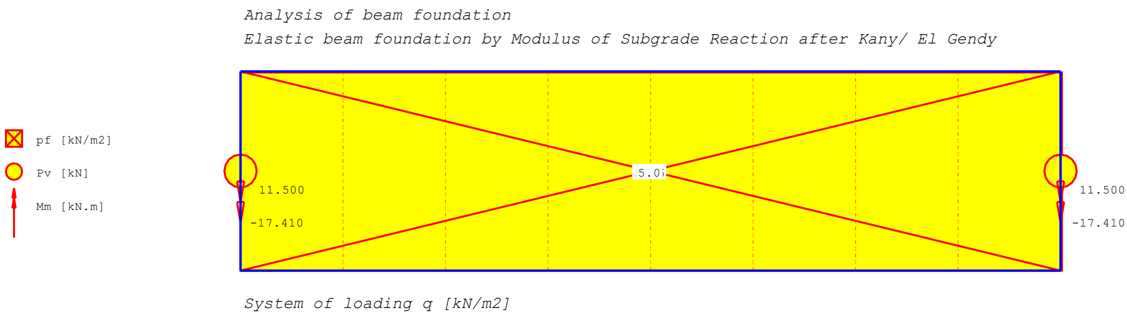
Element No. I [-]	Right sides of the system of equations Rv [m]
1	2.8275E+02
2	1.3464E+02
3	7.8916E-04
4	1.1705E-03
5	1.7496E-03
6	2.4803E-03
7	3.3623E-03
8	4.3959E-03

Settlements/ Contact pressures/ Moduli of subgrade reactions:

Element No. I [-]	Contact pressure q [kN/m2]	Settlement s [cm]	Modulus of subgrade reaction ks [kN/m3]
1	67.3	0.13	50000
2	32.7	0.07	50000
3	16.9	0.03	50000
4	11.0	0.02	50000
5	11.1	0.02	50000
6	17.3	0.03	50000
7	33.6	0.07	50000
8	66.5	0.13	50000

Moments/ Shear Forces:

No. I [-]	Distance x [m]	Distance [-]	Moment Mm [kN.m]	Shear force Qk [kN/m]
1	-0.10	CL	-17.07	-6.7
2	0.00	CC	-17.41	0.0
3	0.00	CC	-17.42	-11.5
4	0.10	CR	-18.36	-7.4
5	0.26		-19.02	-0.8
6	0.52		-17.83	9.9
7	0.79		-15.02	11.5
8	1.05		-11.79	13.1
9	1.31		-8.68	10.6
10	1.57		-6.23	8.0
11	1.84		-4.66	3.9
12	2.09	MM	-4.17	0.0
13	2.10		-4.17	-0.1
14	2.36		-4.74	-4.2
15	2.63		-6.37	-8.3
16	2.89		-8.86	-10.7
17	3.15		-12.00	-13.2
18	3.41		-15.21	-11.3
19	3.67		-17.94	-9.5
20	3.94		-19.05	1.0
21	4.10	CL	-18.36	7.5
22	4.20	CC	-17.41	11.5
23	4.20	CC	-17.41	0.0
24	4.30	CR	-17.41	0.0



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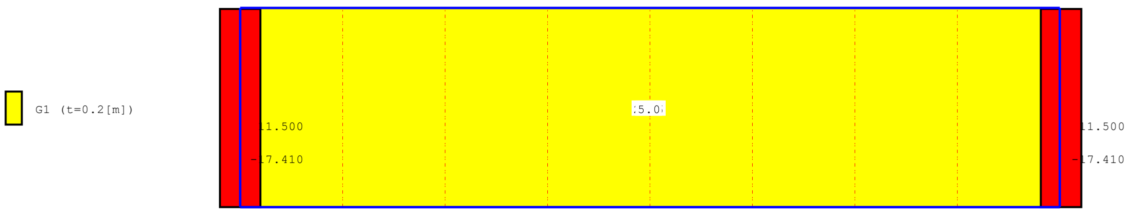
Scale: 21	Title: Beam Foundations after Kany and El Gendy
File: Aqueduct	Date: 05-05-2022
Page No.:	Project: Analysis of a bottom slab for an aqueduct

Analysis of beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Element groups t [m]

No. of element groups = 1

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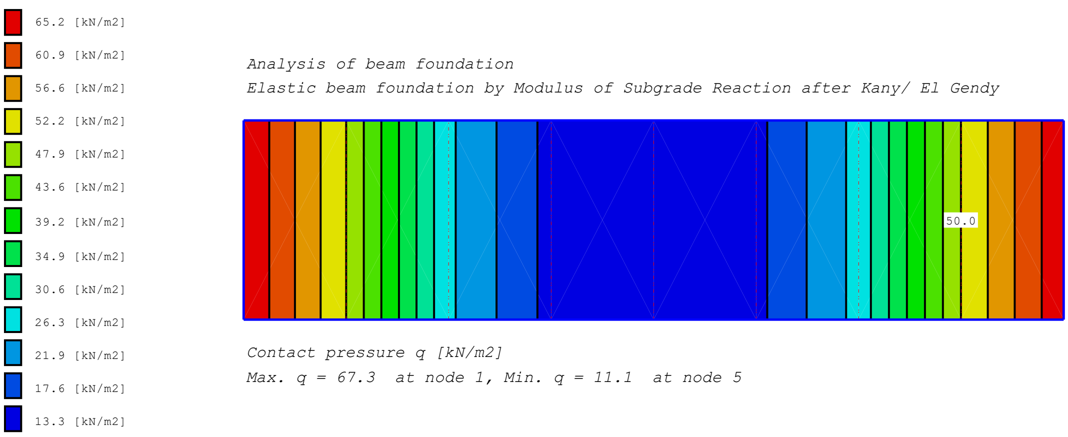

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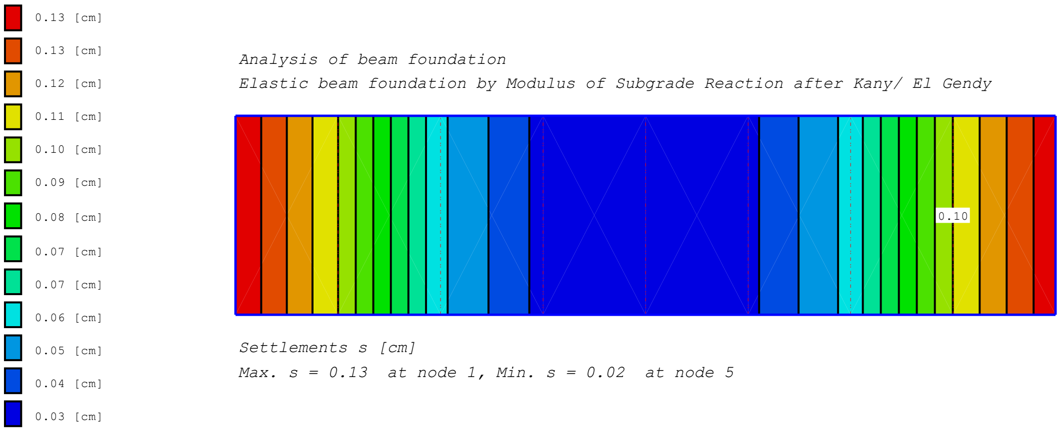

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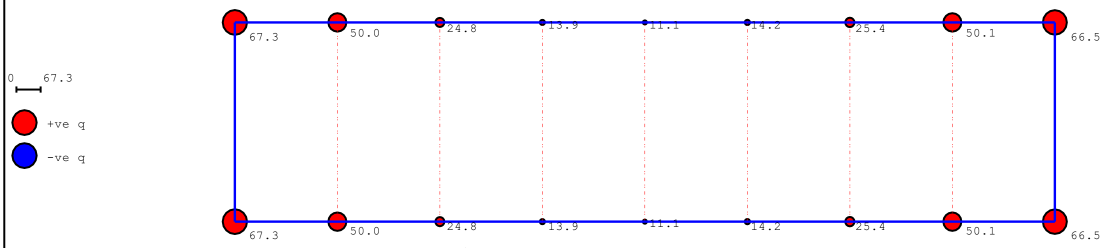

Contact pressure q [kN/m2]

Max. q = 67.3 at node 1, Min. q = 11.1 at node 5

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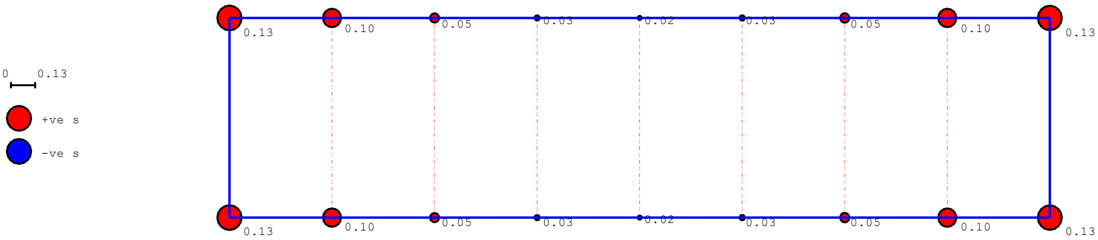
Analysis of beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Settlements s [cm]

Max. s = 0.13 at node 1, Min. s = 0.02 at node 5

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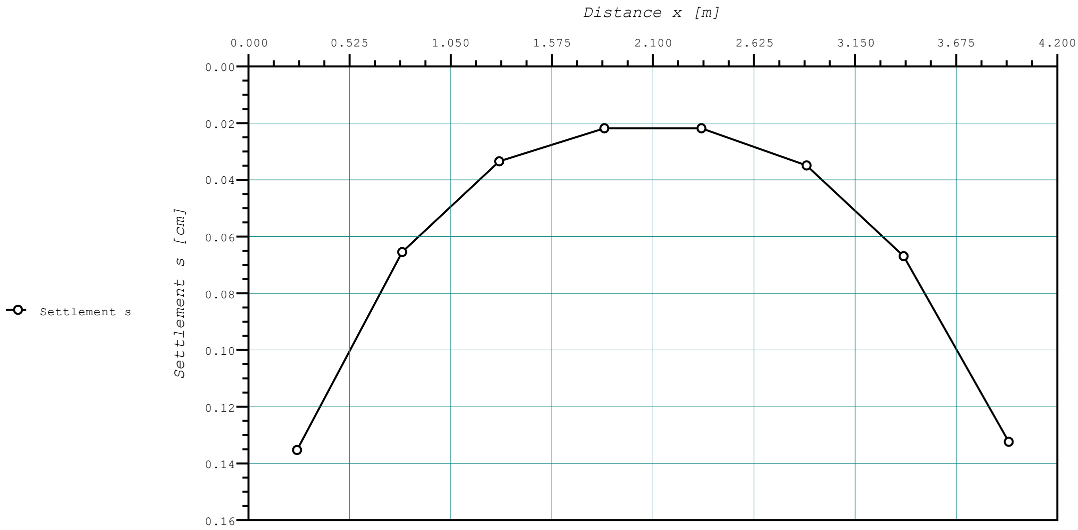
Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Settlements

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Scale: 21	**Title: Beam Foundations after Kany and El Gendy**
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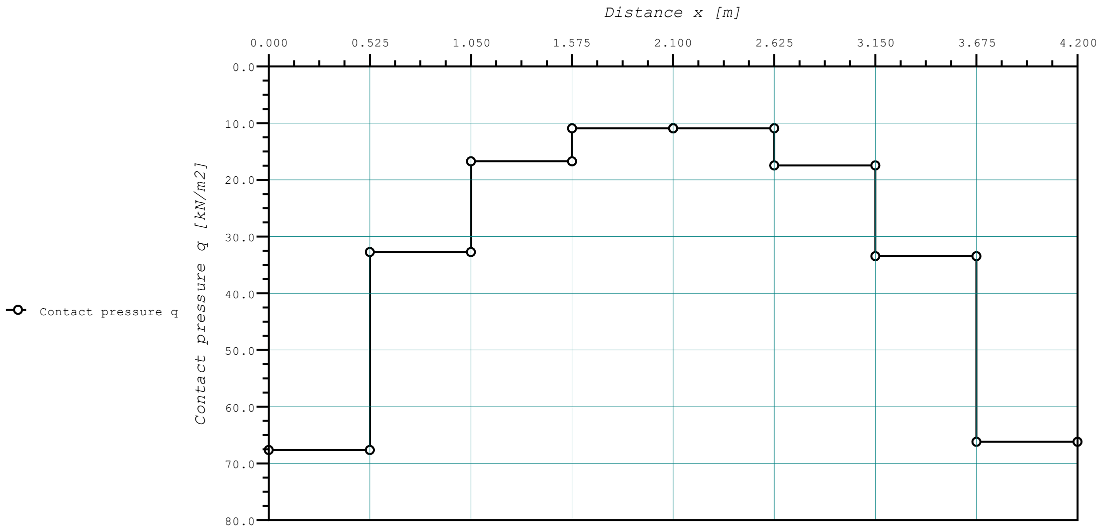
Analysis of a beam foundation
Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Contact pressure

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Analysis of a beam foundation
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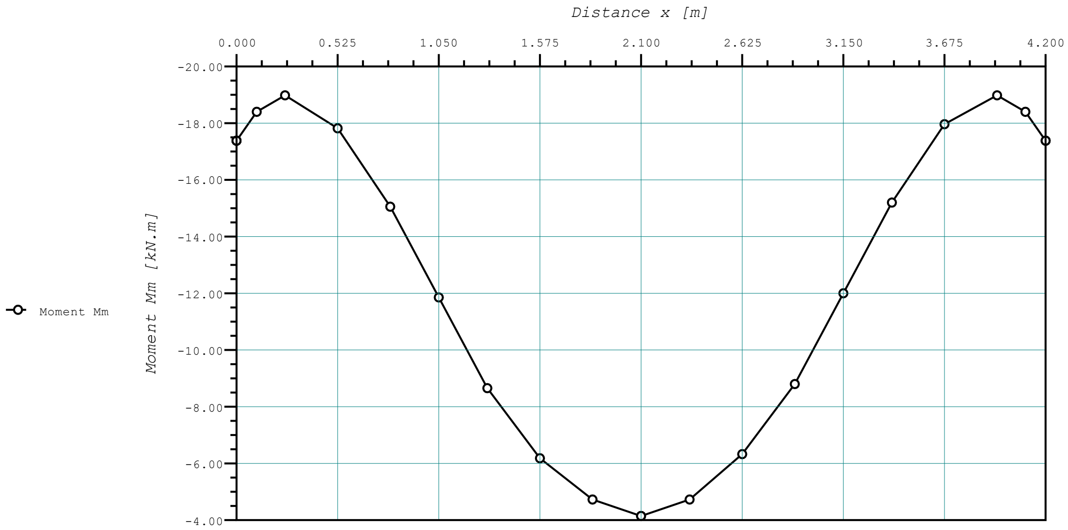

Moments

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Scale: 21	Title: Beam Foundations after Kany and El Gendy
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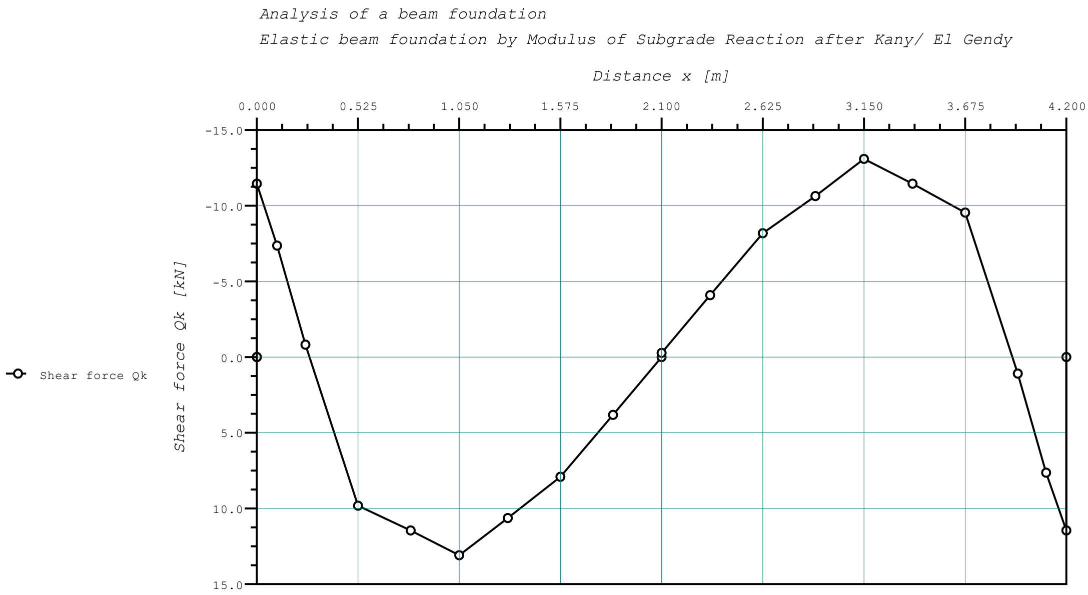

Shear Forces

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File: Aqueduct	Date: 05-05-2022
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10.5 Example 3: Analysis of a raft for two equal walls

10.5.1 Description of the problem

Figure 10.13 shows plan and section of a raft with dimensions and loads. The raft carries loads from two equal walls, each 800 [kN/m]. It is required to find the contact pressure distribution, settlements, moment and shear force diagrams for the raft. The loading and the raft are symmetrical.

Geometry:

Thickness of the raft	d	=0.5	[m]
Dimensions of the raft	A_f	=8×10	[m^2]
Groundwater depth under the ground surface	Tw	=1.75	[m]
Foundation depth under the ground surface	Tf	=2	[m]

Material properties of the concrete and unit weight of the water

Modulus of elasticity of the concrete	$E_b = 2\times10^7$	[kN/m^2]
Unit weight of the concrete	$\gamma_b=25$	[kN/m^3]
Unit weight of the water	$\gamma_w=10$	[kN/m^3]

Soil properties

Modulus of subgrade reaction of the soil k_s = 25000 [kN/m^3].

10.5.2 Preparing the calculation

The raft can be regarded as a beam on elastic foundation subjected to:

- A uniformly distributed loading p_f equal to the weight of the raft itself minus the uplift pressure from the ground water.
- Two concentrated forces from two walls $P_1 = P_2 = 800$ [kN/m].

Computing the uniform load on the raft

Own weight of the raft	$w_o = \gamma_b \times d = 25\times0.5$	=12.5	[kN/m^2]
Up lift pressure	$w_w = \gamma_w\times(T_f\text{-}Tw) = 10(2\text{-}1.75)$	=-2.5	[kN/m^2]
Total	$p_f =$	=10	[kN/m^2]

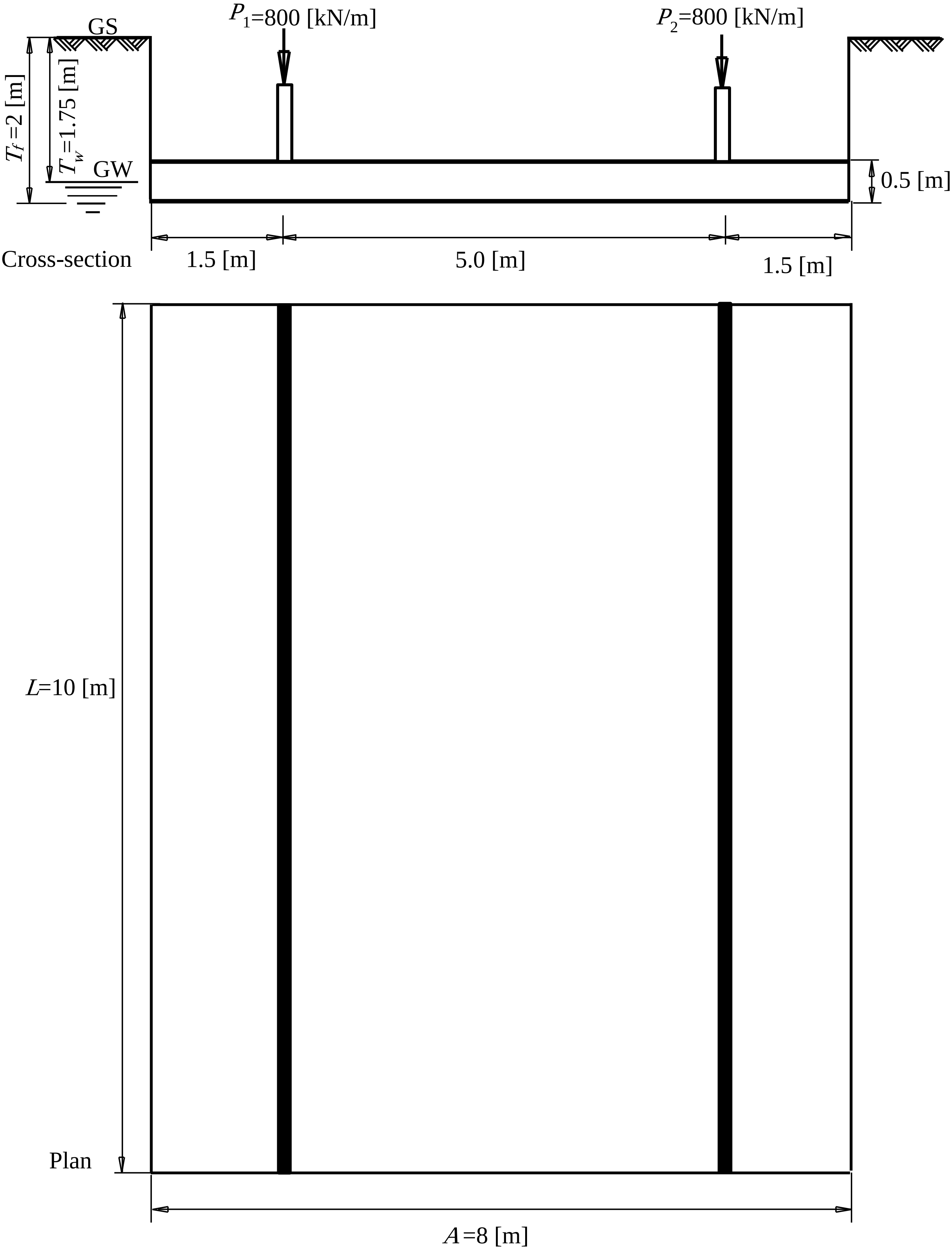


Figure 10.13 Raft with dimensions and loads from two equal walls

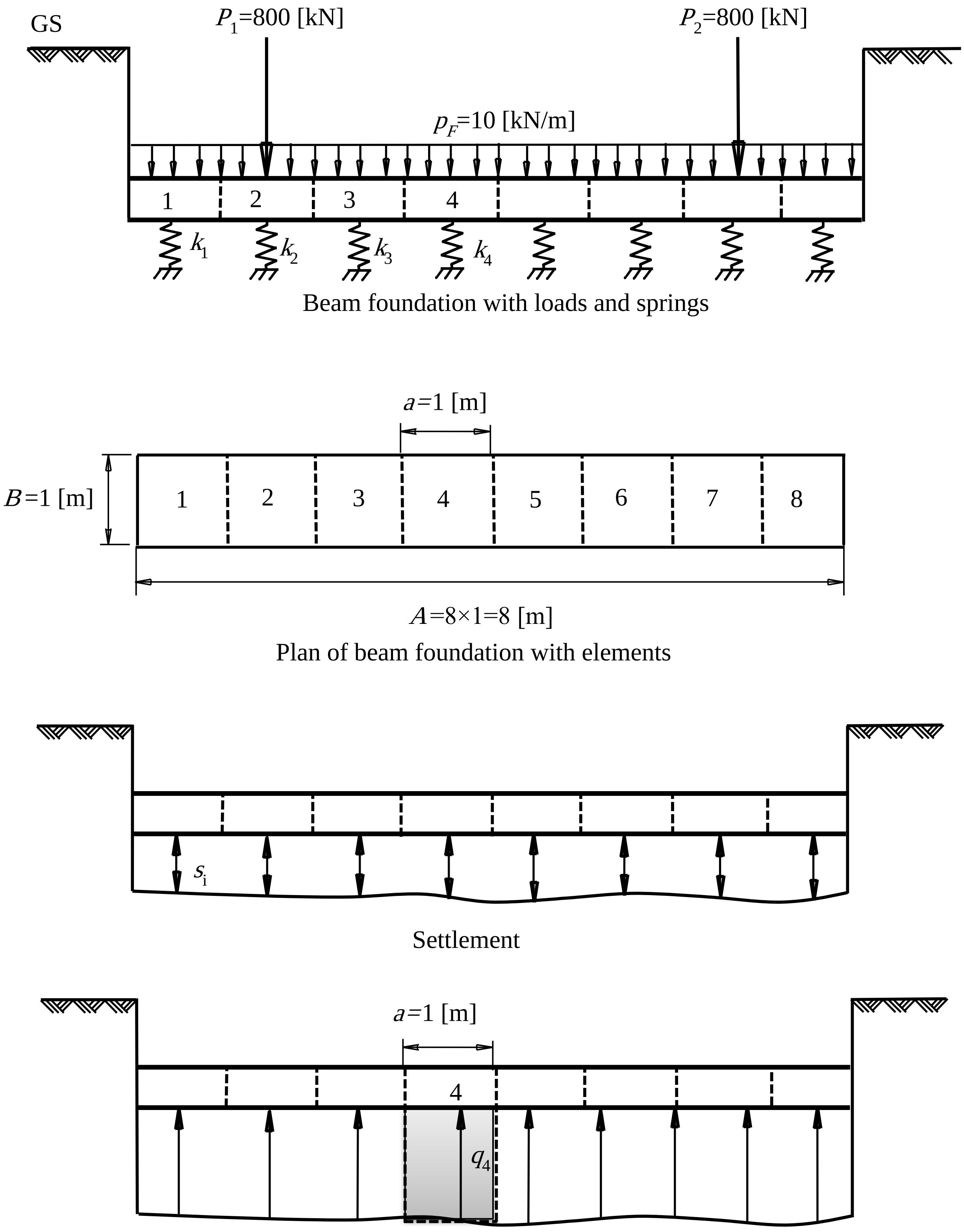


Figure 10.14 One meter strip width of the raft

10.5.3 Hand calculation

Assume one-meter strip width from the raft and consider it as a beam on elastic foundation. The beam is divided into eight equal elements, each 1 [m] long (Figure 10.14). Because of the symmetry of the system, the analysis can be carried out by considering only half of the beam. Hence, the total number of equations is reduced to four.

According to *Kany/ El Gendy* (1995), the analysis of a beam on elastic foundation is carried out in the following steps:

10.5.3.1 Moment of inertia I and beam stiffness α:

$$I = \frac{Bd^3}{12} = \frac{1 \times 0.5^3}{12} = 0.0104 \; [m^4]$$

$$\alpha = \frac{a \times B}{E_b I} = \frac{1^4 \times 1}{20000000 \times 0.0104} = 4.81 \times 10^{-6} \; [\text{m}^3/\text{kN}]$$

10.5.3.2 Determining external moments $M_i^{(l)}$

The external moments $M_i^{(l)}$ at points 2, 3, 4 and 5 are:

$$M_1^{(l)} = zero$$

$$M_2^{(l)} = 10 \times \frac{(1.5 \times 1)^2}{2} = 11.25 \; [\text{kN.m}]$$

$$M_3^{(l)} = 10 \times \frac{(1.5 \times 1)^2}{2} + 800 \times 1 = 831.25 \; [\text{kN.m}]$$

$$M_4^{(l)} = 10 \times \frac{(1.5 \times 1)^2}{2} + 800 \times 2 \times 1 = 1661.25 \; [\text{kN.m}]$$

$$M_5^{(l)} = 10 \times \frac{(1.5 \times 1)^2}{2} + 800 \times 3 = 2501.25 \; [\text{kN.m}]$$

10.5.3.3 Determining the right hand side R_i

The right hand side R_i of the contact pressure equation is:

$$R_i = \left(u_i M_{i-1}^{(l)} + v_i M_i^{(l)} + w_i M_{i+1}^{(l)} \right) \frac{a^2}{6EI_i}$$

$$R_i = \left(1 \times M_{i-1}^{(l)} + 4 \times M_i^{(l)} + 1 \times M_{i+1}^{(l)} \right) \frac{1^2}{6 \times 20000000 \times 0.0104}$$

$$R_i = 8.01 \times 10^{-7} \left(M_{i-1}^{(l)} + 4M_i^{(l)} + M_{i+1}^{(l)} \right)$$

Apply the above equation at points 2, 3 and 4:

$$R_2 = 8.01 \times 10^{-7}(0 + 4 \times 11.25 + 831.25) = 7.02 \times 10^{-4}$$

$$R_3 = 8.01 \times 10^{-7}(11.25 + 4 \times 831.25 + 1661.25) = 4 \times 10^{-3}$$

$$R_4 = 8.01 \times 10^{-7}(831.25 + 4 \times 1661.25 + 2501.25) = 8 \times 10^{-3}$$

10.5.3.4 Determining contact pressures

The contact pressure equation is:

$$\left(\frac{1}{k}\right) q_{i+1} - \left(\frac{2}{k} - \frac{\alpha}{6}\right) q_i + \left(\frac{1}{k} + \alpha\right) q_{i-1} + \alpha \left(\sum_{j=1}^{i-2} (i-j)\, q_j\right) = R_i$$

$$\left(\frac{1}{25000}\right) q_{i+1} - \left(\frac{2}{25000} - \frac{4.81 \times 10^{-6}}{6}\right) q_i + \left(\frac{1}{25000} + 4.81 \times 10^{-6}\right) q_{i-1}$$

$$+ 4.81 \times 10^{-6} \left(\sum_{j=1}^{i-2} (i-j)\, q_j\right) = R_i$$

or

$$q_{i+1} - 1.98\, q_i + 1.12\, q_{i-1} + 0.12 \left(\sum_{j=1}^{i-2} (i-j)\, q_j\right) = 25000 R_i$$

Apply the above equation at points 2, 3 and 4:

$$q_3 - 1.98\, q_2 + 1.12\, q_1 = 17.55$$

$$q_4 - 1.98\, q_3 + 1.12\, q_2 + 0.24\, q_1 = 100$$

$$-0.98\, q_4 + 1.12\, q_3 + 0.36\, q_1 + 0.24\, q_2 = 200$$

There are four unknown q_1, q_2, q_3, and q_4, so a farther equation is required. This can be obtained by considering the overall equilibrium of vertical forces.

$$\sum V = 0$$

$$a \times B(q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8) = P_1 + P_2 + A \times B \times P_f$$

or

$$q_1 + q_2 + q_3 + q_4 = 840$$

Contact pressure equations in matrix form:

$$\begin{bmatrix} 1.12 & -1.98 & 1 & 0 \\ 0.24 & 1.12 & -1.98 & 1 \\ 0.36 & 0.24 & 1.12 & -0.98 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 17.55 \\ 100 \\ 200 \\ 840 \end{bmatrix}$$

Solving the above system of linear equations to obtain the contact pressures q_1, q_2, q_3, and q_4.

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 249.98 \\ 230.44 \\ 193.85 \\ 165.72 \end{bmatrix} \text{[kN/m}^2\text{]}$$

10.5.3.5 Determining settlements s_i

The settlement s_i can be given by:

$$s_i = \frac{q_i}{k_i} = \frac{q_i}{25000} \text{ [m]}$$

$s_1 = 1.00$ [cm]

$s_2 = 0.92$ [cm]

$s_3 = 0.78$ [cm]

$s_4 = 0.66$ [cm]

The contact pressure distribution, settlement, moment and shear force diagrams for the raft are shown in Figure 10.15 to Figure 10.18. Once the internal forces are obtained at various sections, the design of the raft can be completed in the normal manner.

10.5.3.6 Computer calculation

The input data and results of *GEO Tools* are presented on the pages 10.55 to 10.66. By comparison, one can see an agreement with the hand calculation.

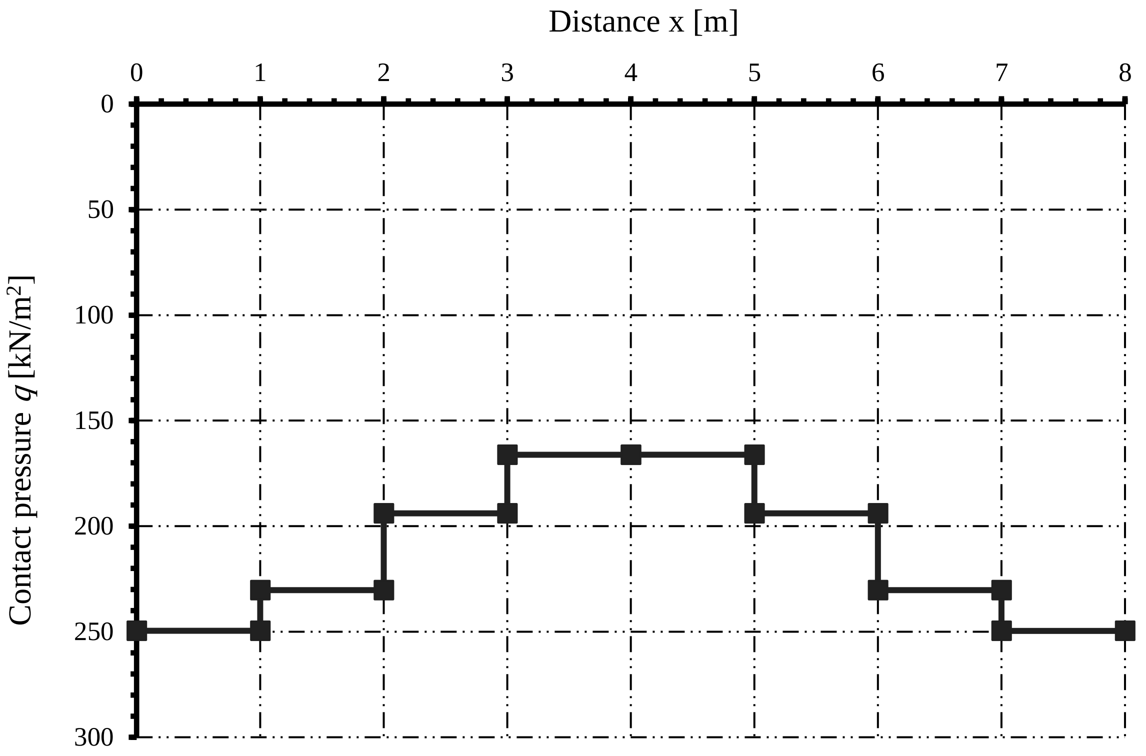


Figure 10.15 Contact pressures

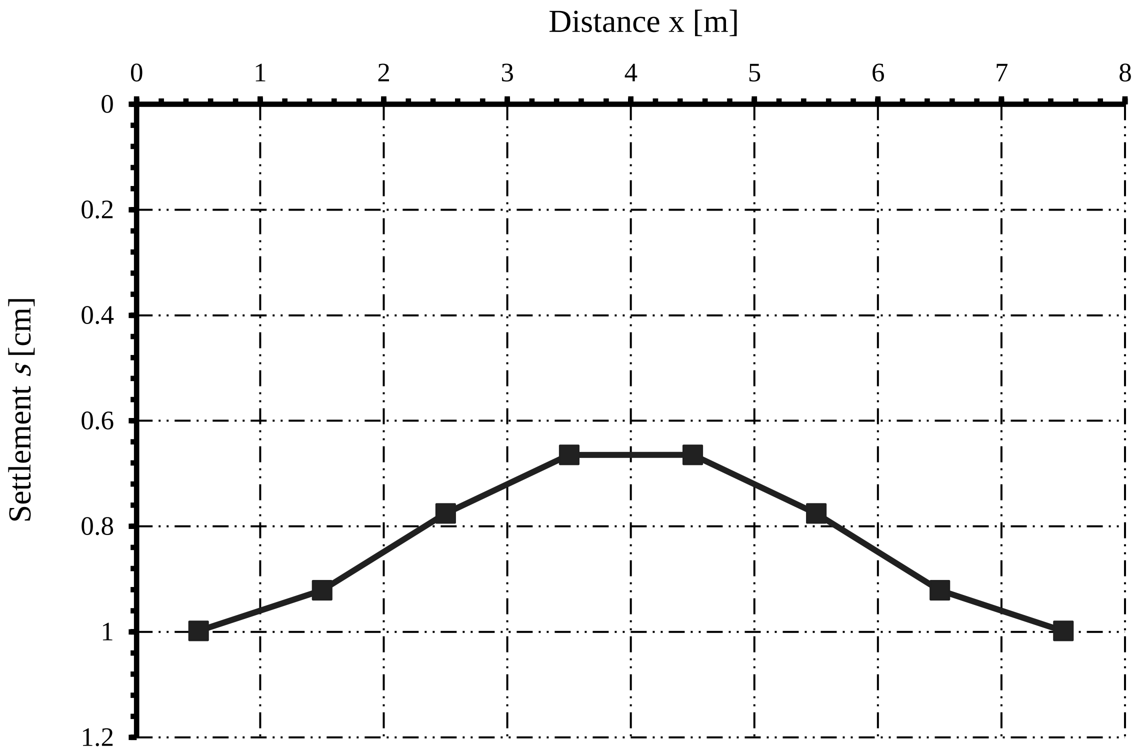


Figure 10.16 Settlements

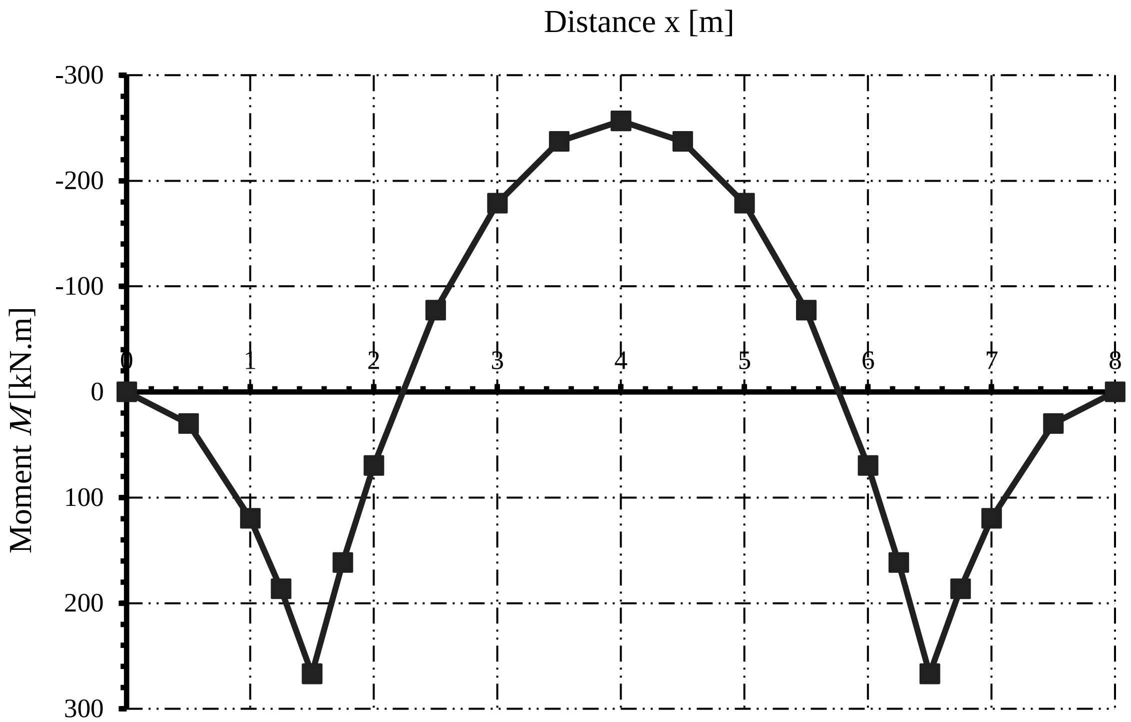


Figure 10.17 Moments

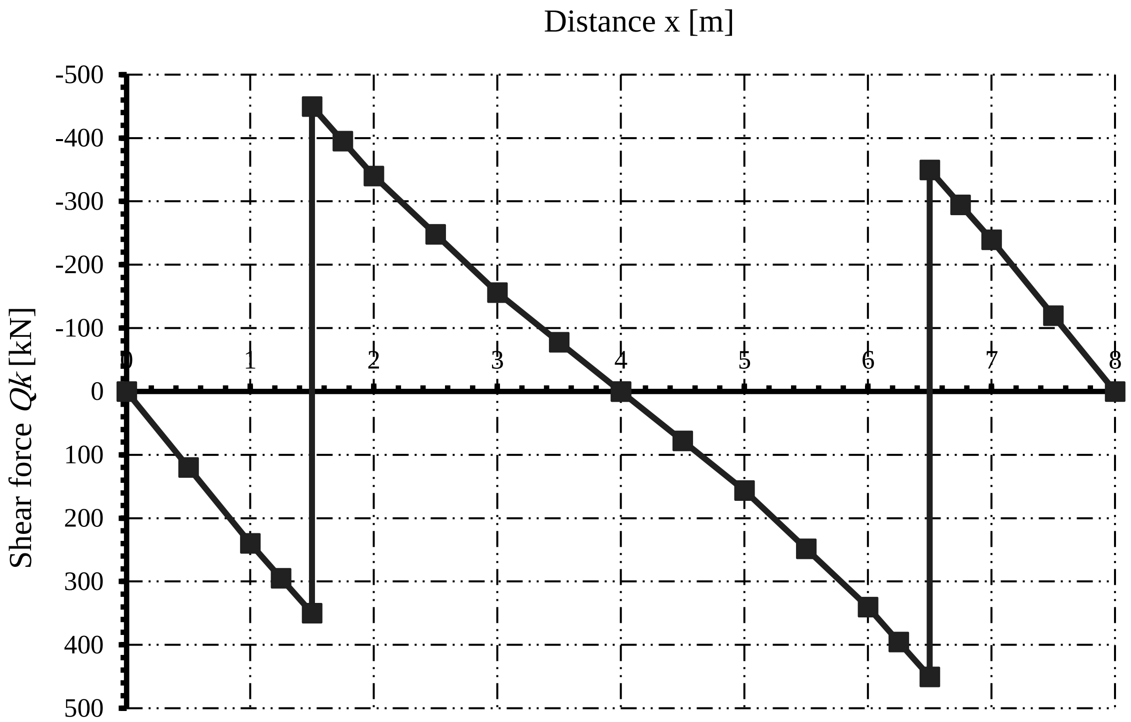


Figure 10.18 Shear forces

```
********************************************************
                        GEO Tools
                        Version 12.3
          Program authors M. El Gendy/ A. El Gendy
********************************************************
Title: Beam Foundations after Kany and El Gendy
Date: 05-05-2022
Project: Analysis of a raft for two equal walls
File: Raft 2Walls

------------------------------------------------------------
Analysis of a beam foundation
Calculation method: Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy
------------------------------------------------------------

Data:

Main Soil Data:

Groundwater depth under the ground surface   Tw          [m]        = 1.75
Foundation depth under ground surface        Tf          [m]        = 2.00

Summary of loading:

Self weight                                  Pe          [kN]       = 100.000
Load on Footing                              Pa          [kN]       = 1580.000
Groundwater force                            Pw          [kN]       = 20.000
Total load                                   Po=Pe+Pa-Pw [kN]       = 1680.000

Groundwater pressure                         Qw          [kN/m2]    = 2.5
Average soil pressure                        Qo          [kN/m2]    = 210.0

Beam Material:

Modulus of elasticity of the concrete        Eb          [kN/m2]    = 20000000.00
Unit weight of footing concrete              γb          [kN/m3]    = 25.0

Dimensions:

Depth of the foundation surface under ground Tk          [m]        = 1.50
Beam thickness                               d           [m]        = 0.50
Moment of inertia of the beam                I           [m4]       = 0.0104
Beam stiffness                               αB          1/[kN/m3]  = 4.80E-06
Beam length (longitudinal)                   A           [m]        = 8.00
Beam width (transversal)                     B           [m]        = 1.00
Length/width ratio                           A/B         [-]        = 8.00
Element size                                 a           [m]        = 1.00
Number of elements of the beam               N           [-]        = 8

Loads:

Point Loads:
-----------------------------------------------------------
No.     Load    Load position    Column    Column    Column
       value         from the      side      side     label
                    left edge
  I        P               Xp         a         b        Lb
[-]     [kN]              [m]       [m]       [m]       [-]
-----------------------------------------------------------
  1  800.000             1.50      0.20      1.00        W1
  2  800.000             6.50      0.20      1.00        W2
-----------------------------------------------------------

Distributed Loads:
-----------------------------------------------------------
No.      Load   Load start from   Load end from   Load type
        value     the left edge   the left edge
  I        Pf               Xpl             Xpr
[-]   [kN/m2]               [m]             [m]         [-]
-----------------------------------------------------------
  1    -2.500              0.00            8.00  (Groundwater pressure)
  2    12.500              0.00            8.00  (Self weight)
-----------------------------------------------------------
```

Right sides of the system of equations:

Element No. I [-]	Right sides of the system of equations Rv [m]
1	6.72E+03
2	1.68E+03
3	7.01E-04
4	3.998E-03
5	7.982E-03
6	1.2014E-02
7	1.6094E-02
8	2.0862E-02

Settlements/ Contact pressures/ Moduli of subgrade reactions:

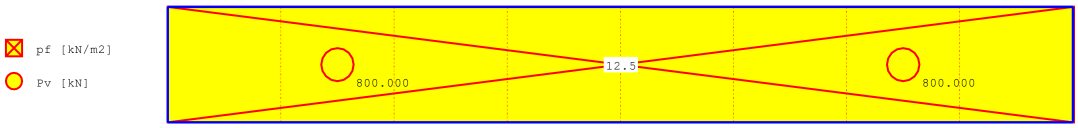
Element No. I [-]	Contact pressure q [kN/m2]	Settlement s [cm]	Modulus of subgrade reaction ks [kN/m3]
1	249.6	1.00	25000
2	230.3	0.92	25000
3	194.0	0.78	25000
4	166.1	0.66	25000
5	166.1	0.66	25000
6	193.9	0.78	25000
7	230.3	0.92	25000
8	249.6	1.00	25000

Moments/ Shear Forces:

No. I [-]	Distance x [m]	Distance [-]	Moment Mm [kN.m]	Shear force Qk [kN/m]
1	0.00		0.00	0.0
2	0.50		29.95	119.8
3	1.00		119.81	239.6
4	1.40	CL	233.28	327.7
5	1.50	CC	267.15	349.8
6	1.50	CC	266.70	-450.0
7	1.60	CR	223.23	-428.2
8	2.00		69.57	-340.1
9	2.50		-77.47	-248.1
10	3.00		-178.53	-156.1
11	3.50		-237.08	-78.1
12	4.00	MM	-256.60	0.0
13	4.00		-256.60	0.0
14	4.50		-237.08	78.1
15	5.00		-178.53	156.1
16	5.50		-77.47	248.1
17	6.00		69.58	340.1
18	6.40	CL	223.24	428.2
19	6.50	CC	267.16	450.2
20	6.50	CC	266.81	-349.6
21	6.60	CR	233.29	-327.7
22	7.00		119.81	-239.6
23	7.50		29.95	-119.8
24	8.00		0.00	0.0

Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



System of loading

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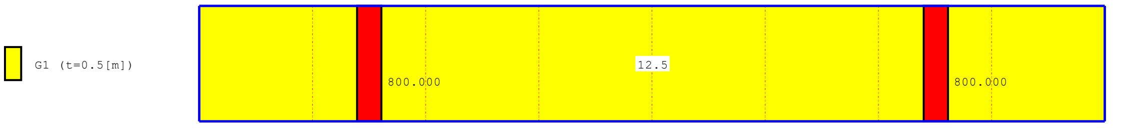
Scale: 36	Title: Beam Foundations after Kany and El Gendy
File: Raft 2Walls	Date: 05-05-2022
Page No.:	Project: Analysis of a raft for two equal walls

Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Element groups t [m]

No. of element groups = 1

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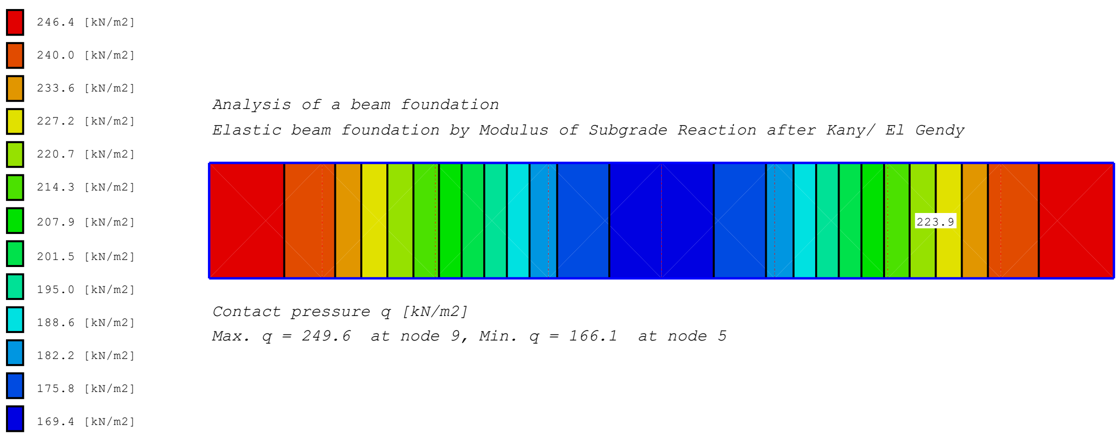

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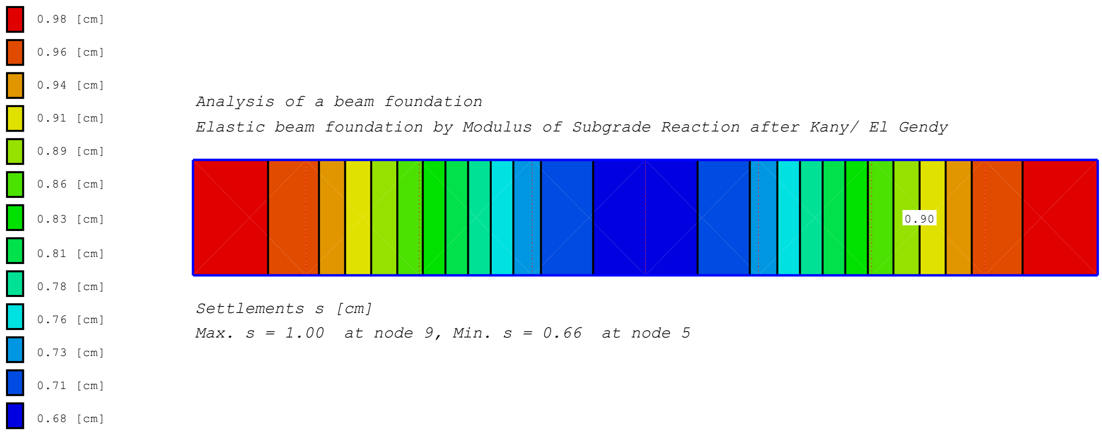

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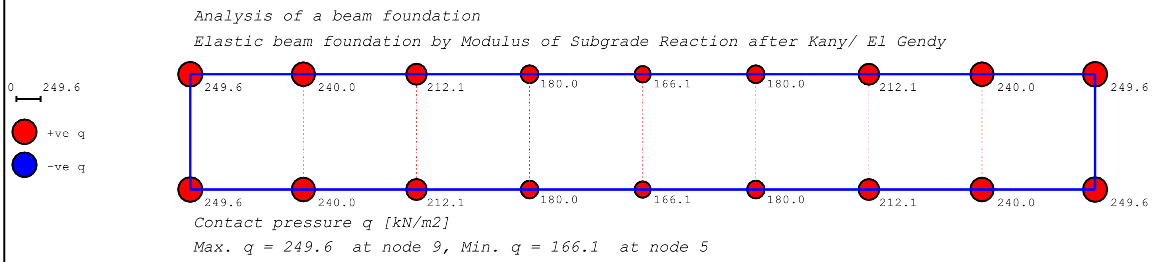
Page No.:	Project: Analysis of a raft for two equal walls



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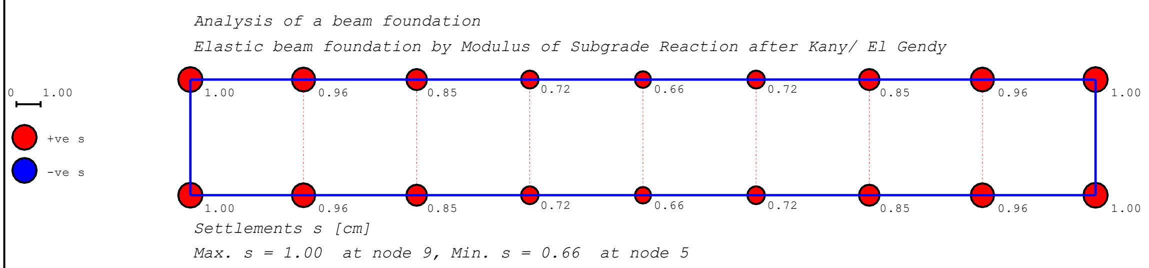
Page No.:	Project: Analysis of a raft for two equal walls



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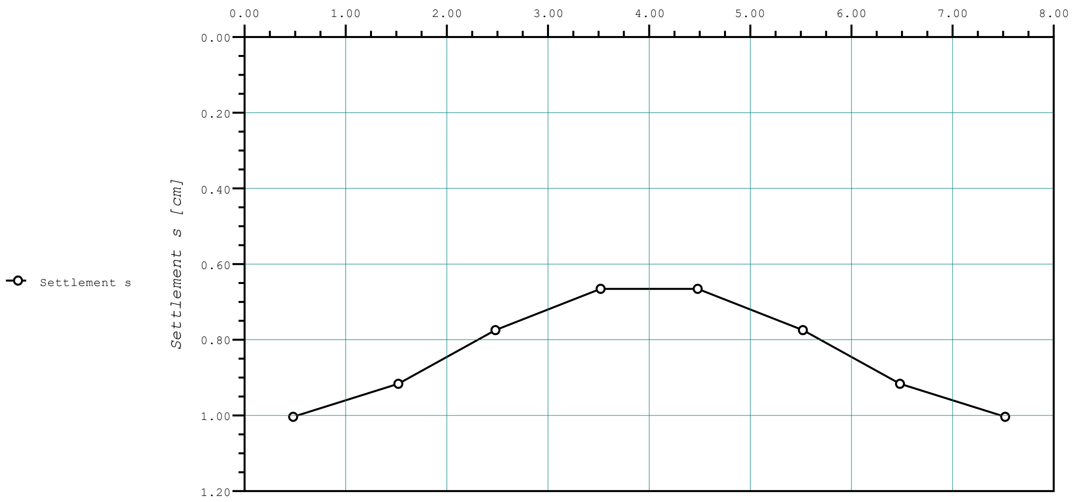
Scale: 36	**Title: Beam Foundations after Kany and El Gendy**
File: Raft 2Walls	**Date: 05-05-2022**
Page No.:	**Project: Analysis of a raft for two equal walls**

Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Settlements

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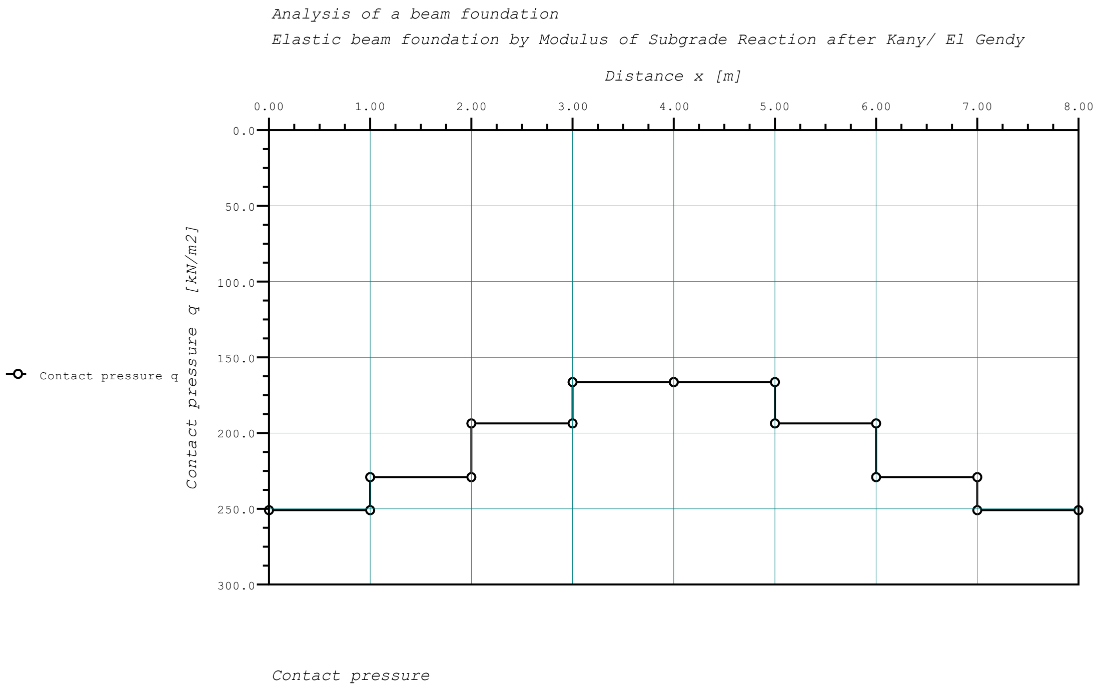
Scale: 40	**Title: Beam Foundations after Kany and El Gendy**
File: Raft 2Walls	**Date: 05-05-2022**
Page No.:	**Project: Analysis of a raft for two equal walls**



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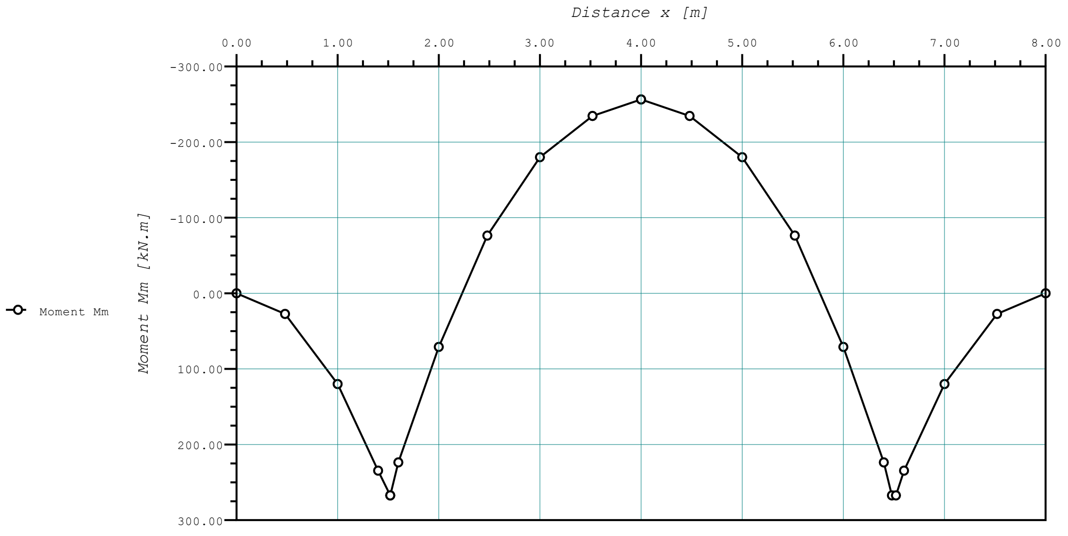
File: Raft 2Walls	Date: 05-05-2022
Page No.:	Project: Analysis of a raft for two equal walls

Analysis of a beam foundation
Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Moments

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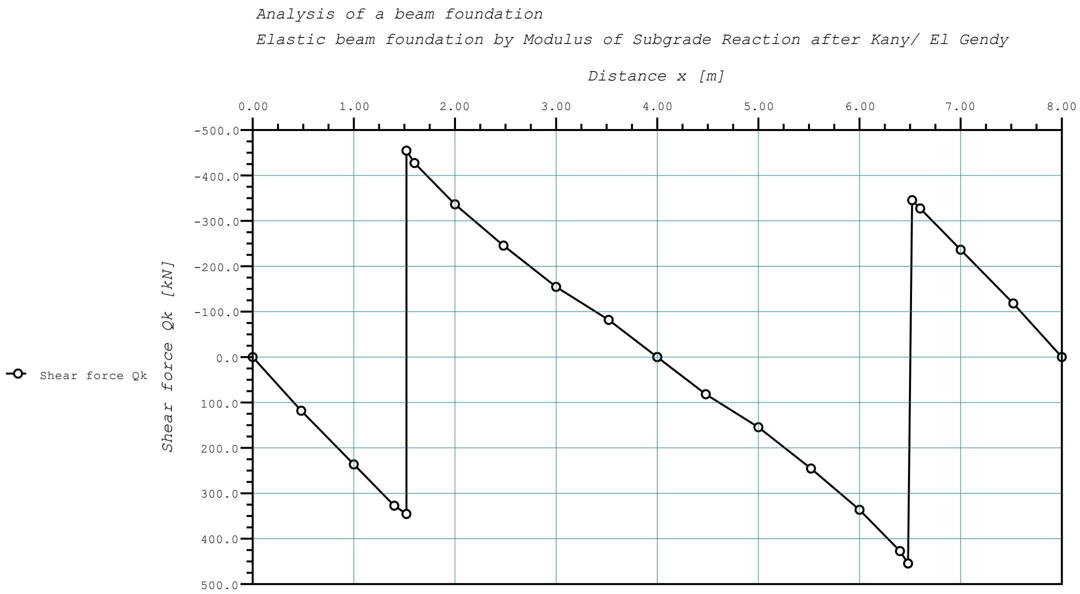
Scale: 40 File: Raft 2Walls Page No.:	Title: Beam Foundations after Kany and El Gendy Date: 05-05-2022 Project: Analysis of a raft for two equal walls



Shear Forces

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File: Raft 2Walls	Date: 05-05-2022
Page No.:	Project: Analysis of a raft for two equal walls

10.6 Example 4: Analysis of a raft for three equal walls

10.6.1 Description of the problem

Figure 10.19 shows plan and section with dimensions and loads for a raft of three equal walls. It is required to find the contact pressure distribution, settlements, moment and shear force diagrams for the raft. The loading and the raft are symmetrical.

Geometry:

Thickness of the raft	d	=0.6	[m]
Dimensions of the raft	A_f	=10×12	[m^2]
Groundwater depth under the ground surface	Tw	=1	[m]
Foundation depth under the ground surface	Tf	=2	[m]

Material properties of the concrete and unit weight of the water

Modulus of elasticity of the concrete	$E_b = 2\times10^7$	[kN/m^2]
Unit weight of the concrete	$\gamma_b=25$	[kN/m^3]
Unit weight of the water	$\gamma_w=10$	[kN/m^3]

Soil properties

Modulus of subgrade reaction of the soil k_s = 40000 [kN/m^3].

10.6.2 Preparing the calculation

The raft can be regarded as a beam on elastic foundation subjected to:

- A uniformly distributed loading p_f equal to the weight of the raft itself minus the uplift pressure from the ground water.
- Three concentrated forces from three walls $P_1 = P_2 = P_3 = 1000$ [kN/m].

Computing the uniform load on the raft

Own weight of the raft	$w_o = \gamma_b \times d = 25\times0.6$	=15	[kN/m^2]
Up lift pressure	$w_w = \gamma_w \times (T_f - Tw) = 10(2-1)$	=-10	[kN/m^2]
Total	$p_f =$	=5	[kN/m^2]

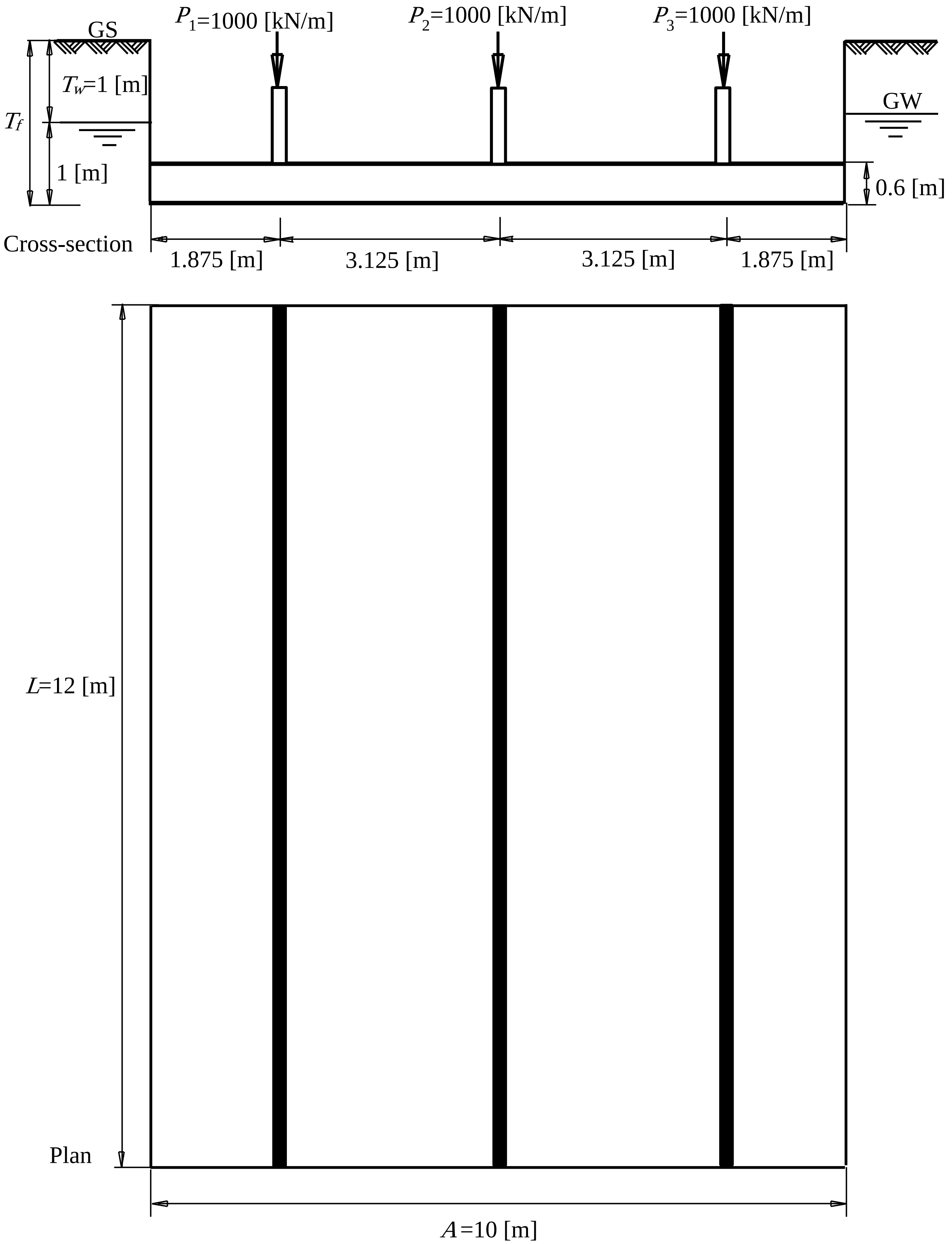


Figure 10.19 Raft of three equal walls

10.6.3 Hand calculation

Assume one-meter strip width from the raft and consider it as a beam on elastic foundation. The beam is divided into eight equal elements, each 1.25 [m] long (Figure 10.20). Because of the symmetry of the system, the analysis can be carried out by considering only half of the beam. Hence, the total number of equations is reduced to four.

According to *Kany/ El Gendy* (1995), the analysis of a beam on elastic foundation is carried out in the following steps:

10.6.3.1 Moment of inertia I and beam stiffness α:

$$I = \frac{Bd^3}{12} = \frac{1\times 0.6^3}{12} = 0.018 [\text{m}^4]$$

and

$$\alpha = \frac{a^4 B}{E_b I} = \frac{1.25^4 \times 1}{(2\times 10^7)(0.018)} = 6.782\times 10^{-6} \ [\text{m}^3/\text{kN}]$$

10.6.3.2 Determining external moments $M_i^{(l)}$

The external moments $M_i^{(l)}$ at points 2, 3, 4 and 5 are:

$$M_1^{(l)} = 0$$

$$M_2^{(l)} = 5\frac{(1.25\times 1.5)^2}{2} = 8.789 \ [\text{kN.m}]$$

$$M_3^{(l)} = 5\frac{(1.25\times 2.5)^2}{2} + 1000\times 1.25\times 1 = 1274.414 \ [\text{kN.m}]$$

$$M_4^{(l)} = 5\frac{(1.25\times 3.5)^2}{2} + 1000\times 1.25\times 2 = 2547.852 \ [\text{kN.m}]$$

$$M_5^{(l)} = 5\frac{(1.25\times 4.5)^2}{2} + 1000\times 1.25\times 3 + 1000\times 1.25\times 0.5 = 4454.102 \ [\text{kN.m}]$$

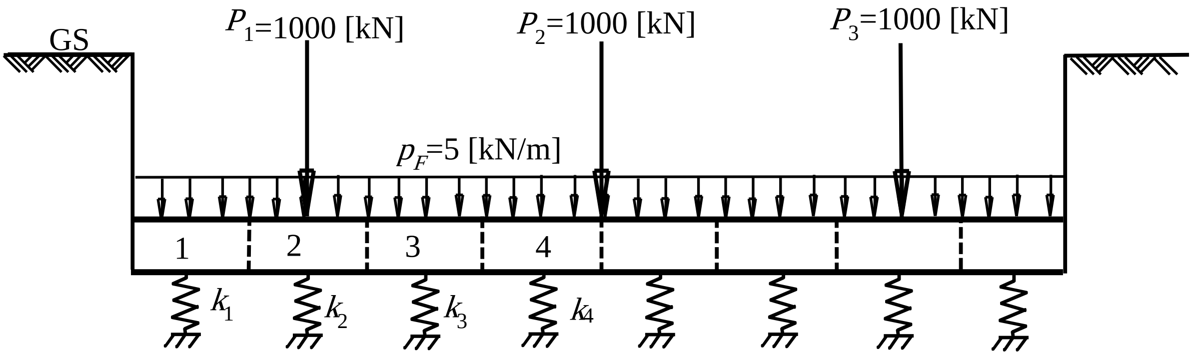
10.6.3.3 Determining the right hand side R_i

The right hand side R_i of the contact pressure equation is:

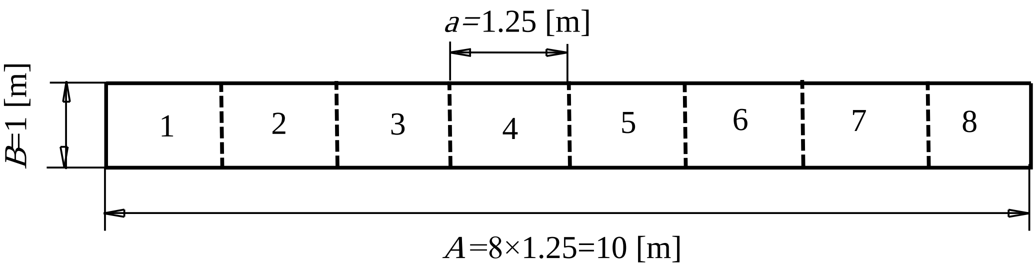
$$R_i = \left(u_i M^{(l)}_{i-1} + v_i M^{(l)}_i + w_i M^{(l)}_{i+1}\right)\frac{a^2}{6E\, I_i}$$

$$R_i = \left(M^{(l)}_{i-1} + 4\, M^{(l)}_i + M^{(l)}_{i+1}\right)\frac{1.25^2}{6\times 2\times 10^7.\times 0.018}$$

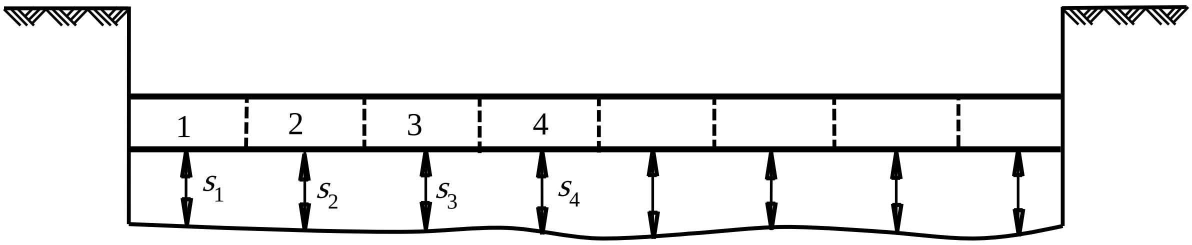
$$R_i = \frac{1.5625}{2160000}\left(M^{(l)}_{i-1} + 4\, M^{(l)}_i + M^{(l)}_{i+1}\right)$$



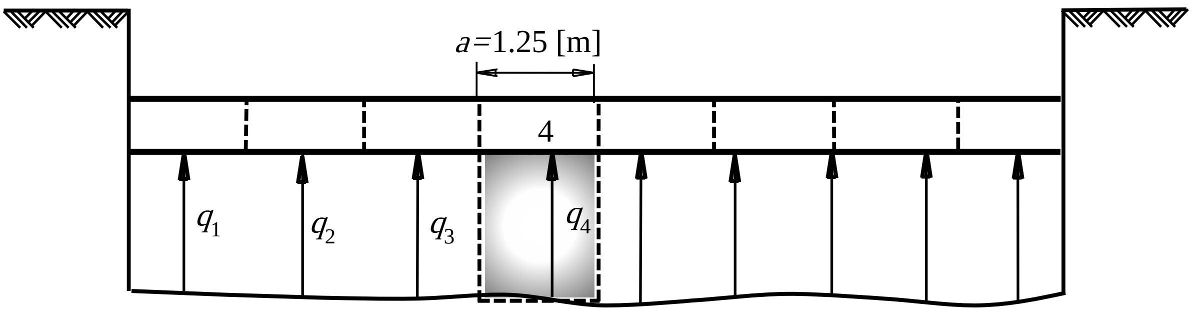
Beam foundation with loads and springs



Plan of beam foundation with elements



Settlement



Contact pressure

Figure 10.20   One meter strip width of the raft

Apply the above equation at points 2, 3 and 4:

$$R_2 = \frac{1.5625}{2160000}(0 + 4\times 8.789 + 1274.414) = \frac{2046.203}{216000}$$
$$R_3 = \frac{1.5625}{2160000}(8.789 + 4\times 1274.414 + 2547.852) = \frac{11959.839}{216000}$$
$$R_4 = \frac{1.5625}{2160000}(1274.414 + 4\times 2547.852 + 4454.102) = \frac{24874.881}{216000}$$

10.6.3.4 Determining contact pressures

The contact pressure equation is:

$$\left(\frac{1}{k}\right) q_{i+1} - \left(\frac{2}{k} - \frac{\alpha}{6}\right) q_i + \left(\frac{1}{k} + \alpha\right) q_{i-1} + \alpha\left(\sum_{j=1}^{i-2} (i-j)\, q_j\right) = R_i$$

$$\left(\frac{1}{40000}\right) q_{i+1} - \left(\frac{2}{40000} - \frac{6.782\times 10^{-6}}{6}\right) q_i + \left(\frac{1}{40000} + 6.782\times 10^{-6}\right) q_{i-1} + 6.782\times 10^{-6}\left(\sum_{j=1}^{i-2} (i-j) q_j\right) = R_i$$

or

$$\left(\frac{1}{40000}\right) q_{i+1} - 4.887\times 10^{-5}\, q_i + 3.178\times 10^{-5}\, q_{i-1} + 6.782\times 10^{-6}\left(\sum_{j=1}^{i-2} (i-j) q_j\right) = R_i$$

or

$$54\ q_{i+1} - 105.559\ q_i + 68.645\ q_{i-1} + 14.649\left(\sum_{j=1}^{i-2} (i-j) q_j\right) = 2160000\ R_i$$

Apply the above equation at points 2, 3 and 4:

$$54\ q_3 - 105.559\ q_2 + 68.645\ q_1 = 2046.203$$
$$54\ q_4 - 105.559\ q_3 + 68.645\ q_2 + 29.298\ q_1 = 11959.839$$
$$54\ q_5 - 105.559\ q_4 - 68.645\ q_3 + 29.298\ q_2 + 43.947\ q_1 = 24874.881$$

Substituting q_5 by q_4

$$54\ q_3 - 105.559\ q_2 + 68.645\ q_1 = 2046.203$$
$$54\ q_4 - 105.559\ q_3 + 68.645\ q_2 + 29.298\ q_1 = 11959.839$$
$$-51.559\ q_4 + 68.645\ q_3 + 29.298\ q_2 + 43.947\ q_1 = 24874.881$$

There are four unknown q_1, q_2, q_3, and q_4, so a farther equation is required. This can be obtained by considering the overall equilibrium of vertical forces.

$$a\times B(q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8) = P_1 + P_2 + P_3 + A\times B\times P_f$$

or

$$q_1 + q_2 + q_3 + q_4 = 1220$$

Contact pressure equations in matrix form:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 68.645 & -105.559 & 54 & 0 \\ 29.298 & 68.645 & -105.559 & 54 \\ 43.947 & 29.298 & 68.645 & -51.559 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 1220 \\ 2046.203 \\ 2888.75 \\ 5078.75 \end{Bmatrix}$$

Solving the above system of linear equations to obtain the contact pressures q_1, q_2, q_3, and q_4.

$q_1 =$ 245.2 [kN/m^2]
$q_2 =$ 208.0 [kN/m^2]
$q_3 =$ 328.2 [kN/m^2]
$q_4 =$ 338.6 [kN/m^2]

10.6.3.5 Determining settlements s_i

The settlement s_i can be given by:

$$s_i = \frac{q_i}{k_i} = \frac{q_i}{40000} \text{ [m]}$$

$s_1 = 0.61$ [cm]
$s_2 = 0.77$ [cm]
$s_3 = 0.82$ [cm]
$s_4 = 0.85$ [cm]

The contact pressure distribution, settlement, moment and shear force diagrams for the raft are shown in Figure 10.21 to Figure 10.24. Once the internal forces are obtained at various sections, the design of the raft can be completed in the normal manner.

10.6.3.6 Computer calculation

The input data and results of *GEO Tools* are presented on the pages 10.75 to 10.86. By comparison, one can see an agreement with the hand calculation.

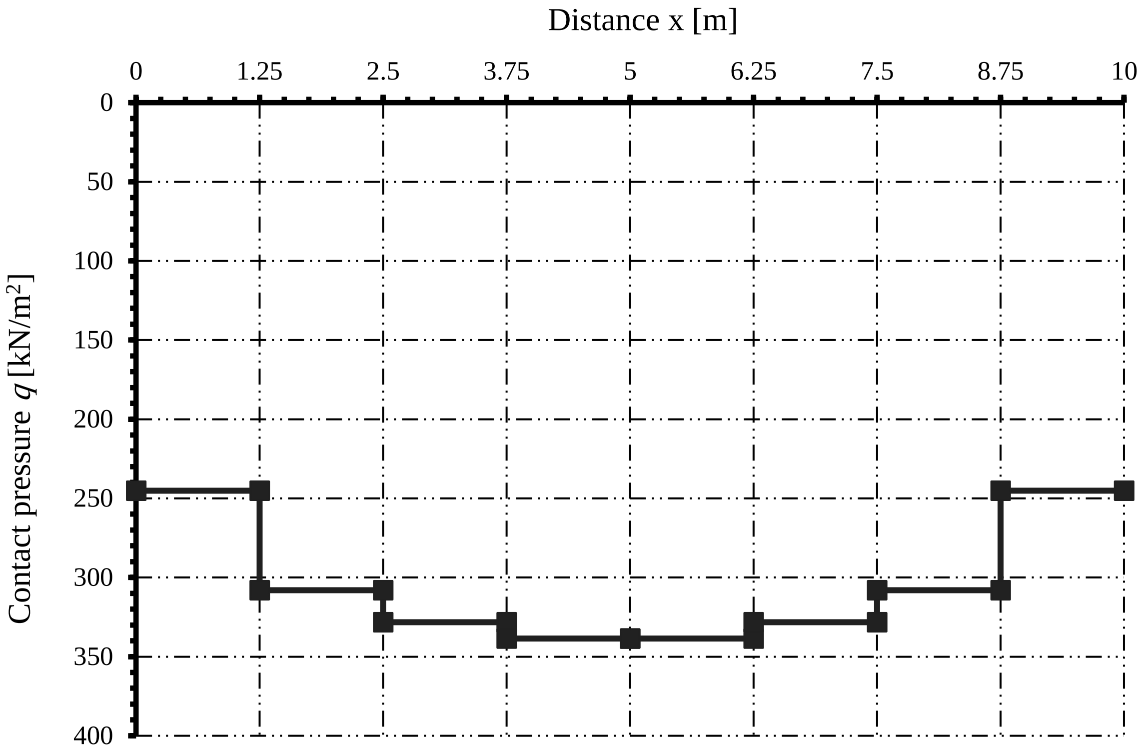


Figure 10.21 Contact pressures

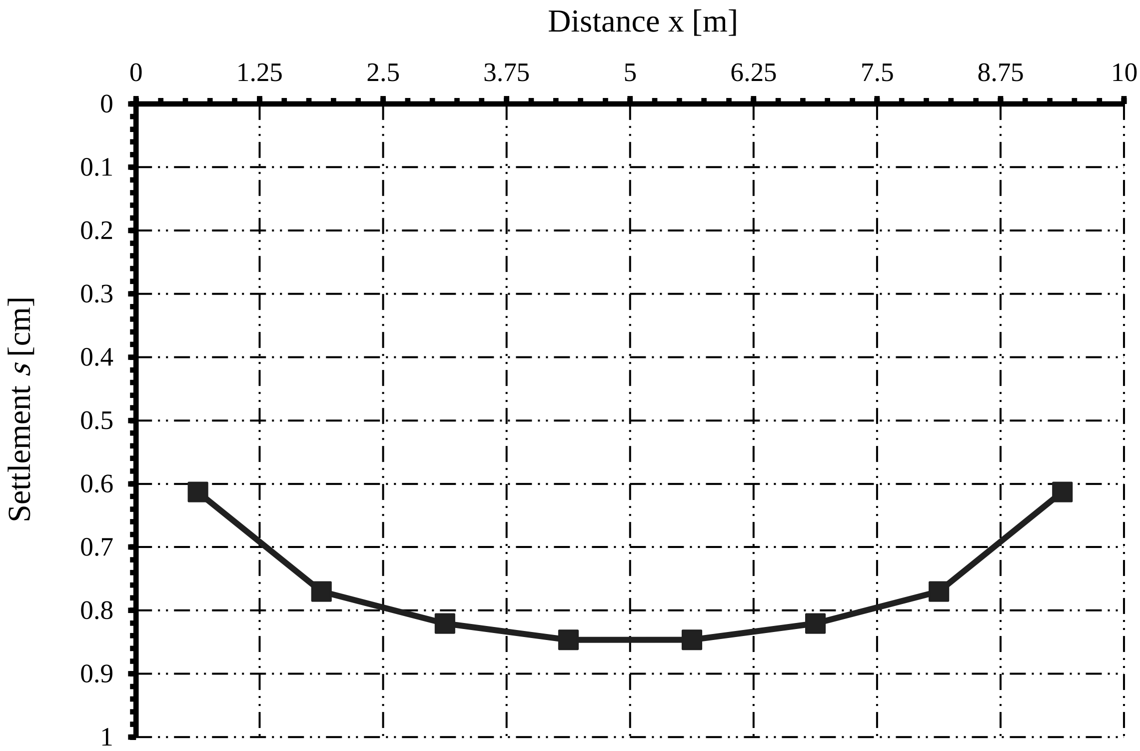


Figure 10.22 Settlements

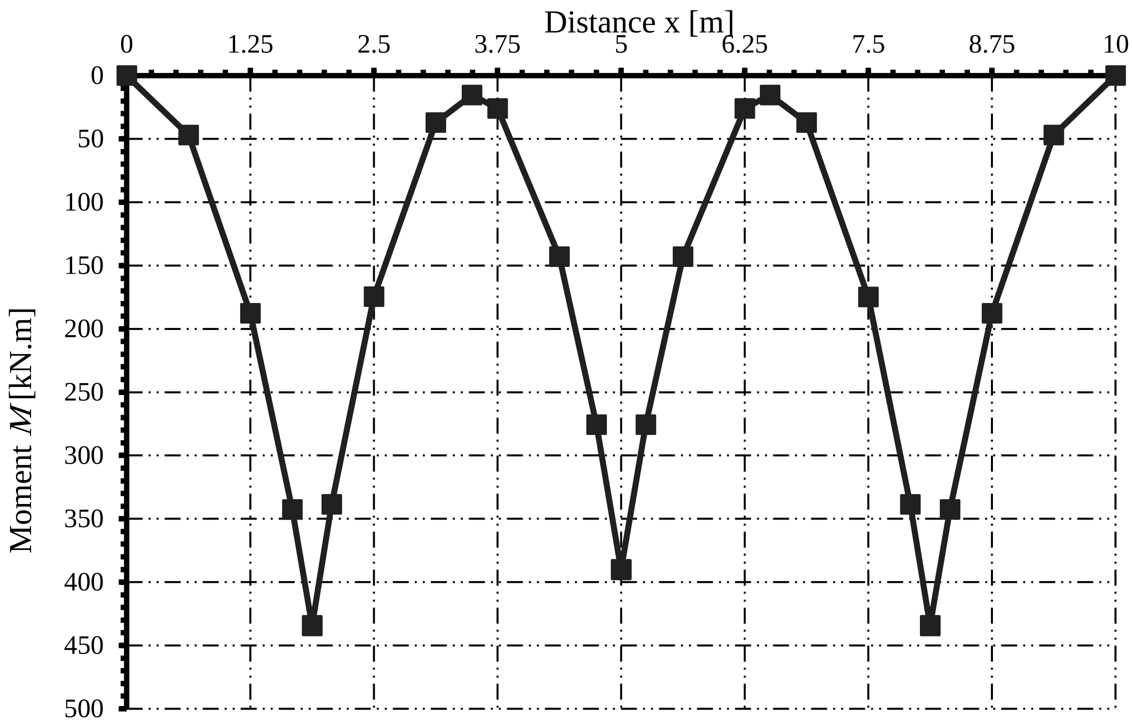


Figure 10.23 Moments

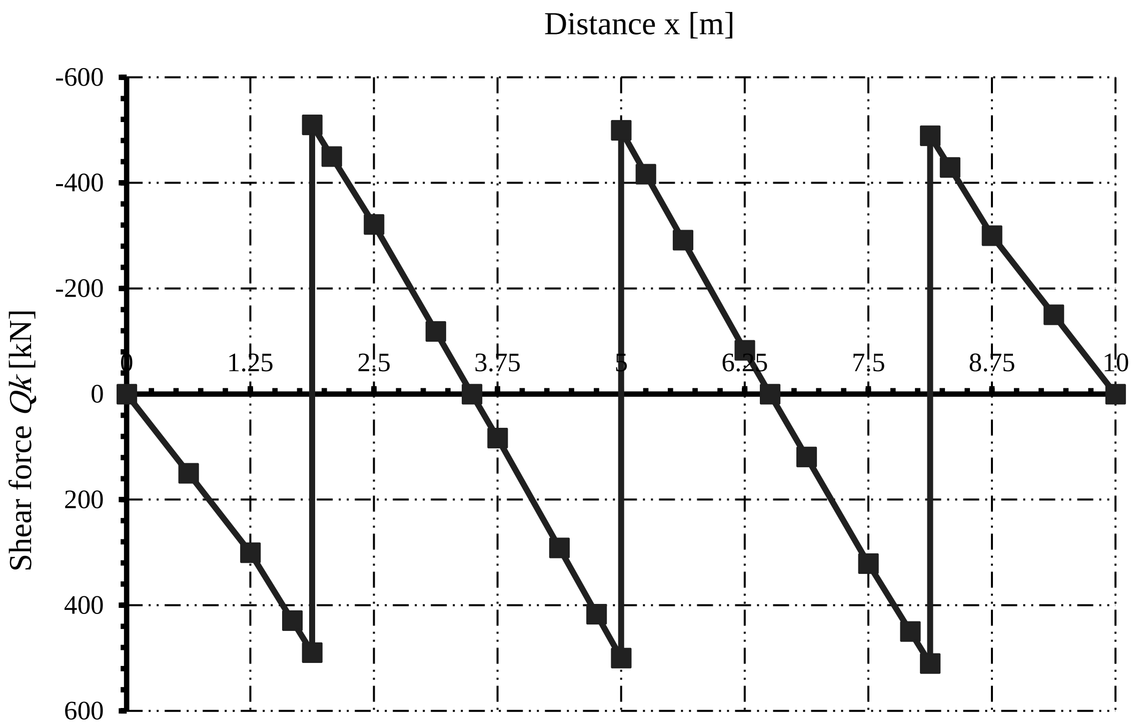


Figure 10.24 Shear forces

```
********************************************************
                        GEO Tools
                        Version 12.3
          Program authors M. El Gendy/ A. El Gendy
********************************************************
Title: Beam Foundations after Kany and El Gendy
Date: 05-05-2022
Project: Analysis of a raft for three equal walls
File: Raft 3Walls

--------------------------------------------------------
Analysis of a beam foundation
Calculation method: Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy
--------------------------------------------------------

Data:

Main Soil Data:

Groundwater depth under the ground surface   Tw           [m]        = 1.000
Foundation depth under ground surface        Tf           [m]        = 2.000

Summary of loading:

Self weight                                  Pe           [kN]       = 150.000
Load on Footing                              Pa           [kN]       = 2900.000
Groundwater force                            Pw           [kN]       = 100.000
Total load                                   Po=Pe+Pa-Pw  [kN]       = 3050.000

Groundwater pressure                         Qw           [kN/m2]    = 10.0
Average soil pressure                        Qo           [kN/m2]    = 305.0

Beam Material:

Modulus of elasticity of the concrete        Eb           [kN/m2]    = 20000000.000
Unit weight of footing concrete              γb           [kN/m3]    = 25.0

Dimensions:

Depth of the foundation surface under ground Tk           [m]        = 1.400
Beam thickness                               d            [m]        = 0.600
Moment of inertia of the beam                I            [m4]       = 0.018
Beam stiffness                               αB           1/[kN/m3]  = 6.78E-06
Beam length (longitudinal)                   A            [m]        = 10.000
Beam width (transversal)                     B            [m]        = 1.000
Length/width ratio                           A/B          [-]        = 10.00
Element size                                 a            [m]        = 1.250
Number of elements of the beam               N            [-]        = 8

Loads:

Point Loads:
-----------------------------------------------------------
No.     Load    Load position    Column    Column    Column
        value        from the      side      side     label
                    left edge
  I         P              Xp         a         b        Lb
[-]      [kN]             [m]       [m]       [m]       [-]
-----------------------------------------------------------
  1  1000.000           1.875     0.200     1.000        W1
  2  1000.000           5.000     0.200     1.000        W2
  3  1000.000           8.125     0.200     1.000        W3
-----------------------------------------------------------

Distributed Loads:
-----------------------------------------------------------
No.      Load    Load start from   Load end from   Load type
         value     the left edge   the left edge
  I         Pf               Xpl             Xpr
[-]    [kN/m2]               [m]             [m]         [-]
-----------------------------------------------------------
  1    -10.000             0.000          10.000   (Groundwater pressure)
  2     15.000             0.000          10.000   (Self weight)
-----------------------------------------------------------
```

Right sides of the system of equations:

Element No. I [-]	Right sides of the system of equations Rv [m]
1	1.525E+04
2	3.05E+03
3	9.4732E-04
4	5.537E-03
5	1.1516E-02
6	1.979E-02
7	3.0358E-02
8	4.2316E-02

Settlements/ Contact pressures/ Moduli of subgrade reactions:

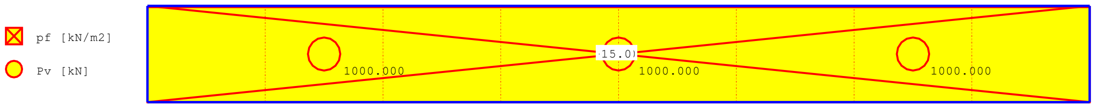
Element No. I [-]	Contact pressure q [kN/m2]	Settlement s [cm]	Modulus of subgrade reaction ks [kN/m3]
1	245.2	0.61	40000
2	308.0	0.77	40000
3	328.2	0.82	40000
4	338.6	0.85	40000
5	338.6	0.85	40000
6	328.2	0.82	40000
7	308.0	0.77	40000
8	245.2	0.61	40000

Moments/ Shear Forces:

No. I [-]	Distance x [m]	Distance [-]	Moment Mm [kN.m]	Shear force Qk [kN/m]
1	0.000		0.00	0.0
2	0.625		46.91	150.1
3	1.250		187.65	300.2
4	1.775	CL	387.02	459.3
5	1.875	CC	434.47	489.6
6	1.876	CC	433.96	-510.1
7	1.975	CR	384.94	-480.1
8	2.500		174.65	-321.0
9	3.125		37.13	-119.0
10	3.493	MM	15.23	0.0
11	3.750		25.89	83.0
12	4.375		142.94	291.5
13	4.900	CL	341.96	466.6
14	5.000	CC	390.29	500.0
15	5.001	CC	389.79	-499.7
16	5.100	CR	341.96	-466.6
17	5.625		142.95	-291.5
18	6.250		25.91	-83.0
19	6.507	MM	15.24	0.0
20	6.875		37.15	119.0
21	7.500		174.66	321.0
22	8.025	CL	384.96	480.1
23	8.125	CC	434.49	510.4
24	8.126	CC	434.00	-489.3
25	8.225	CR	387.04	-459.3
26	8.750		187.66	-300.2
27	9.375		46.92	-150.1
28	10.000		0.00	0.0

Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy

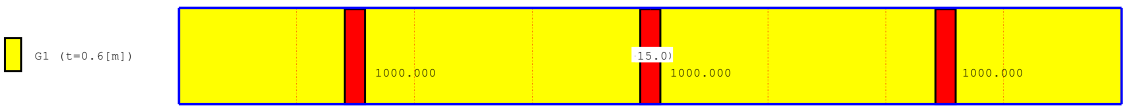


System of loading

GEOTEC Software Inc PO Box 14001 Richmond Road PO, Calgary AB, Canada T3E 7Y7	
Scale: 43	Title: Beam Foundations after Kany and El Gendy
File: Raft 3Walls	Date: 05-05-2022
Page No.:	Project: Analysis of a raft for three equal walls

Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Element groups t [m]

No. of element groups = 1

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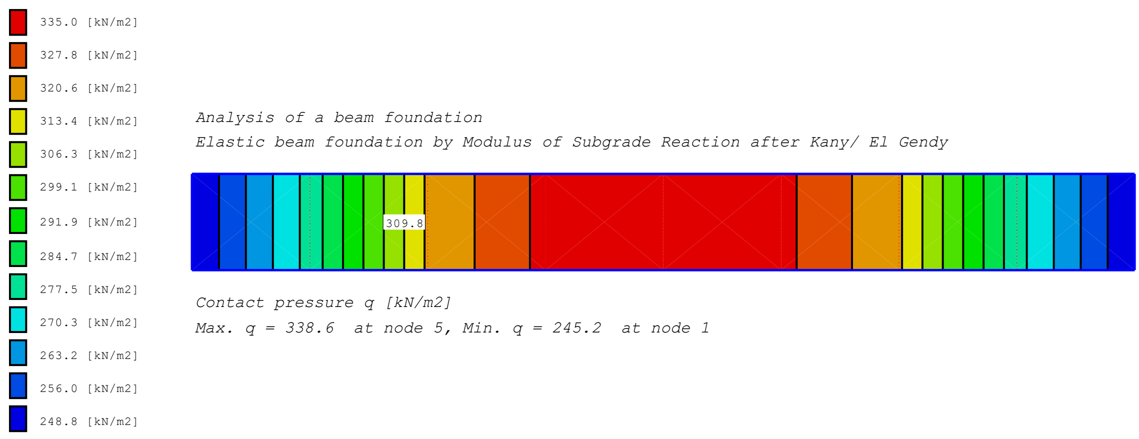

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Scale: 43	Title: Beam Foundations after Kany and El Gendy
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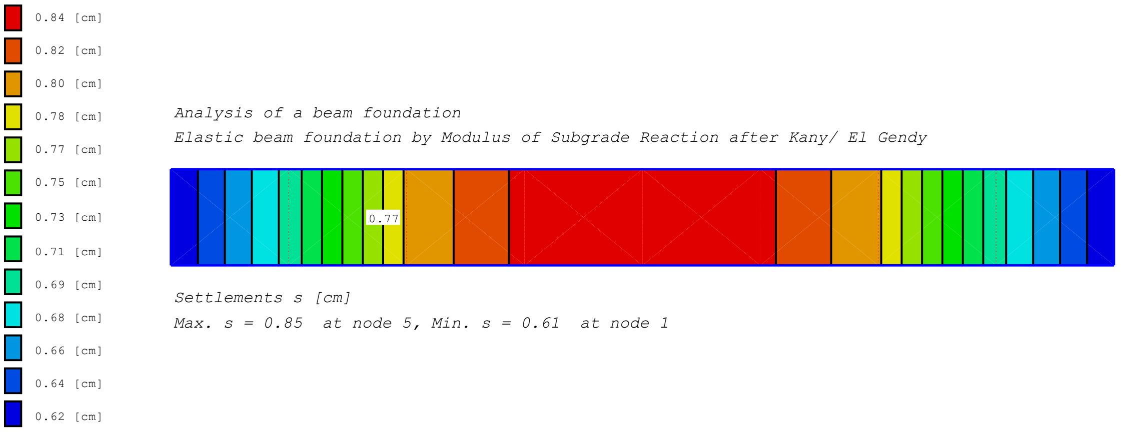
Page No.:	Project: Analysis of a raft for three equal walls



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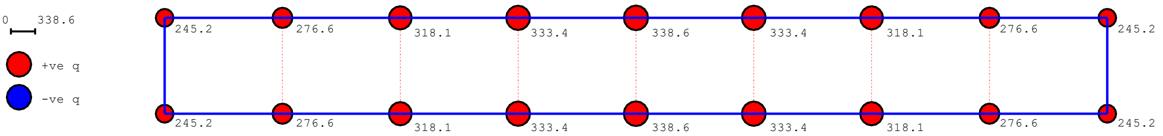
Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Contact pressure q [kN/m2]

Max. q = 338.6 at node 5, Min. q = 245.2 at node 1

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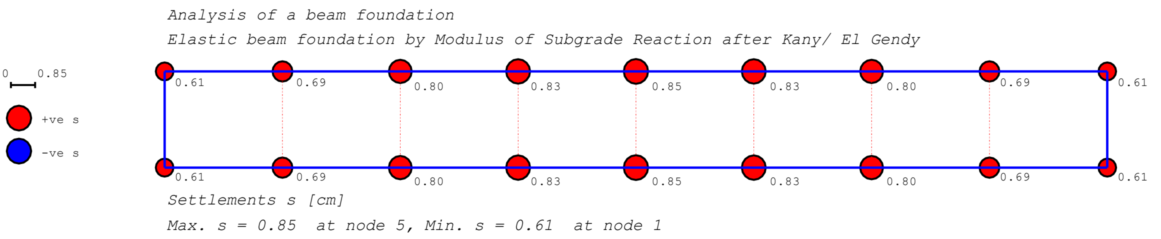

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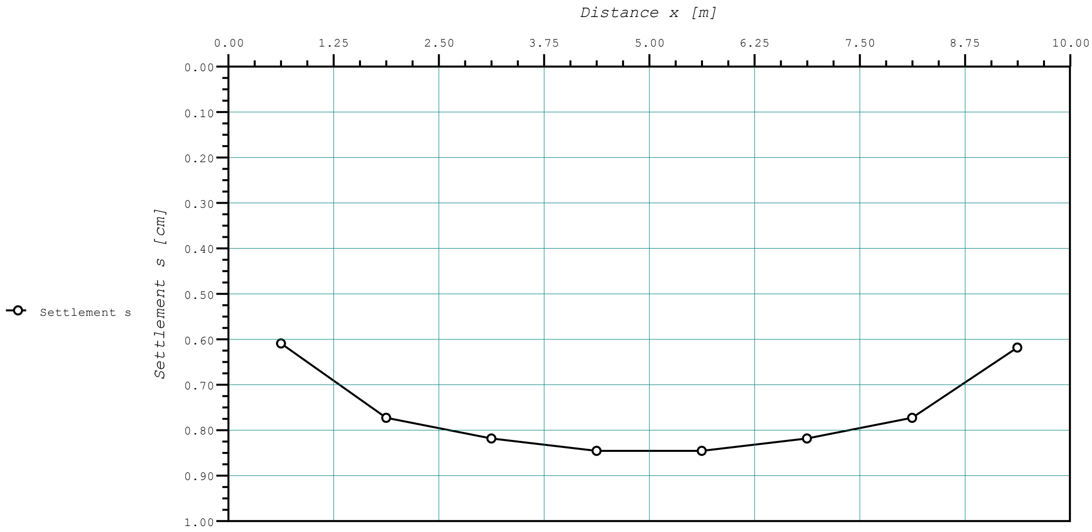
Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Settlements

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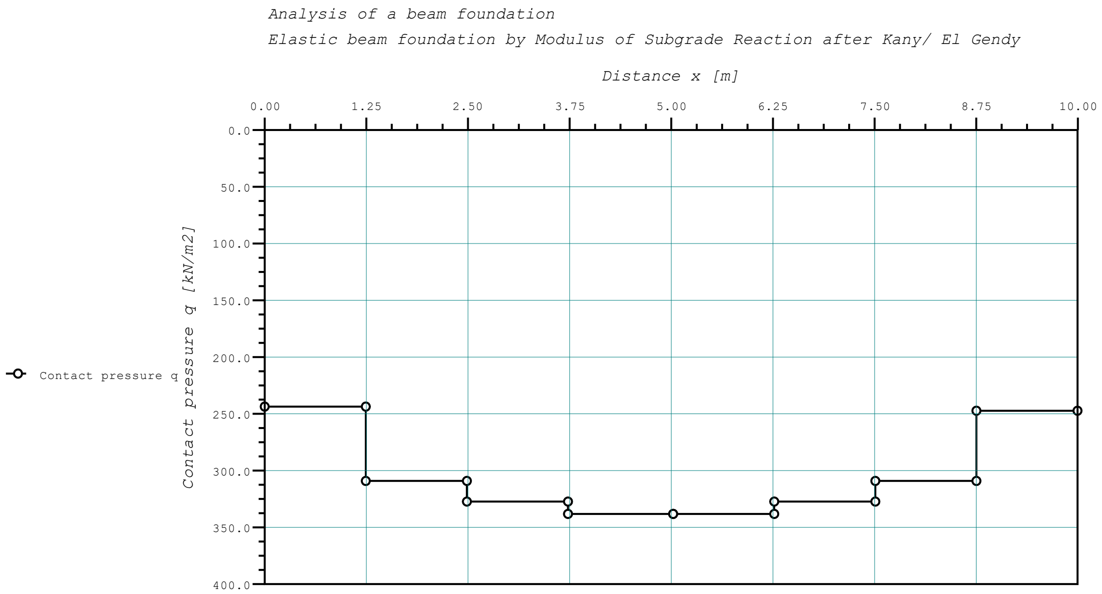
Page No.:	**Project: Analysis of a raft for three equal walls**



Contact pressure

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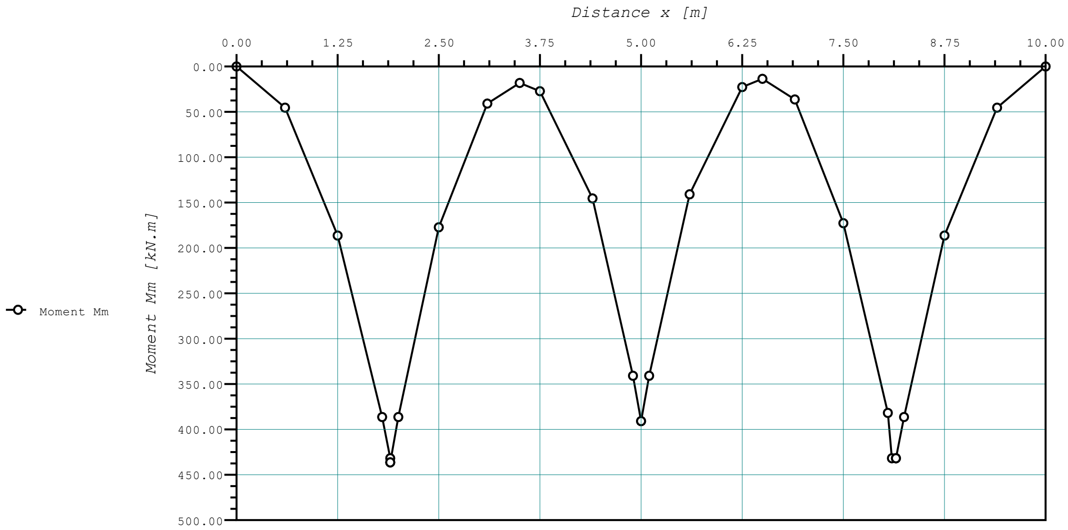
Scale: 50	Title: Beam Foundations after Kany and El Gendy
File: Raft 3Walls	Date: 05-05-2022
Page No.:	Project: Analysis of a raft for three equal walls

Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Moments

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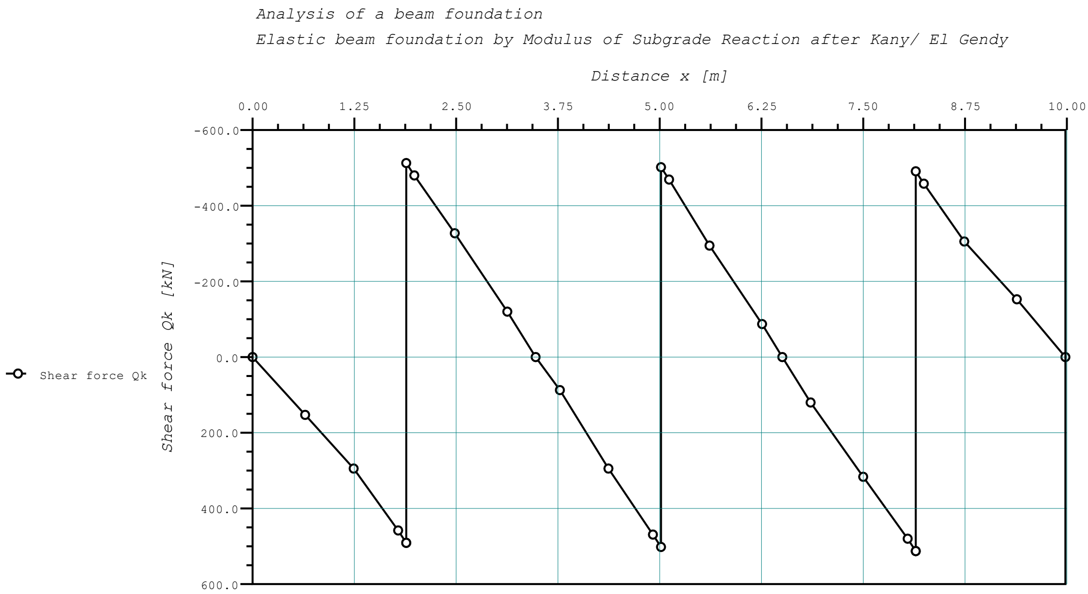
File: Raft 3Walls	**Date: 05-05-2022**
Page No.:	**Project: Analysis of a raft for three equal walls**



Shear Forces

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File: Raft 3Walls	Date: 05-05-2022
Page No.:	Project: Analysis of a raft for three equal walls

10.7 Example5: Analysis of a raft for four equal walls

10.7.1 Description of the problem

Figure 10.1 shows plan and section with dimensions and loads for a raft of four equal walls. It is required to find the contact pressure distribution, settlements, moment and shear force diagrams for the raft. The loading and the raft are symmetrical.

Geometry:

Thickness of the raft	d	=0.6	[m]
Dimensions of the raft	A_f	=8×10	[m^2]
Groundwater depth under the ground surface	Tw	=1	[m]
Foundation depth under the ground surface	Tf	=2	[m]

Material properties of the concrete and unit weight of the water

Modulus of elasticity of the concrete	$E_b = 2\times10^7$	[kN/m^2]
Unit weight of the concrete	γ_b=25	[kN/ m^3]
Unit weight of the water	γ_w=10	[kN/ m^3]

Soil properties

Modulus of subgrade reaction of the soil k_s = 20000 [kN/m^3].

10.7.2 Preparing the calculation

The raft can be regarded as a beam on elastic foundation subjected to:

- A uniformly distributed loading p_f equal to the weight of the raft itself minus the uplift pressure from the ground water.
- Four concentrated forces from four walls $P_1 = P_4$ = 200 [kN/m] and $P_2 = P_3$ = 300 [kN/m].

Computing the uniform load on the raft

Own weight of the raft	$w_o = \gamma_b \times d = 25\times0.6$	=15	[kN/m^2]
Up lift pressure	$w_w = \gamma_w \times (T_f - Tw) = 10(2-1)$	=-10	[kN/m^2]
Total	p_f =	=5	[kN/m^2]

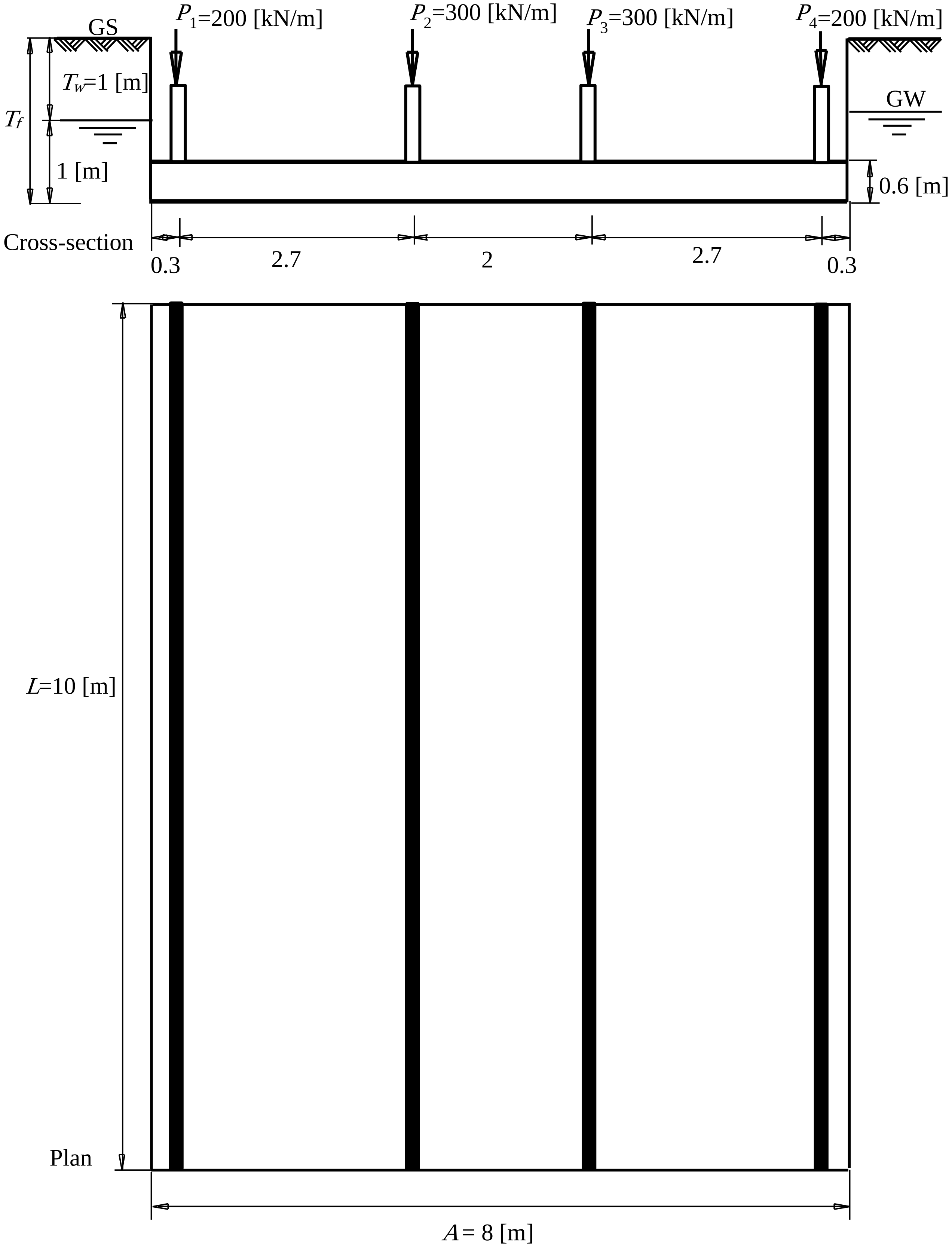


Figure 10.25 Raft of four equal walls

10.7.3 Hand calculation

Assume one-meter strip width from the raft and consider it as a beam on elastic foundation. The beam is divided into eight equal elements, each 1 [m] long (Figure 10.2). Because of the symmetry of the system, the analysis can be carried out by considering only half of the beam. Hence, the total number of equations is reduced to four.

According to *Kany/ El Gendy* (1995), the analysis of a beam on elastic foundation is carried out in the following steps:

10.7.3.1 Calculation of u_i, v_i and w_i:

$$u_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i-1}}\right)$$

$$v_i = \frac{1}{4}\left(\frac{I_i}{I_{i-1}} + 14 + \frac{I_i}{I_{i+1}}\right)$$

$$w_i = \frac{1}{2}\left(1 + \frac{I_i}{I_{i+1}}\right)$$

For a constant beam, moment of inertia $I_i = I$, then

$$u_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

$$v_i = \frac{1}{4}\left(\frac{I}{I} + 14 + \frac{I}{I}\right) = \frac{1}{4} \times 16 = 4$$

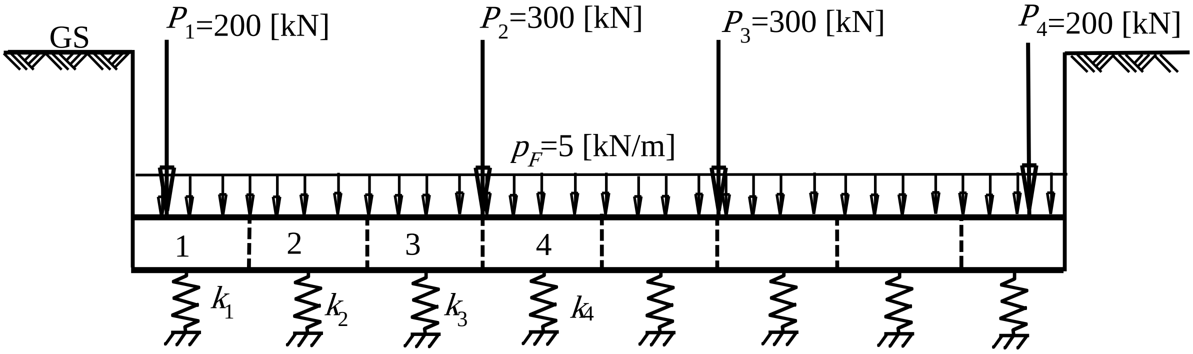
$$w_i = \frac{1}{2}\left(1 + \frac{I}{I}\right) = \frac{1}{2} \times 2 = 1$$

10.7.3.2 Moment of inertia I_i and beam stiffness α_i:

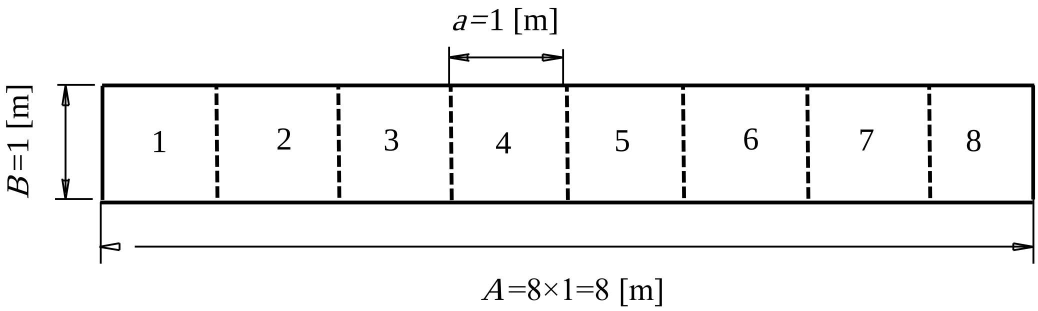
$$I_i = I = \frac{B d_i^3}{12} = \frac{1 \times 0.6^3}{12} = 0.018 [\mathrm{m}^4]$$

and

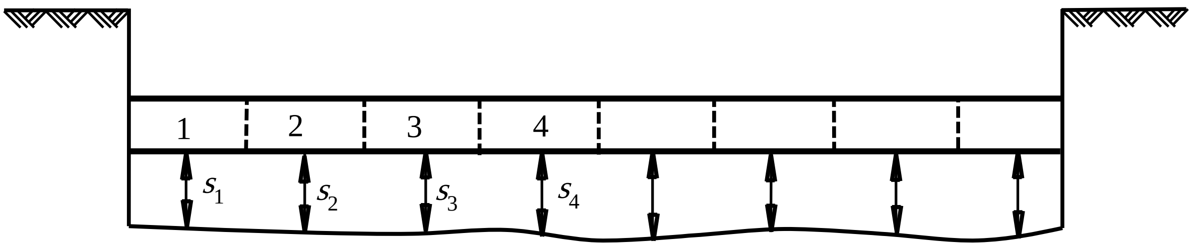
$$\alpha_i = \alpha = \frac{a^4 B}{E_b I} = \frac{1^4 \times 1}{(2 \times 10^7)(0.018)} = \frac{1}{360000} [\mathrm{m}^3/\mathrm{kN}]$$



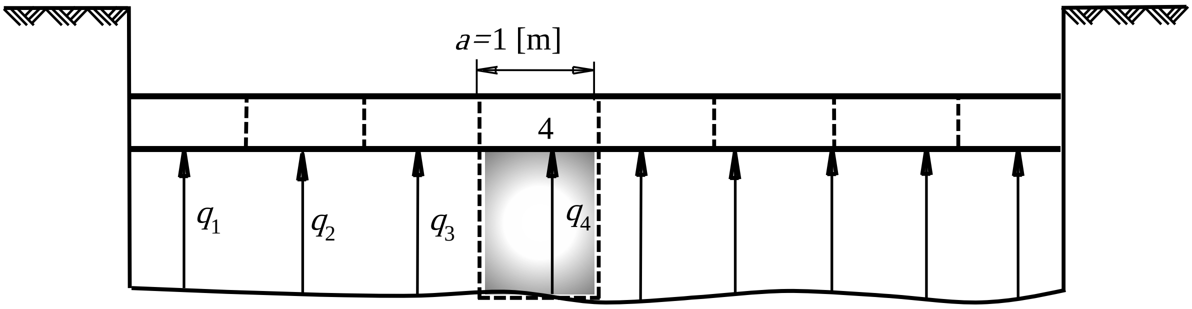
Beam foundation with loads and springs



Plan of beam foundation with elements



Settlement



Contact pressure

Figure 10.26 One meter strip width of the raft

10.7.3.3 Determining external moments $M_i^{(l)}$

The external moments $M_i^{(l)}$ at points 2, 3, 4 and 5 are:

$$M_1^{(l)} = 0$$
$$M_2^{(l)} = 200(1.5 - 0.3) + 5\frac{1.5^2}{2} = 245.625 \text{ [kN.m]}$$
$$M_3^{(l)} = 200(2.5 - 0.3) + 5\frac{2.5^2}{2} = 455.625 \text{ [kN.m]}$$
$$M_4^{(l)} = 200(3.5 - 0.3) + 5\frac{3.5^2}{2} + 300 \times 0.5 = 820.625 \text{ [kN.m]}$$
$$M_5^{(l)} = 200(4.5 - 0.3) + 5\frac{4.5^2}{2} + 300 \times 1.5 = 1340.625 \text{ [kN.m]}$$

10.7.3.4 Determining the right hand side R_i

The right hand side R_i of the contact pressure equation is:

$$R_i = \left(M^{(l)}{}_{i-1} + 4\, M^{(l)}{}_i + M^{(l)}{}_{i+1}\right)\frac{a^2}{6E\, I_i}$$
$$R_i = \left(M^{(l)}{}_{i-1} + 4\, M^{(l)}{}_i + M^{(l)}{}_{i+1}\right)\frac{1^2}{6 \times 2 \times 10^7 . \times 0.018}$$
$$R_i = \frac{1}{2160000}\left(M^{(l)}{}_{i-1} + 4\, M^{(l)}{}_i + M^{(l)}{}_{i+1}\right)$$

Apply the above equation at points 2, 3 and 4:

$$R_2 = \frac{1}{2160000}(0 + 4 \times 245.625 + 455.625) = \frac{1438.125}{216000}$$
$$R_3 = \frac{1}{2160000}(245.625 + 4 \times 455.625 + 820.625) = \frac{2888.75}{216000}$$
$$R_4 = \frac{1}{2160000}(455.625 + 4 \times 820.625 + 1340.625) = \frac{5078.75}{216000}$$

10.7.3.5 Determining contact pressures

The contact pressure equation is:

$$\left(\frac{1}{k_{i+1}}\right) q_{i+1} - \left(\frac{2}{k_i} - \frac{\alpha_i}{6} w_i\right) q_i + \left(\frac{1}{k_{i-1}} + \frac{\alpha_i}{6}(v_i + 2w_i)\right) q_{i-1} + \frac{\alpha_i}{6}\left(\sum_{j=1}^{i-2}[(i-j-1)u_i + (i-j)v_i + (i-j+1)w_i]\, q_j\right) = R_i$$

$$\left(\frac{1}{k}\right) q_{i+1} - \left(\frac{2}{k} - \frac{\alpha}{6}\right) q_i + \left(\frac{1}{k} + \alpha\right) q_{i-1} + \alpha\left(\sum_{j=1}^{i-2} (i-j)\, q_j\right) = R_i$$

$$\left(\frac{1}{20000}\right) q_{i+1} - \left(\frac{2}{20000} - \frac{1}{360000\times 6}\right) q_i + \left(\frac{1}{20000} + \frac{1}{360000}\right) q_{i-1} + \frac{1}{360000}\left(\sum_{j=1}^{i-2} (i-j) q_j\right) = R_i$$

or

$$108\ q_{i+1} - 215\ q_i + 114\ q_{i-1} + 6\left(\sum_{j=1}^{i-2} (i-j) q_j\right) = 2160000\ R_i$$

Apply the above equation at points 2, 3 and 4:

$$108\ q_3 - 215\ q_2 + 114\ q_1 = 1438.125$$

$$108\ q_4 - 215\ q_3 + 114\ q_2 + 12\ q_1 = 2888.75$$

$$-107\ q_4 - 114\ q_3 + 12\ q_2 + 18\ q_1 = 5078.75$$

There are four unknown q_1, q_2, q_3, and q_4, so a farther equation is required. This can be obtained by considering the overall equilibrium of vertical forces.

$$a \times B\left(q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8\right) = P_1 + P_2 + P_3 + P_4 + A \times B \times P_f$$

or

$$q_1 + q_2 + q_3 + q_4 = 520$$

Contact pressure equations in matrix form:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 114 & -215 & 108 & 0 \\ 12 & 114 & -215 & 108 \\ 18 & 12 & 114 & -107 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 520 \\ 1438.125 \\ 2888.75 \\ 5078.75 \end{Bmatrix}$$

Solving the above system of linear equations using *Gaussian's* elimination to obtain the contact pressures q_1, q_2, q_3, and q_4.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2.886 & -0.053 & -1 \\ 0 & 8.5 & -18.917 & 8 \\ 0 & -0.333 & 5.333 & -6.944 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 520 \\ -507.385 \\ -279.271 \\ -237.845 \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0.982 & 0.653 \\ 0 & 1 & 0.018 & 0.347 \\ 0 & 0 & -2.244 & 0.594 \\ 0 & 0 & -16.018 & 20.486 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 344.191 \\ 175.809 \\ -208.664 \\ 537.739 \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0.913 \\ 0 & 1 & 0 & 0.352 \\ 0 & 0 & 1 & -0.265 \\ 0 & 0 & 0 & -1.014 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 252.877 \\ 174.135 \\ 92.988 \\ -126.559 \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} 138.925 \\ 130.202 \\ 126.063 \\ 124.811 \end{Bmatrix}$$

q_1 =	139.925	[kN/m^2]
q_2=	130.202	[kN/m^2]
q_3=	126.063	[kN/m^2]
q_4=	124.811	[kN/m^2]

10.7.3.6 Determining settlements s_i

The settlement s_i can be given by:

$$s_i = \frac{q_i}{k_i} = \frac{q_i}{20000} \text{ [m]}$$

s_1 = 0.70 [cm]
s_2= 0.65 [cm]
s_3= 0.63 [cm]
s_4= 0.62 [cm]

The contact pressure distribution, settlement, moment and shear force diagrams for the raft are shown in Figure 10.3 to Figure 10.6. Once the internal forces are obtained at various sections, the design of the raft can be completed in the normal manner.

10.7.3.7 Computer calculation

The input data and results of *GEO Tools* are presented on the pages 10.15 to 10.26. By comparison, one can see an agreement with the hand calculation.

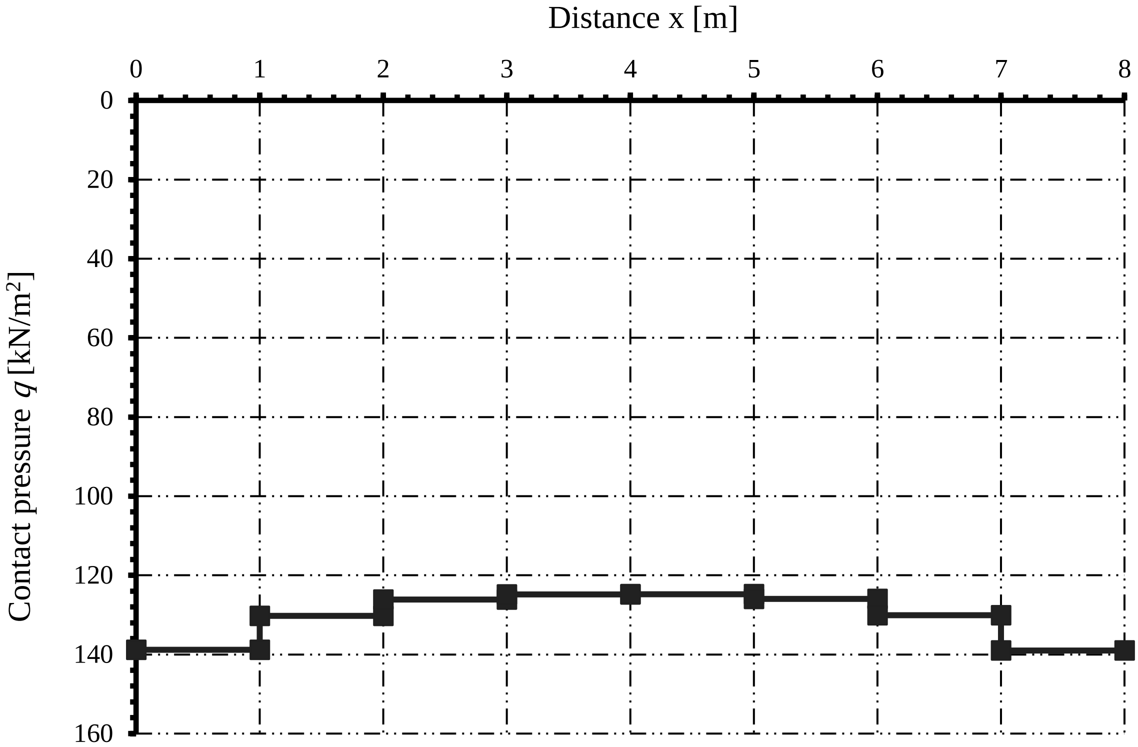


Figure 10.27 Contact pressures

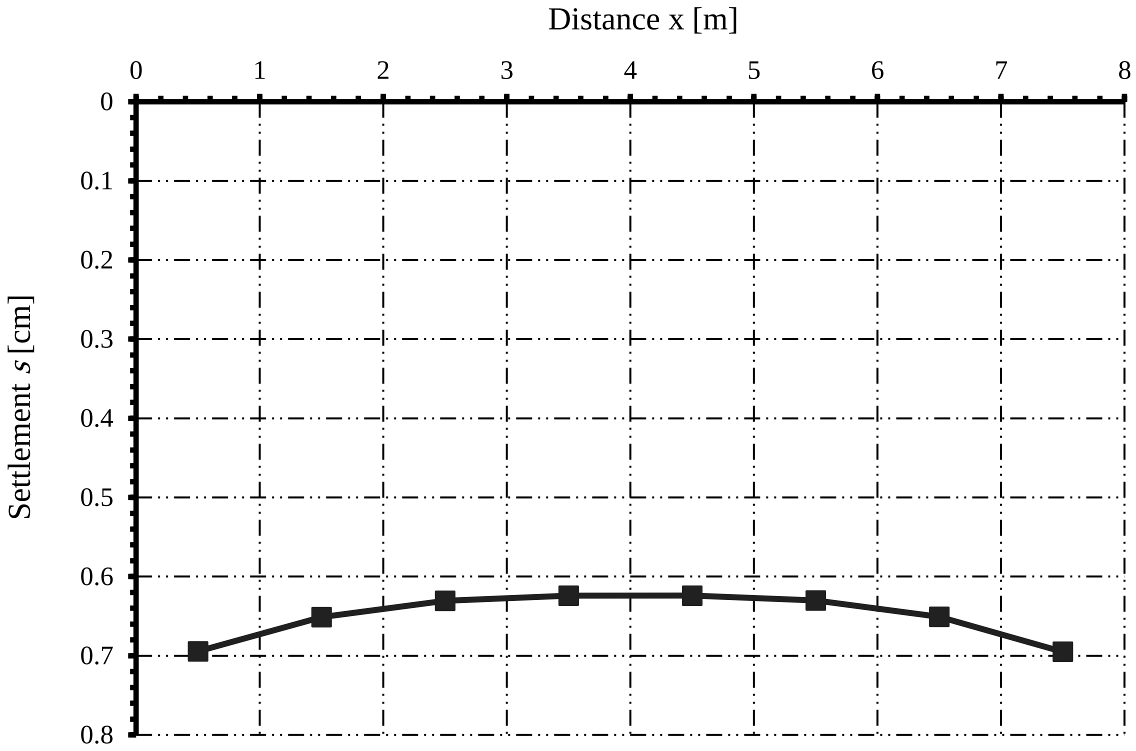


Figure 10.28 Settlements

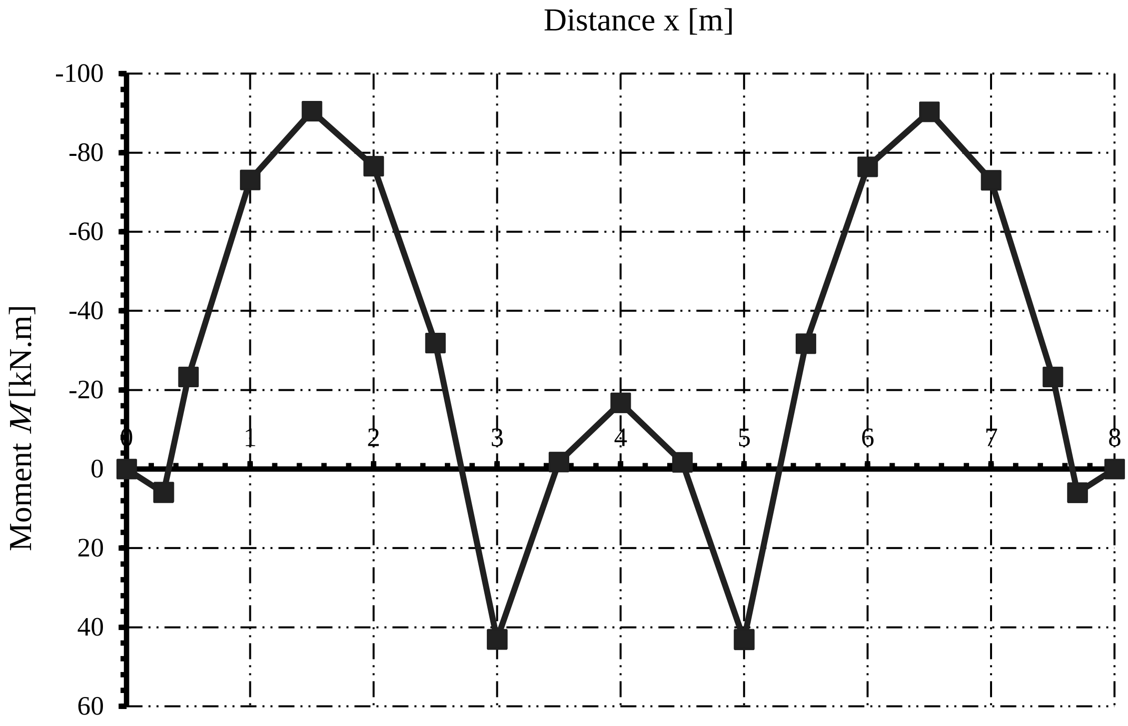


Figure 10.29 Moments

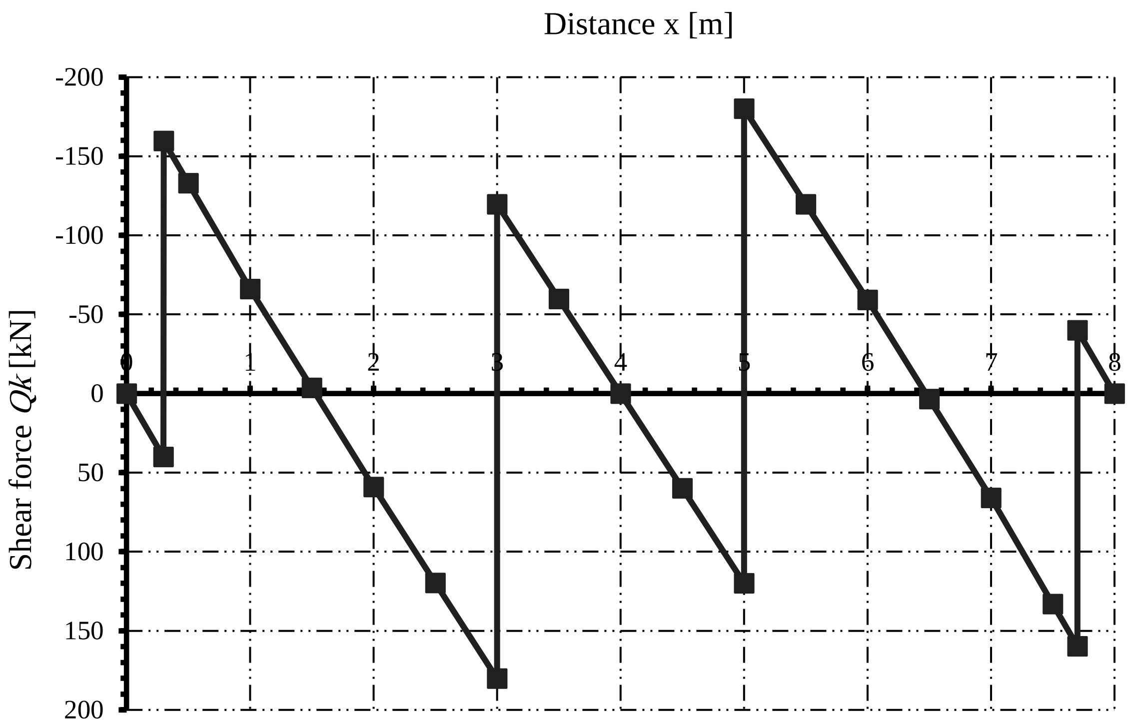


Figure 10.30 Shear forces

```
*************************************************************
                        GEO Tools
                        Version 12.3
          Program authors M. El Gendy/ A. El Gendy
*************************************************************
Title: Beam Foundations after Kany and El Gendy
Date: 05-05-2022
Project: Analysis of a raft for four equal walls
File: Raft 4Walls

-------------------------------------------------------------
Analysis of beam a foundation
Calculation method: Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy
-------------------------------------------------------------

Data:

Main Soil Data:

Groundwater depth under the ground surface   Tw           [m]       = 1.00
Foundation depth under ground surface        Tf           [m]       = 2.00

Summary of loading:

Self weight                                  Pe           [kN]      = 120
Load on Footing                              Pa           [kN]      = 920
Groundwater force                            Pw           [kN]      = 80
Total load                                   Po=Pe+Pa-Pw  [kN]      = 1040

Groundwater pressure                         Qw           [kN/m2]   = 10.0
Average soil pressure                        Qo           [kN/m2]   = 130.0

Beam Material:

Modulus of elasticity of the concrete        Eb           [kN/m2]   = 20000000.00
Unit weight of footing concrete              γb           [kN/m3]   = 25.0

Dimensions:

Depth of the foundation surface under ground Tk           [m]       = 1.40
Beam thickness                               d            [m]       = 0.60
Moment of inertia of the beam                I            [m4]      = 0.018
Beam stiffness                               αB           1/[kN/m3] = 2.78E-06
Beam length (longitudinal)                   A            [m]       = 8.00
Beam width (transversal)                     B            [m]       = 1.00
Length/width ratio                           A/B          [-]       = 8.00
Element size                                 a            [m]       = 1.00
Number of elements of the beam               N            [-]       = 8

Loads:

Point Loads:
```

No. I [-]	Load value P [kN]	Load position from the left edge Xp [m]	Column side a [m]	Column side b [m]	Column label Lb [-]
1	200	0.30	0.20	1.00	W1
2	300	3.00	0.20	1.00	W2
3	300	5.00	0.20	1.00	W3
4	200	7.70	0.20	1.00	W4

Distributed Loads:

No. I [-]	Load value Pf [kN/m2]	Load start from the left edge Xpl [m]	Load end from the left edge Xpr [m]	Load type [-]
1	-10	0.00	8.00	(Groundwater pressure)
2	15	0.00	8.00	(Self weight)

```
Right sides of the system of equations:
--------------------------------
```

Element No. I [-]	Right sides of the system of equations Rv [m]
1	4.16E+03
2	1.04E+03
3	6.658E-04
4	1.3374E-03
5	2.3513E-03
6	3.7957E-03
7	5.6707E-03
8	7.9068E-03

Settlements/ Contact pressures/ Moduli of subgrade reactions:

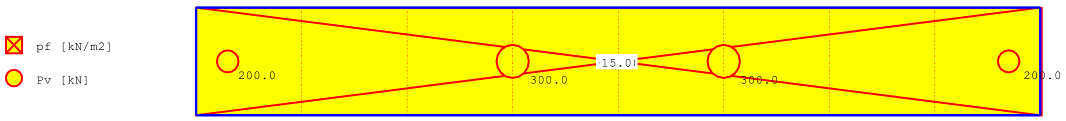
Element No. I [-]	Contact pressure q [kN/m2]	Settlement s [cm]	Modulus of subgrade reaction ks [kN/m3]
1	138.9	0.69	20000
2	130.3	0.65	20000
3	126.1	0.63	20000
4	124.8	0.62	20000
5	124.8	0.62	20000
6	126.0	0.63	20000
7	130.1	0.65	20000
8	139.0	0.70	20000

Moments/ Shear Forces:

No. I [-]	Distance x [m]	Distance [-]	Moment Mm [kN.m]	Shear force Qk [kN/m]
1	0.00		0.0	0.0
2	0.20	CL	2.7	26.8
3	0.30	CC	6.0	40.2
4	0.30	CC	5.9	-159.7
5	0.40	CR	-9.3	-146.5
6	0.50		-23.3	-133.1
7	1.00		-73.1	-66.1
8	1.50		-90.5	-3.5
9	1.53	MM	-90.5	0.0
10	2.00		-76.6	59.1
11	2.50		-31.9	119.7
12	2.90	CL	25.7	168.1
13	3.00	CC	43.1	180.2
14	3.00	CC	43.0	-119.6
15	3.10	CR	31.7	-107.8
16	3.50		-1.8	-59.9
17	4.00	MM	-16.7	0.0
18	4.00		-16.7	0.1
19	4.50		-1.7	60.0
20	4.90	CL	31.8	107.9
21	5.00	CC	43.2	119.9
22	5.00	CC	43.0	-180.0
23	5.10	CR	25.8	-168.0
24	5.50		-31.7	-119.6
25	6.00		-76.4	-59.1
26	6.47	MM	-90.4	0.0
27	6.50		-90.3	3.4
28	7.00		-73.0	66.0
29	7.50		-23.2	133.0
30	7.60	CL	-9.3	146.4
31	7.70	CC	6.0	159.8
32	7.70	CC	6.0	-40.1
33	7.80	CR	2.7	-26.8
34	8.00		0.0	0.0

Analysis of beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



System of loading

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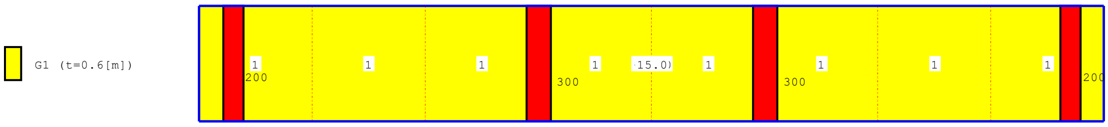
Scale: 38	**Title: Beam Foundations after Kany and El Gendy**
File: Raft 4Walls	**Date: 05-05-2022**
Page No.:	**Project: Analysis of a raft for four equal walls**

Analysis of beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Element groups t [m]

No. of element groups = 1

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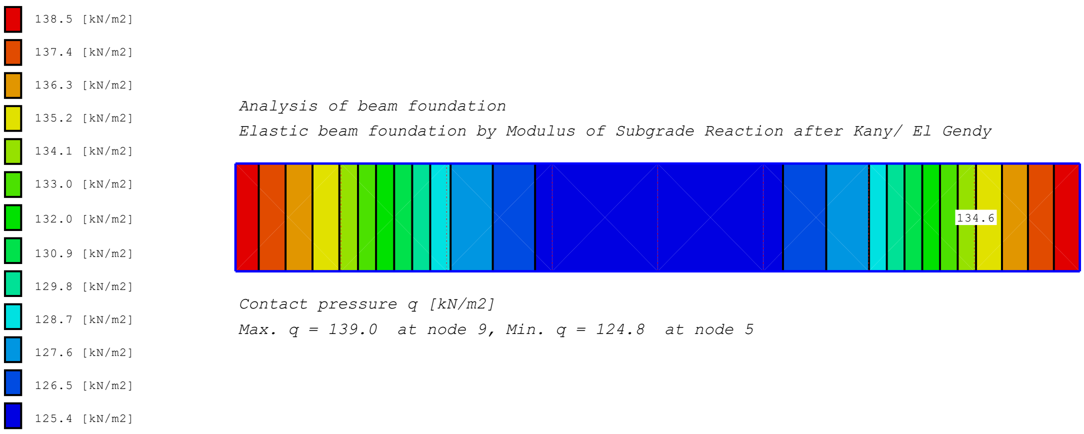
Scale: 36	**Title: Beam Foundations after Kany and El Gendy**
File: Raft 4Walls	**Date: 05-05-2022**
Page No.:	**Project: Analysis of a raft for four equal walls**



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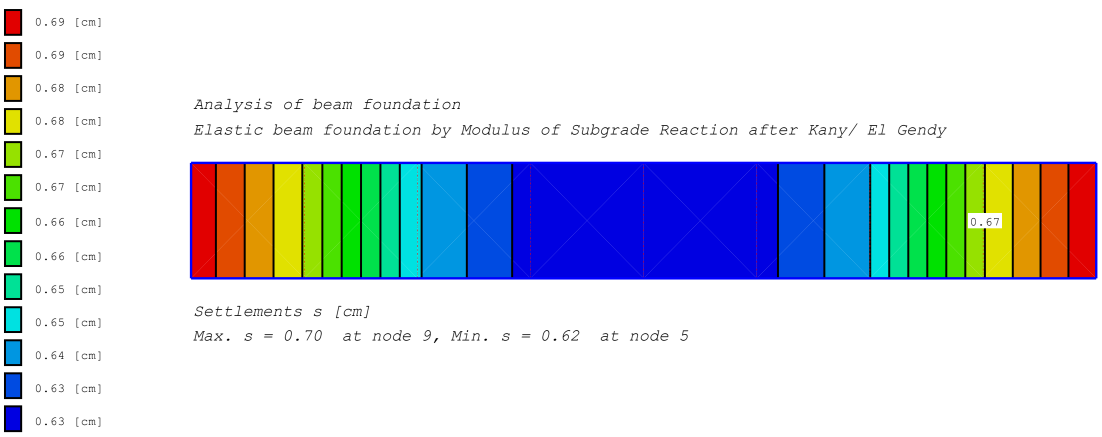
Scale: 38	Title: Beam Foundations after Kany and El Gendy
File: Raft 4Walls	Date: 05-05-2022
Page No.:	Project: Analysis of a raft for four equal walls



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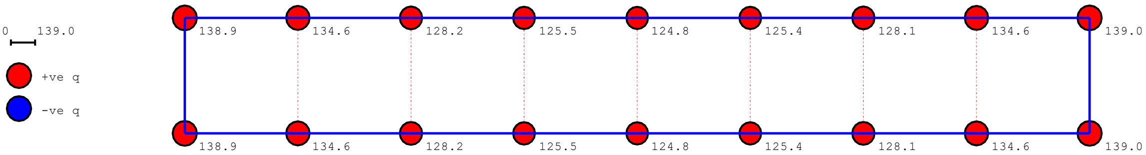
Analysis of beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Contact pressure q [kN/m2]

Max. q = 139.0 at node 9, Min. q = 124.8 at node 5

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Scale: 36	**Title: Beam Foundations after Kany and El Gendy**
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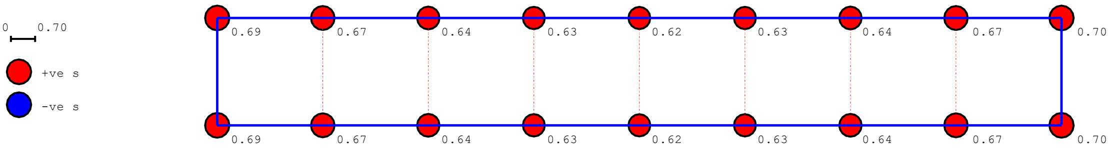
Page No.:	**Project: Analysis of a raft for four equal walls**

Analysis of beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Settlements s [cm]

Max. s = 0.70 at node 9, Min. s = 0.62 at node 5

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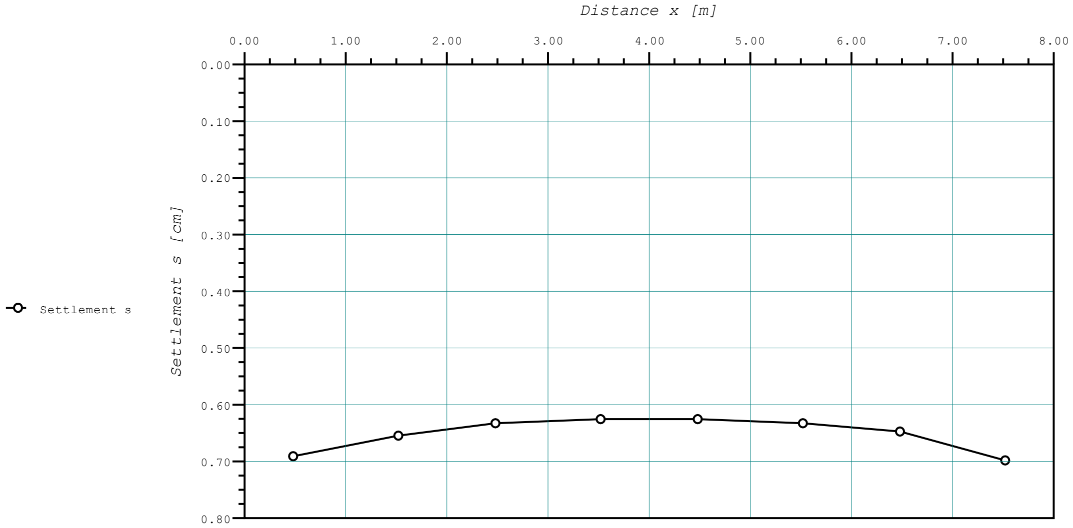
Page No.:	Project: Analysis of a raft for four equal walls

Analysis of a beam foundation
Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Settlements

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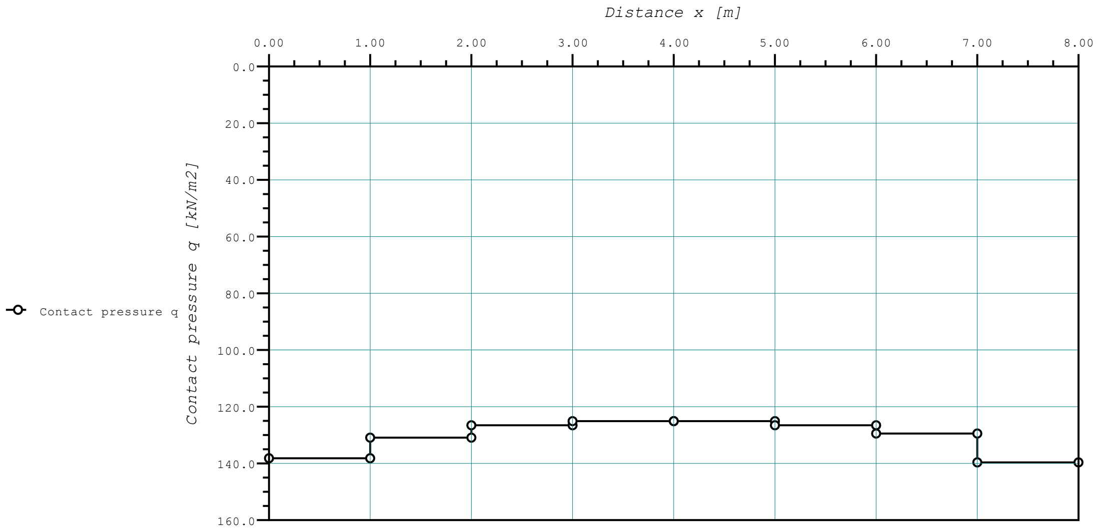
File: Raft 4Walls	**Date: 05-05-2022**
Page No.:	**Project: Analysis of a raft for four equal walls**

Analysis of a beam foundation

Elastic beam foundation by Modulus of Subgrade Reaction after Kany/ El Gendy



Contact pressure

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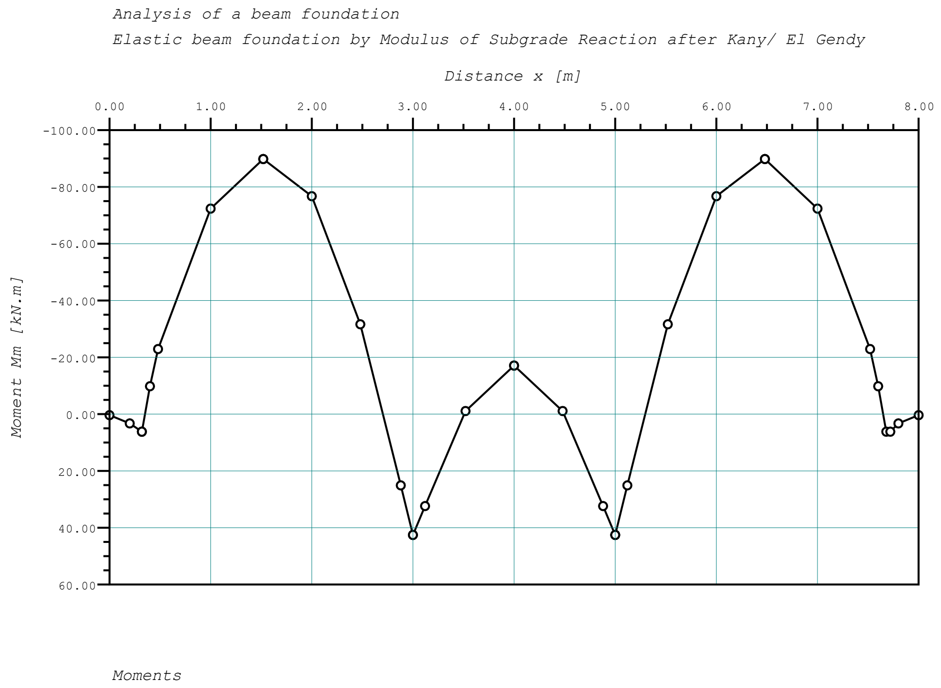
Page No.:	Project: Analysis of a raft for four equal walls



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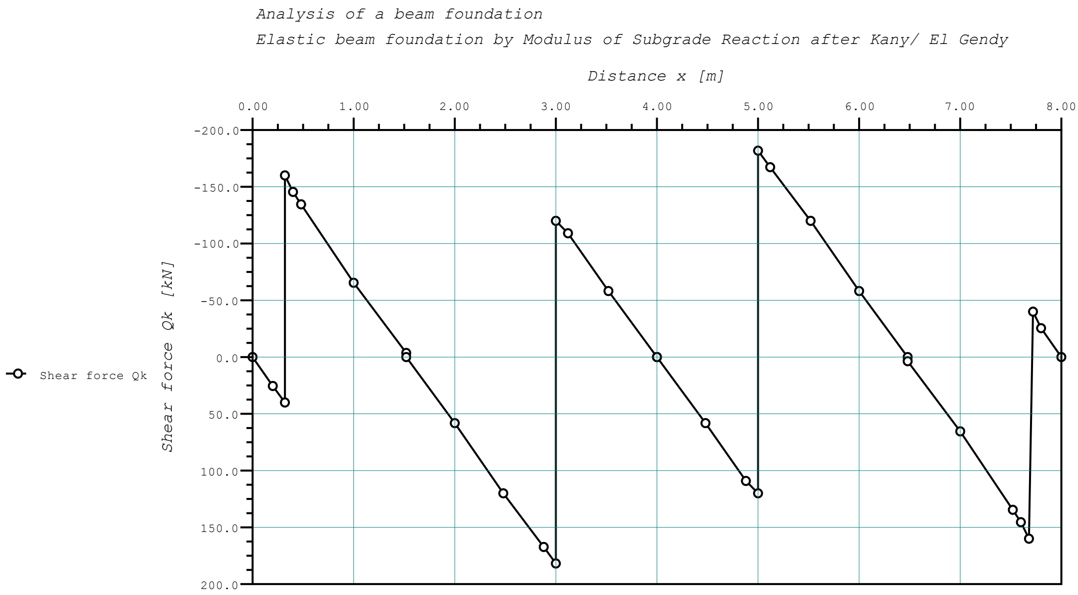
Scale: 40 File: Raft 4Walls Page No.:	Title: Beam Foundations after Kany and El Gendy Date: 05-05-2022 Project: Analysis of a raft for four equal walls



Shear Forces

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File: Raft 4Walls	Date: 05-05-2022
Page No.:	Project: Analysis of a raft for four equal walls

10.8 Example 6: Analysis of flexible and rigid beams

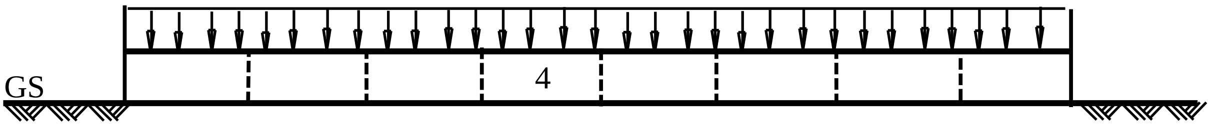
10.8.1 Description of the problem

Error! Reference source not found. shows a beam foundation having dimensions of 1×8 [m^2] and a uniform load of 100 [kN/m^2]. The subsoil under the beam is an Isotropic Elastic Half Space Soil Medium with Modulus of Compressibility of E_s = 5000 [kN/m^2] and *Poission's* ratio ν_s=0.

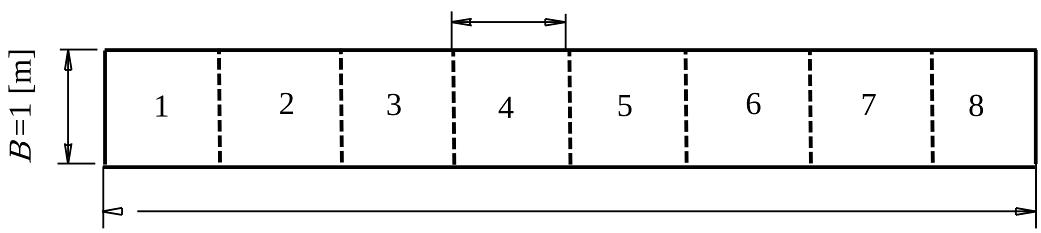
It is required to determine numerically:

a) The settlement under the flexible beam.

b) The settlement and contact pressure under the rigid beam.



Beam foundation with loads



Plan of beam foundation with elements

Figure 10.31 Beam foundation on an Isotropic Elastic Half Space Soil Medium

Geometry:

Dimensions of the beam = 8.0 [m] × 1.0 [m]

Soil properties

Modulus of elasticity of the soil E_s =5000 [kN/m^2]

Loads on the beam

Uniform load p_f=100[kN/m^2]

10.8.2 Hand calculation

Consider the footing as a rigid beam on elastic foundation. The beam is divided into eight equal elements, each 1.0 [m] long (Figure 10.31). Because of the symmetry of the system, the analysis can be carried out by considering only half of the beam. Hence, the total number of equations is reduced to four.

The analysis of a rigid and flexible beam on elastic foundation is carried out in the following steps:

10.8.2.1 Determining flexibility coefficients

10.8.2.1.1 Flexibility coefficients *co, o* of point *o* due to a load at that point *o*

The settlement $s_{o,o}$ at the center of a circular element o of a radius r_o [m] having a circular loaded area of intensity q_o [kN/m^2] $= Q_o / \pi r_o^2$ acting on the surface as shown in Figure 10.32 is given by:

$$s_{o,o} = \frac{2 q_o \left(1 - \nu_s^2\right) r_o}{E_s} = \frac{2 Q_o \left(1 - \nu_s^2\right)}{\pi r_o E}$$

or

$$s_{o,o} = c_{o,\,o} Q_o$$

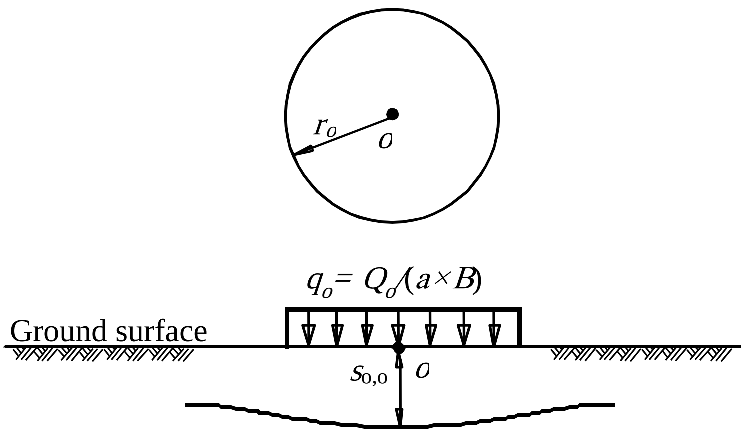


Figure 10.32 Circular loaded area on the surface

where $c_{o,o}$ is the flexibility coefficient of point o due to a load at that point o [m/kN].

This coefficient is given by:

$$c_{o,o} = \frac{2\left(1 - \nu_s^2\right)}{\pi r_o E} = \frac{2\left(1 - 0\right)}{\pi\, 5000\, r_o}$$

For simplicity, the rectangular element of size $B \times a = 1 \times 1$ is converted to an equivalent circular area.

$$\pi\, r_0^2 = a \times 1\, m \quad so \quad r_0 = 0.5642 \text{ [m]}$$

Flexibility coefficient $c_{o,o}$ [m/kN] due to contact force Q_o [kN] under the same point

$$C_{o,o} = \frac{2\,(1-\nu_s{}^2)}{\pi\, r_0\, E_s} = \frac{2(1-0)}{\pi \times 0.5642 \times 5000} = 22.5672 \times 10^{-5}\ [\mathrm{m/kN}]$$

10.8.2.1.2 Flexibility coefficients *ci, j* of point *i* due to a concentrated load at point *j*

The settlement $s_{i,j}$ [m] at point i due to a concentrated load Q_j [kN] at point j for isotropic elastic half-space soil medium (Figure 10.33) is given by:

$$s_{i,j} = \frac{Q_j\,(1-\nu_s^2)}{\pi\, E\, r_{i,j}}$$

or

$$s_{i,j} = c_{i,j}\, Q_j$$

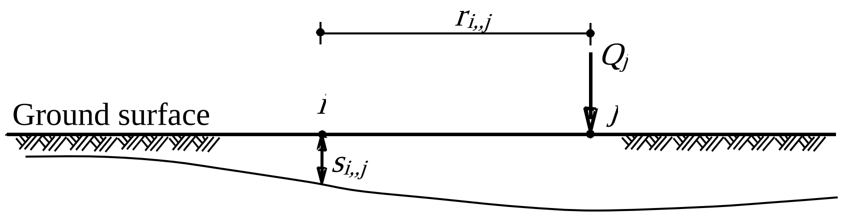


Figure 10.33 Isotropic elastic half-space soil medium

where $r_{i,j}$ [m] is the radial distance between points i and j [m] and $c_{i,j}$ [m/kN] is the flexibility coefficient of a point i due to a load Q_j [kN] at point j.

This coefficient is given by:

$$c_{i,j} = \frac{(1-\nu_s^2)}{\pi\, E\, r_{i,j}} = \frac{1}{\pi\, 5000\, r_{i,j}}$$

The flexibility coefficients $c_{i,j}$ and c_i are calculated in Table 10.1.

Table 10.1 Flexibility coefficients c_i and $c_{i,j}$

Flexibility coefficient ci	$c_{i,j}$= $c_{j,i}$	$c_{i,j}$= $c_{j,i}$	$c_{i,j}$= $c_{j,i}$	$c_{i,j}$= $c_{j,i}$	Radial distance $r_{i,j}$ [m]	Flexibility coefficient $c_{i,j}=\frac{6.3662 * 10^{-5}}{r_{i,j}}$ [m/kN]
c_0	$c_{1,1}$	$c_{2,2}$	$c_{3,3}$	$c_{4,4}$	0	22.5672×10^{-5}
c_1	$c_{1,2}$	$c_{2,3}$	$c_{3,4}$	$c_{4,5}$	a=1	6.3662×10^{-5}
c_2	$c_{1,3}$	$c_{2,4}$	$c_{3,5}$	$c_{4,6}$	2a=2	3.1831×10^{-5}
c_3	$c_{1,4}$	$c_{2,5}$	$c_{3,6}$	$c_{4,7}$	3a=3	2.1221×10^{-5}
c_4	$c_{1,5}$	$c_{2,6}$	$c_{3,7}$	$c_{4,8}$	4a=4	1.5916×10^{-5}
c_5	$c_{1,6}$	$c_{2,7}$	$c_{3,8}$		5a=5	1.2732×10^{-5}
c_6	$c_{1,7}$	$c_{2,8}$			6a=6	1.061×10^{-5}
c_7	$c_{1,8}$				7a=7	9.0946×10^{-6}

10.8.2.1.3 Settlement-contact pressure equations

For B=1 [m] and a=1 [m]

$$Q_j = q_j a\, b = q_j$$

And due to the symmetry

$$q_1 = q_8, q_2 = q_7,\ q_3 = q_6, q_4 = q_5$$

The settlement at the center of the element is given by:

$$s_i = \sum_{j=1}^{i} c_{i-j}\, q_j + \sum_{j=i+1}^{n} c_{j-i}\, q_j$$

Applying above equation at points 1 to 4

$$s_1 = (c_0 + c_7)\, q_1 + (c_1 + c_6)\, q_2 + (c_2 + c_5)\, q_3 + (c_3 + c_4)\, q_4$$

$$s_2 = (c_1 + c_6)\, q_1 + (c_0 + c_5)\, q_2 + (c_1 + c_4)\, q_3 + (c_2 + c_3)\, q_4$$

$$s_3 = (c_2 + c_5)\, q_1 + (c_1 + c_4)\, q_2 + (c_0 + c_3)\, q_3 + (c_1 + c_2)\, q_4$$

$$s_4 = (c_3 + c_4)\, q_1 + (c_2 + c_3)\, q_2 + (c_1 + c_2)\, q_3 + (c_0 + c_1)\, q_4$$

or

$$s_1 = 23.477 \times 10^{-5}\, q_1 + 7.427 \times 10^{-5} q_2 + 4.456 \times 10^{-5}\, q_3 + 3.714 \times 10^{-5}\, q_4$$

$$s_2 = 7.427 \times 10^{-5} q_1 + 23.840 \times 10^{-5}\, q_2 + 7.958 \times 10^{-5} q_3 + 5.305 \times 10^{-5}\, q_4$$

$$s_3 = 4.456 \times 10^{-5} q_1 + 7.958 \times 10^{-5} q_2 + 24.689 \times 10^{-5} q_3 + 9.549 \times 10^{-5}\, q_4$$

$$s_4 = 3.714 \times 10^{-5} q_1 + 5.305 \times 10^{-5} q_2 + 9.549 \times 10^{-5} q_3 + 28.933 \times 10^{-5} q_4$$

Settlement-contact pressure in matrix form:

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = 10^{-5} \begin{bmatrix} 23.477 & 7.427 & 4.456 & 3.714 \\ 7.427 & 23.840 & 7.958 & 5.305 \\ 4.456 & 7.958 & 24.689 & 9.549 \\ 3.714 & 5.305 & 9.549 & 28.933 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

10.8.2.1.4 Determining flexible settlements si

For flexible beam analysis q_1, q_2, q_3, and q_4 are known, while s_1, s_2, s_3, and s_4 are required to determine.

Substituting q_1=q_2=q_3=q_4=100 [kN/m^2] in matrix equation of the settlement-contact pressure:

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = 10^{-5} \begin{bmatrix} 23.477 & 7.427 & 4.456 & 3.714 \\ 7.427 & 23.840 & 7.958 & 5.305 \\ 4.456 & 7.958 & 24.689 & 9.549 \\ 3.714 & 5.305 & 9.549 & 28.933 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}$$

Gives the flexible settlements at point 1 to 4:

s_1 = 3.91[cm]

s_2 = 4.45 [cm]

s_3 = 4.67 [cm]

s_4 = 4.75 [cm]

10.8.2.1.5 Determining rigid settlements so

For rigid beam analysis s_1, s_2, s_3, and s_4 are equal and have the same value s_o. The unknown of the problem are s_o, q_1, q_2, q_3, and q_4.

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = 10^{-5} \begin{bmatrix} 23.477 & 7.427 & 4.456 & 3.714 \\ 7.427 & 23.840 & 7.958 & 5.305 \\ 4.456 & 7.958 & 24.689 & 9.549 \\ 3.714 & 5.305 & 9.549 & 28.933 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Inversing the flexibility matrix, gives:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 4782.052 & -1319.26 & -337.01 & -260.732 \\ -1319.26 & 5108.15 & -1274.31 & -346.683 \\ -337.01 & -1274.31 & 5060.878 & -1393.37 \\ -260.732 & -346.683 & -1393.37 & 4013.162 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$$

For rigid beam $s_1 = s_2 = s_3 = s_4 = s_o$ [m]

Then

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 4782.052 & -1319.26 & -337.01 & -260.732 \\ -1319.26 & 5108.15 & -1274.31 & -346.683 \\ -337.01 & -1274.31 & 5060.878 & -1393.37 \\ -260.732 & -346.683 & -1393.37 & 4013.162 \end{bmatrix} \begin{bmatrix} s_o \\ s_o \\ s_o \\ s_o \end{bmatrix}$$

Multiplying both sides by *a.B*

$$\begin{bmatrix} a.B.q_1 \\ a.B.q_2 \\ a.B.q_3 \\ a.B.q_4 \end{bmatrix} = a.B \text{ x} \begin{bmatrix} 4782.052 & -1319.26 & -337.01 & -260.732 \\ -1319.26 & 5108.15 & -1274.31 & -346.683 \\ -337.01 & -1274.31 & 5060.878 & -1393.37 \\ -260.732 & -346.683 & -1393.37 & 4013.162 \end{bmatrix} \begin{bmatrix} s_o \\ s_o \\ s_o \\ s_o \end{bmatrix}$$

or

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} 4782.052 & -1319.26 & -337.01 & -260.732 \\ -1319.26 & 5108.15 & -1274.31 & -346.683 \\ -337.01 & -1274.31 & 5060.878 & -1393.37 \\ -260.732 & -346.683 & -1393.37 & 4013.162 \end{bmatrix} \begin{bmatrix} s_o \\ s_o \\ s_o \\ s_o \end{bmatrix}$$

Expanding the above equation matrix for all elements and equating all settlements by uniform rigid body translation s_o, yields to the contact forces as a function in s_o as follows:

$$Q_1 = 4782.052 s_o - 1319.26 s_o - 337.01 s_o - 260.732 s_o$$

$$Q_2 = -1319.26 s_o + 5108.15 s_o - 1274.31 s_o - 346.683 s_o$$

$$Q_3 = -337.01 s_o - 1274.31 s_o + 5060.878 s_o - 1393.37 s_o$$

$$Q_4 = -260.732 s_o - 346.683 s_o - 1393.37 s_o + 4013.162 s_o$$

Carrying out the summation of all contact forces, leads to:

$$\sum_{i=1}^{4} Q_i \quad = 9101.498 s_o$$

Replacing the sum of all contact forces by the resultant force *N/2*=100×1×8/2=400 [kN], gives rigid body translation s_o, which equals to the settlement s_i at all elements.

$$400 = 9101.498 s_o$$

The rigid settlement is

$$s_o = 4.3949 \text{ [cm]}$$

10.8.2.1.6 Determining rigid contact pressures qi

Substituting the uniform rigid body translation s_o=0.043949 gives the *n* unknown contact pressures q_i by:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 4782.052 & -1319.26 & -337.01 & -260.732 \\ -1319.26 & 5108.15 & -1274.31 & -346.683 \\ -337.01 & -1274.31 & 5060.878 & -1393.37 \\ -260.732 & -346.683 & -1393.37 & 4013.162 \end{bmatrix} \begin{bmatrix} 0.043949 \\ 0.043949 \\ 0.043949 \\ 0.043949 \end{bmatrix}$$

$$q_1 = 125.92 \text{ [kN/m}^2\text{]}$$

$$q_2 = 95.28 \text{ [kN/m}^2\text{]}$$

$$q_3 = 90.36 \text{ [kN/m}^2\text{]}$$

$$q_4 = 88.44 \text{ [kN/m}^2\text{]}$$

10.8.2.2 Computer calculation

The input data and results of *GEO Tools* are presented on the pages 10.114 to 10.115. By comparison, one can see an agreement with the hand calculation.

```
********************************************************
                        GEO Tools
                        Version 12.3
          Program authors M. El Gendy/ A. El Gendy
********************************************************
Title: Beam Foundations after Kany and El Gendy
Date: 05-05-2022

Data:

Main Soil Data:

Groundwater depth under the ground surface   Tw          [m]     = 0.00
Foundation depth under ground surface        Tf          [m]     = 0.00
Settlement reduction factor                  α           [-]     = 1.00

Summary of loading:

Self weight                                  Pe          [kN]    = 0.000
Load on Footing                              Pa          [kN]    = 800.000
Groundwater force                            Pw          [kN]    = 0.000
Total load                                   Po=Pe+Pa-Pw [kN]    = 800.000

Groundwater pressure                         Qw          [kN/m2] = 0.0
Average soil pressure                        Qo=Qu+Qe    [kN/m2] = 100.0

Dimensions:

Beam length (longitudinal)                   A           [m]     = 8.00
Beam width (transversal)                     B           [m]     = 1.00
Length/width ratio                           A/B         [-]     = 8.00
Element size                                 a           [m]     = 1.00
Number of elements of the beam               N           [-]     = 8

Loads:

Distributed Loads:
--------------------------------------------------------
No.         Load      Load start from      Load end from
           value       the left edge       the left edge
  I           Pf                 Xpl                 Xpr
[-]      [kN/m2]                 [m]                 [m]
--------------------------------------------------------
  1      100.000                0.00                8.00
--------------------------------------------------------

Boring:

Modulus of compressibility                     Es           [kN/m2] = 5000


Results:


Flexibility coefficients of the soil:
-------------------------------------------------
Element      Flexibility coefficients of the soil
    No.
      I                                      Ci,j
    [-]                                 1/[kN/m3]
-------------------------------------------------
      1                                 .2257E-03
      2                                 .6366E-04
      3                                 .3183E-04
      4                                 .2122E-04
      5                                 .1592E-04
      6                                 .1273E-04
      7                                 .1061E-04
      8                                 .9095E-05
-------------------------------------------------
```

```
------------------------------------------------------------
Analysis of a beam foundation
Calculation method: Flexible beam foundation by Modulus of Compressibility
------------------------------------------------------------


Settlements/ Contact pressures/ Moduli of subgrade reactions:
-----------------------------------------------------
Element       Contact      Settlement      Modulus of
    No.      pressure                        subgrade
                                             reaction
      I             q               s              ks
    [-]       [kN/m2]            [cm]         [kN/m3]
-----------------------------------------------------
      1         100.0            3.91            2559
      2         100.0            4.45            2246
      3         100.0            4.67            2143
      4         100.0            4.75            2105
      5         100.0            4.75            2105
      6         100.0            4.67            2143
      7         100.0            4.45            2246
      8         100.0            3.91            2559
-----------------------------------------------------

------------------------------------------------------------
Analysis of a beam foundation
Calculation method: Rigid beam foundation by Modulus of Compressibility after Kany
------------------------------------------------------------


Settlements/ Contact pressures/ Moduli of subgrade reactions:
-----------------------------------------------------
Element       Contact      Settlement      Modulus of
    No.      pressure                        subgrade
                                             reaction
      I             q               s              ks
    [-]       [kN/m2]            [cm]         [kN/m3]
-----------------------------------------------------
      1         125.9            4.39            2865
      2          95.3            4.39            2168
      3          90.4            4.39            2056
      4          88.4            4.39            2012
      5          88.4            4.39            2012
      6          90.4            4.39            2056
      7          95.3            4.39            2168
      8         125.9            4.39            2865
-----------------------------------------------------
```